DYNAMICAL ORIGIN OF SYMMETRY

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Abstract:

It is argued that the majority of broken symmetries observed in elementary particle physics - in particular symmetries connected with strangeness, charm and the higher symmetries SU_3, SU_4 - should not be interpreted as spontaneously broken fundamental symmetries but as structural symmetries which arise from the many-body structure of the particles. Quarks in this interpretation would have only phenomenological significance as 'quasi-particles' in the sense of many-body physics. In the framework of a nonlinear spinor theory of the Nambu-Jona-Lasinio type it is indicated how such structural symmetries can be explicitly obtained as a consequence of a spontaneously broken isospin SU_2 symmetry.

Introduction

I am afraid my contribution will be somewhat out of context with the general line of the Colloquium which emphasizes the mathematical aspects of the group theoretical methods in physics, because I cannot offer anything interesting from the mathematical point of view. On the other hand I consider it very important, as long as this Colloquium is also concerned with physics, that one has a good glance at the phenomenological situation in physics to see what structures evolve and require mathematical formulation. My lecture, therefore, will not be aimed at providing new mathematical tools but in directing attention to problems which appear to me of decisive importance but still await satisfactory solution.

Although not indicated in the title the emphasis of my talk will be on elementary particle physics. The general framework of my arguments will, however, originate from our knowledge of atomic and nuclear physics. Despite of its many unsolved problems elementary particle physics, up to now, has not come up with any phenomenon which, in one form or the other, was not already familiar to us before. In particular particle physics appears void of any inconsistencies of the form which tantalized physics before the advent of quantum mechanics. Hence, one may hope that the problems of particle physics can be solved without similar dramatic changes of well established principles.

My lecture will have essentially two parts:

In the first part I will present some general considerations on the general situation in elementary particle physics. I will start to characterize the similarities and dissimilarities between atomic and nuclear physics on the one hand, and elementary particle physics on the other hand. Then I will concentrate on the important problem of broken symmetries which phenomenologically exhibit themselves in the hierarchy of interactions of various strength and structure. I will offer possible interpretations for this asymmetry, in particular in view of the crucial question, which of these symmetries should be regarded as fundamental. On the basis of empirical and theoretical criteria I will propose a choice which is in contrast to the widely accepted quark model. The majority of the broken symmetries will be interpreted as structural symmetries without fundamental significance. General mechanisms for the occurrence of such structural symmetries in relativistic dynamics will be discussed.

A second part of my lecture will be concerned with an explicit construction of structural symmetries as a consequence of a spontaneous symmetry breakdown. This will be done in the framework of a Nambu-Jona-Lasinio model with a broken chiral symmetry and also with an broken isospin SU_2 symmetry. The possible relevance of the latter results for particle physics will be shortly indicated at the end of the lecture.

1. General Scope

In atomic and nuclear physics the forces between the particles are sufficiently weak that binding energies result which are small compared to the mass of the constituents. Hence the dynamics can be treated essentially in nonrelativistic approximation. This has the important consequence that real and virtual pair creation of particles can be neglected. All (finite) dynamical systems can be described in terms of a certain finite number of constituents (electrons, protons, neutrons, etc.) which interact in more or less complicated ways.

For deciphering the dynamical code one suitably starts by investigating dynamical systems with only a small number of constituents, e.g. the H-atom in atomic physics or the deuteron in nuclear physics, and with this knowledge then proceeds to unravel the dynamics of the more complex systems. With increasing number of constituents, however, one very soon gets into great complications. An explicit mathematical solùtion of such complex systems usually proves impossible, and for practical purposes one has strongly to rely on approximation methods which in many instances are more supported by empirical evidence than mathematical rigour. In all these approximations of dynamical systems with many degrees of freedom the system is subdivided into, at least, two subsystems, where one, containing the majority of the dynamical degrees of freedom, is only treated in some gross fashion, whereas the other one, containing merely very few degrees of freedom, is treated explicitly.

Important examples of this type are the optical electron approximation in atomic physics where the atomic nucleus with all electrons of closed shells are amalgamated into an effective classical potential which is used for explicitly solving the dynamics of the outer electrons. The shell model of the nuclides is another important example which demonstrates that the approximate separation of a few degrees of freedom from the remainder can even be successful for systems with strongly interacting constituents. The main difference to the weak coupling case seems to be that the separated subsystems have less similarity to the actual constituents. Instead of the original constituents certain correlated sets of degrees of freedom, so-called 'quasi-particles', become instrumental for the phenomenological description, e.g. for the description of the observed energy spectrum. Mathematically the quasi-particle description constitutes the problem of finding approximate linearizations of a highly nonlinear dynamics by selecting appropriate dynamical variables. The nature of the most appropriate quasi-particles will, in general, not only depend on the particular dynamics of the system and its structure but also on the particular problem investigated. An approximate solution of high energy problems, for example, will require different linearizations than the calculation of the stationary states, the energy spectrum.

Elementary particle physics seems to differ distinctly from the situation in atomic and nuclear physics: Because the binding energies become comparable to the masses of the particles the relativistic features of the dynamics should become fully effective. As a consequence one would expect it impossible to describe elementary particles in terms of a finite number of constituents. This, however, is not the generally accepted point of view in elementary particle physics.

Nowadays the majority of particle physicists believe that the relativistic features of the dynamics of elementary particles are effectively suppressed at small distances by some kind of mechanism, e.g. 'asymptotic freedom', such that elementary particles - similar to atoms and nuclides - can again be imagined as systems of a finite and, in fact, a very small number of particle-like constituents, the 'quarks'(1). A 'bag model'(2) where three quarks or a quark-antiquark move in a square-well type potential (the 'bag') seems to be rather successful for a qualitative and quantitative description of hadronic fermions and bosons, respectively.

In contrast to this conventional point of view I will take here the position(3) that the relativistic features of the dynamics are fundamentally, i.e. locally, fully effective. Hence the number of the dynamical degrees of freedom of elementary particles is virtually infinite. The concept of a 'constituent' becomes meaningless in this case, a distinction between an elementary and a complex system principally impossible. As a consequence a dynamical description of elementary particles should be viewed more in analogy to non-relativistic many-body systems(4). In particular, one may expect that even in case of virtually infinite many degrees of freedom a dynamical description in terms of a finite number of quasi-particles moving in an effective potential may emerge as a reasonable approximation for the energy spectrum. The constituent quarks then should be identified with such quasi-particles which have no meaning as real particles. The quarks, in this case, would only have phenomenological significance and would only relate in a rather complicated way to the underlying basic dynamical variables.

Symmetry considerations have only played a secondary role at the very start of atomic physics, because the existence of exactly solvable simple dynamical systems, like the Coulomb system or the harmonic oscillator, allowed to explicitly derive the energy spectrum without any knowledge of group theory. In more complex systems, however, group theoretical considerations provided an essential guide for approximations and often remained the only effective tools for extracting physically relevant quantities from the complicated dynamics.

In elementary particle physics there are principally no simple systems which could be used as a starting point for unravelling the basic dynamics. The situation there hence may be compared to the task for extracting the Schrödinger equation merely from the knowledge of the energy spectrum of higher atoms or molecules. With the basic dynamics of particle physics unknown it is not surprising that we have to rely very heavily on group theoretical considerations. But in contrast to non-relativistic many-body systems we are even at the great disadvantage of not knowing off-hand which symmetry transformations are the invariance groups of the unknown basic dynamics. In the following I will mainly deal with this problem, i.e. how, from the particle spectrum and the conservation laws in particle reactions, one may learn something about the basic symmetry structure.

2. Broken symmetries and the hierarchy of interactions

Invariance properties of the dynamics under symmetry transformations lead to conservation laws. Hence the observation of certain conservation laws in processes of elementary particles is a strong indication for the existence of a symmetry of the underlying dynamical law. This, however, is not conclusive, because from particular models we know that selection rules may also originate in other ways. They may, for example, be connected with topological properties(5) or with particular properties of the quantum mechanical state space(6).

The main difficulty, however, in clearly deciding on the symmetries of the basic dynamics of elementary particles relates to the fact that there exists a large number of conservation laws which are not strictly valid but only hold to a certain degree.

These approximate conservation laws give rise phenomenologically to the well-known hierarchy of interactions.

Interaction	Degree of symmetry	Broken symmetry
strong	highest	SU_3, SU_4, ...
electromagnetic	↓	isospin
weak		parity, strangeness, charm
superweak		PC
gravitational	lowest	Poincaré

The complicated broken symmetry structure is also reflected in the mass spectrum of elementary particles.

For hadrons we have the following general situation regarding their symmetry properties baryon number N_F

flavours:
- isospin charge $I_3 \longrightarrow SU_2$ strong isospin $\} \rightarrow SU_3$
- strangeness S $\} \rightarrow SU_4$
- charm C $\} \rightarrow SU_n$
- .
- .
- .

There exist only two strictly conserved quantum numbers baryon or fermion number N_F and electric charge number $Q = \frac{1}{2} N_F + I_3$, usually ascribed to two independent U_1 symmetry transformations. The isospin part I_3 of the charge is incorporated into an approximately valid SU_2 symmetry group defining 'isospin'. By adding the rather well conserved quantity strangeness S this group can be enlarged to a badly broken SU_3, and even, as suggested by recent experiments, by including another well-conserved 'flavour' quantity C, charm, to a very badly broken SU_4[7]. Some physicists[8] suspect that this still does not exhaust the list of possible 'flavours' but that there may be more (e.g. like 'top', 'bottom' ...) which may allow to enlarge the unitary symmetry groups to a very badly broken SU_6 or even higher groups. The higher symmetry groups SU_2, SU_3, SU_4, ... allow to arrange the elementary particles into larger und larger multiplets where the mass splittings of their components reflect the violation of these symmetries.

Among the non-hadrons there are bosons with a rather strange mass spectrum: On the one hand there are the neutral massless particles 'photon' and possibly 'graviton', on the other hand - still hypothetically - some rather heavy charged and neutral vector bosons $W^{\pm}$, Z^{O} with masses probably above 30 GeV. The fermions, the leptons, seem to group into doublets (e, ν_e), (μ, ν_μ), to which, according to new experiments[9] one also has to add the heavy lepton τ at 1.9 GeV which is assumed to have also a neutral partner ν_τ. Besides electric charge Q the leptons carry different charge-like quantum numbers N_e, N_μ, N_τ which seem to be all strictly conserved and are usually interpreted to be a consequence of independent U_1 symmetries. The doublets are thought to reflect a broken 'weak isospin' SU_2 group.

Hence one has the following structure

electron number	N_e	:	U_1
muon number	N_μ	:	U_1
tauon number	N_τ(?)	:	U_1
weak isospin charge	I_3	:	$U_1 \longrightarrow SU_2$ weak isospin

Remarkable and perhaps disturbing is the abundance of different U_1 symmetries the number of which may still increase with the discovery of new 'heavy leptons'.

The general situation regarding the hierarchy of symmetries may be visualized in the way of fig. 1. Nature appears to us phenomenologically like a 'potato',

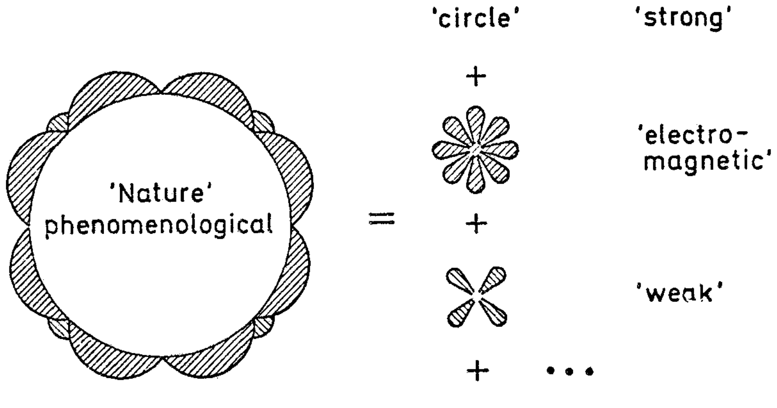


Fig. 1: Illustration of the hierarchy of symmetries.

i.e. like a somewhat distorted sphere or the distorted circle on the left of the figure. The decomposition of this 'potato' into a sum of symmetrical shapes with decreasing symmetry, as schematically indicated on the right, would correspond to our phenomenological description in terms of various interactions of decreasing symmetry and strength. Hence an explanation of the origin of these distortions would at the same time also render an explanation for the hierarchy of interactions. But, why is nature a 'potato' and not a sphere ? What is the origin of this apparent asymmetry ?

3. Possible interpretations of the asymmetry

There are various possible interpretations for the observed asymmetry:

I.) The basic dynamics is asymmetric. In this oldest interpretation the Lagrangian describing the dynamics of elementary particles is conceived of containing a finite sum of interaction terms $\mathcal{L}_{int} = \sum_i g_i \mathcal{L}^{(i)}$ of appropriate forms and decreasing symmetry, the strength of which are characterized by certain dimensionless coupling parameters g_i, as for example the square root of the

Sommerfeld finestructure constant $\alpha = 1/137$ which measures the strength of the electromagnetic interaction and the violation of isospin symmetry.

II.) The basic dynamics is symmetric but the ground state is asymmetric[10,11,12]. This nowadays is called a 'spontaneous breakdown' of the symmetry. It is well-known in many-body physics, for example in case of superconductivity[13] and ferromagnetism[14]. The asymmetry appears here in all physical processes despite of the full symmetry of the dynamics because the 'world', on the background of which these processes are observed, is not a 'vacuum', a 'nothing state' anymore, but is 'warped'.

III.) The basic dynamics is not symmetric, at all. The approximate symmetries are only structural symmetries which arise as a consequence of the many-body nature of the dynamical systems[4].
Such structural symmetries play an important role in atomic and nuclear physics. The symmetry generating mechanism is connected with an approximate dynamical independence of certain subsystems of the total system. If, for example, a system, with a basic dynamics invariant under a symmetry group G, can be approximated by two very loosely coupled subsystems, the dynamics will effectively exhibit the higher symmetry $G \otimes G$, because the symmetry operations can be approximately performed independently on both subsystems.
An important example of this kind in atomic physics is the decoupling of the orbital and spin degrees of freedom leading to an approximate higher symmetry $SO_3 \otimes SU_2$. A more elaborate example for a hierarchy of higher symmetries is given in fig. 2, which schematically indicates the consecutive splittings of the 56-plet energy 'ground state' of the B-atom and is discussed in some detail in ref.[4].

Interpretation I is not only philosophically unsatisfactory and aesthetically ugly but also unattractive from a pragmatic point of view because of the numerical arbitrariness of the dimensionless coupling parameters. Of course, there may be some ultimate explanation for these, for example, on cosmological grounds, or perhaps even a 'Darwinistic' interpretation[15] in the sense that our existence as physicists, who ask this question - i.e. as human beings consisting of extremely complicated organic systems - requires a particular structure of atoms and molecules which drastically limits the admissable numerical range of the fine structure constant.

In the following I will, nevertheless, only consider interpretations II and III, and accept interpretation I, or any other interpretation for the asymmetry which may still exist, only as a last retreat.

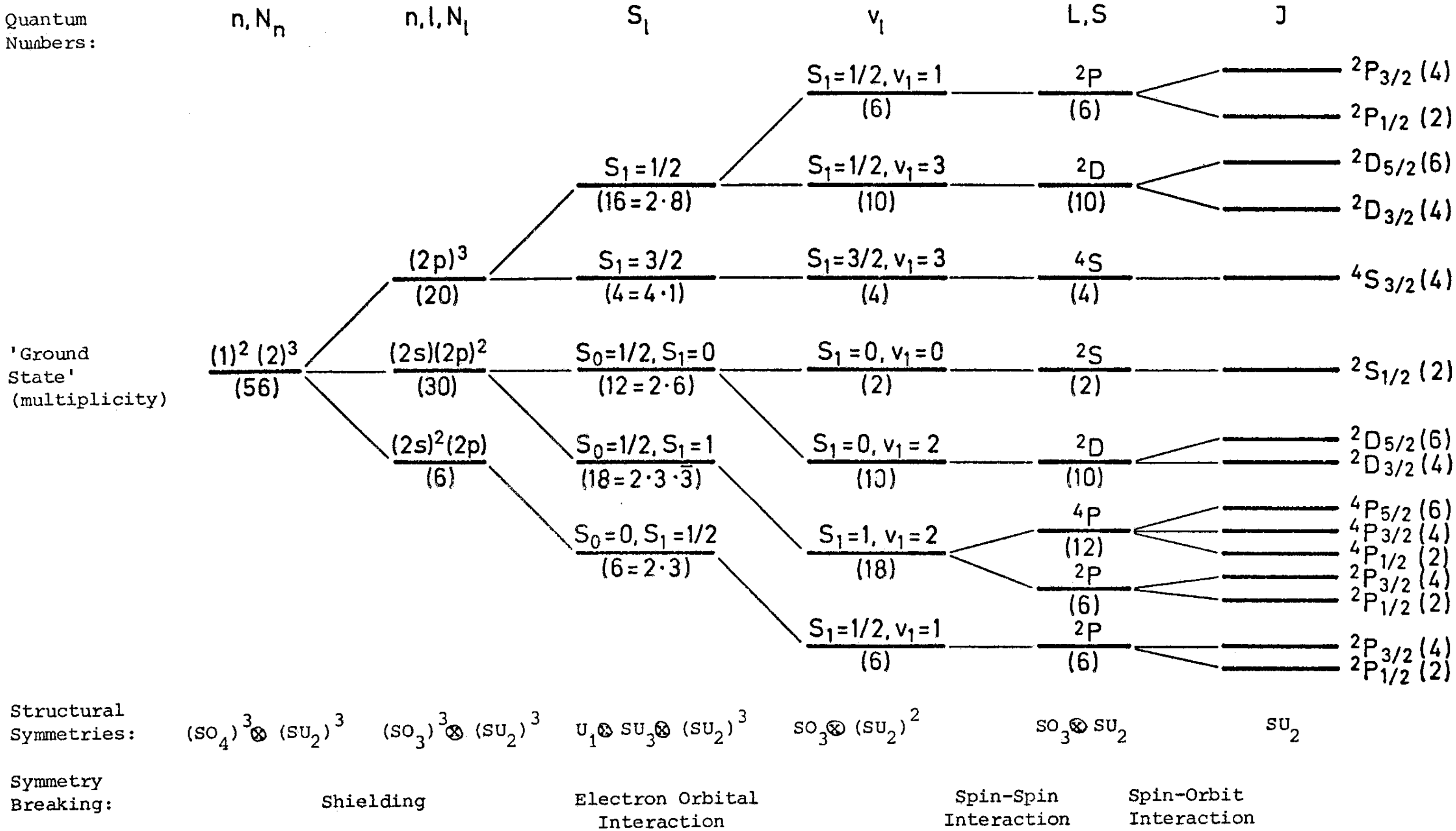


Fig. 2: Spectrum of Boron Atom (schematic).

4. Which symmetries are fundamental ?

For the formulation of a fundamental theory it is important to know which symmetry transformations have to be required as invariance groups of the basic dynamics. This means, in particular, that one has to make up one's mind which of the broken symmetries observed in nature one should interprete as spontaneously broken symmetries - i.e. as symmetries with fundamental significance - and which merely as structural symmetries having no local correspondence.

There are two extreme positions on this question:

Standpoint I:

All broken symmetries are considered spontaneously broken symmetries, i.e. they correspond to exact invariance properties of the local Lagrangian in the sense of a substitution transformation and give rise to exact local conservation laws

(1) $$\partial_\mu j^\mu(x) = 0$$

They cannot, however, be unitarily implemented.

To incorporate all the broken symmetries in a nontrivial fashion in the local Lagrangian a large number of independent fields have to be introduced from the beginning. Hence this point of view leads to a terrible inflation of the number of basic dynamical degrees of freedom.

In the generally accepted quark theory of elementary particles[1,7], a flavour-SU_4, or sometimes even a flavour-SU_6[8] is introduced on the fundamental level which require at least four, respectively six, different Dirac-type quark fields. In the actual case these numbers have to be further increased by introducing additional symmetries connected with 'colour', a quality which is supposed to be strictly conserved and only trivially represented in all observables. In addition 15, respectively 35, vector gauge fields coupled to the flavour currents, colour vector gauge fields (gluons) and a large number of Higgs boson fields have to be introduced by hand to complete these models.

Standpoint II:

Only a bare minimum of broken symmetries are connected with symmetries of the fundamental dynamics. The majority of the broken symmetries is interpreted as structural symmetries as described before. This will be the point of view I will advocate in my lecture[10,16].

How can one empirically select this bare minimum of fundamental symmetries ?

Reactions at very high energy and momentum transfer, i.e. very hard collisions should actually probe the short distance behaviour of interactions. Spontaneously broken symmetries then should eventually appear as perfect symmetries, structural symmetries, on the other hand, completely disappear in this limiting case. To actually make a reliable decision could, however, require experimentally unattainable high energies.

Hence one may prefer to use theoretical arguments to support this decision. From a general theorem, given in the non-relativistic case by Goldstone and in the relativistic case by Goldstone-Salam-Weinberg[(11)] one learns that a spontaneously broken symmetry is always accompanied by certain mass-zero particles (Goldstone particles) if no long range forces are acting[(17)]. Since the empirically known mass-zero particles do not have the required symmetry properties, one has to conclude that spontaneous symmetry breakdown can only be admitted if long range forces or, in the relativistic case, gauge-type interactions[(17)] are admitted at the same time (Higgs mechanism). Hence the existence of a local generalization of a symmetry transformation, a gauge symmetry, can be considered as a reliable indication for a fundamental symmetry. Gauge symmetries can only hold if there exist corresponding local conservation laws (1).

How can one, however, empirically recognize a gauge symmetry ?

From general considerations one learns that in a gauge invariant dynamics

- the corresponding currents occur explicitly in the interaction (charge number gets a dynamical meaning as a coupling strength, a charge)
- the interaction goes via vector fields
- the coupling is universal
- the vector fields have zero mass if the symmetry is not spontaneously broken.

If the symmetries are not internal symmetries, as for example for Poincaré transformation, this has to be phrased somewhat differently.

Empirically one observes a universal neutral vector-type electromagnetic interaction of zero mass, a (up to a Cabbibo angle) universal vector-type charged and neutral weak interaction, which are supposed to be weak because of their extremely short range connected with a large mass of the responsible vector bosons. Strong interactions by first evidence do not appear to arise from vector-type interactions, they are also not universal, because only hadrons and not leptons are engaged. The colour theory of strong interactions does, however, postulate such an interaction in a hidden form and explains their absence for leptons by assuming them colourless.

Using above arguments as guiding principle and by considering only the immediate empirical evidence then only a SU_2, isospin, or a $U_1 \otimes SU_2$ should be admitted as fundamental symmetry which is spontaneously broken except for a U_1 subgroup as required by the strict conservation of electric charge and the mass zero of the photon.

If further refinements connected with space-parity are taken into account this essentially ties on to the unified theory of weak and electromagnetic interactions of Glashow, Weinberg and Salam[18].

As a consequence we propose: All the approximate internal symmetries of hadronic physics, like strangeness, charm, and perhaps other flavours, and all the higher symmetries SU_3, SU_4, ... built upon these flavours but not the isospin SU_2 subgroup shall be regarded as structural symmetries. Also space-parity P, but not PC, shall be counted as a structural symmetry[19].

Is there a realistic chance for such an interpretation ?

At first sight this seems rather unlikely because strangeness, charm and parity, designated as structural symmetries, are conserved to extremely high accuracy - they are only broken by weak interactions - whereas isospin, regarded as fundamental, is violated by the much stronger electromagnetic forces.

One should note, however, that the apparent strength of an interaction does not necessarily reflect its real strength at small distances. Weak interactions do not seem to be really weak but only of very short range as in the Glashow-Weinberg-Salam theory[18] dictated by the large mass of the $W^{\pm}$, Z^0 vector mesons. Strong interactions, on the other hand, may only be strong at intermediate range. In the colour-gauge models of strong interactions such a weakening of the strong interaction is expected as a consequence of 'asymptotic freedom'[20] in the bag-model[2] it is put in by hand by assuming a square-well-type bag leading to free motion of the quarks at small distances. Also in the Heisenberg unified theory[19] the strong interaction is supposed to die away with the mass terms and hence should resemble a superrenormalizable interaction.

If such a situation really holds the gauge-type isospin interactions should eventually overcome the strong interactions at small distances and hence render our interpretation quite natural.

In Heisenberg's unified theory[19] one attempts to derive the dynamics of elementary particles from a nonlinear field equation for a 2x2-component isospinor-Weylspinor field which is only invariant under a global U_1 and a local SU_2 internal symmetry group[21]. This theory would suggest that at small distances only a (parity violating) left-handed, SU_2 Yang-Mills-gauge type interaction with strong coupling survives.

5. Preconditions for structural symmetries

Before dealing with particular examples of structural symmetries let me shortly remark on the general preconditions for structural symmetries.

As stated before structural symmetries are a consequence of approximate dynamical decouplings of certain subsystems of a complex system. This decoupling becomes apparent only if appropriate parametrizations are used. The main art of finding solutions for complex systems consists therefore in making an educated guess for such a parametrization.

In atomic physics the shell structure, which in turn is a consequence of the Pauli principle, facilitates greatly such decouplings because of the insufficient overlap of the wave functions of electrons in different orbitals.

In molecular physics the time constants of various motions (orbital, vibrational, rotational) are very different. This allows adiabatic approximations where by taking appropriate time averages an effective decoupling of the motions is achieved.

In nuclear physics the Pauli principle is again the main reason why a shell model with its effective decouplings constitutes a reasonable approximation despite of the short range and the great strength of the interaction. The complicated dynamical effects of the large number and variety of mesons generated by the nucleons can be essentially incorporated into an effective classical potential which can be characterized by a very few parameters like depth,range and perhaps some multipole moments, and which is roughly independent of the quasi-nucleons moving in it. The quantum aspects of the 'potential' can be taken into account at a second step by including vibrational and rotational excitations.

For elementary particles as relativistic systems with virtually an infinite number of dynamical degrees of freedom one has good reason to believe that similar linearizations should occur, in particular if the dynamics is based on fermions which obey the Pauli principle. To construct stationary solutions for a relativistic dynamical system it is mandatory to search for an appropriate approximation of the infinitely many degrees of freedom. This perhaps can be achieved by taking into account certain classical contributions in the construction of the ground state (e.g. as in case of the spontaneous symmetry breakdown) and also in the one-particle configurations. The 'bags'[(2)] and 'solitons'[(22)] or similar classical localized states may perhaps prove useful as zero-order approximations of hadrons, the Regge spectrum $m^2 \sim j$ indicate a quantized vibrational spectrum as expected at the next step of the approximation procedure.

The approximate separation of a classical part - simulating the many-quantum effects - from the quantum part may be an important source for structural symmetries. Let me expand on this aspect a little more in the remaining part of my lecture.

6. Structural symmetries as a consequence of spontaneous symmetry breakdown

An asymmetry of the physical ground state with regard to a certain symmetry of the dynamical law has several important consequences:

I.) The asymmetry of the ground state reflects a condensation phenomenon which attributes classical components to some local bose field operators.

II.) The globally broken symmetry is locally restored by virtue of mass-zero particles (Goldstone particles) if no long-range forces or gauge-type interactions are present.

III.) The particle spectrum may contain particles with anomalous transformation properties in the sense that these particles do not transform simply as states generated from the ground state by applying products of the basic local field operators[23,24].

Intuitively the latter means that in case of an asymmetric ground state there exists, in principle, a larger variety in 'dressing up' the bare fields to particles because the dressing can utilize different portions of the asymmetric condensate, i.e. the dressed states, the particles, may differ from the bare configurations by properties which are carried by the ground state.

In case of an isospin asymmetric ground state, which will be of main interest to us, the particles may, for example, obtain different isospin properties than the states generated from the basic fields, but preserve their charge if the I_3 subgroup is not violated. In a theory containing only a basic spinor-isospinor field, for example,[29] states with half-integral spin and integral isospin and vice versa can now occur. They are, so-to-say, isospin-wave polarons.

Because different Goldstone dressings lead to different physical particles a single bare field may generate several particle states. They may constitute the hadron multiplets.

To exemplify the possible occurrence of such anomalous states I will shortly indicate their formal construction at first in the familiar chiral model of Nambu and Jona-Lasinio[12], and then in the more interesting case of an isospin-SU_2 generalization of this model. An explicit account of these investigations was recently published by H. Saller and myself[24, 25].

7. Dynamical rearrangement in models of the Nambu-Jona-Lasinio type

1.) The chiral U_1 model[24]:

The chiral model of Nambu and Jona-Lasinio[12] is based on a chiral (Touschek-U_1) invariant nonlinear Lagrangian

(2)
$$\mathcal{L} = \frac{i}{2}\bar{\psi}\gamma^{\mu}\overleftrightarrow{\partial}_{\mu}\psi - \frac{g}{2M^2}(\bar{\psi}\overset{i}{\Gamma}\psi)(\bar{\psi}\overset{i}{\Gamma}\psi)$$

with $\overset{i}{\Gamma}\otimes\overset{i}{\Gamma}$ some appropriate sum of Dirac matrices, g a dimensionless coupling parameter (characterizing the strength of the scalar coupling), M some mass scale.

If the chiral Touschek-U_1 transformation

(3) $$\Psi(x) \longmapsto e^{-\frac{i}{2}\gamma_5\alpha}\,\Psi(x)$$

is spontaneously broken as exhibited by the nonvanishing ground state expectation value

(4) $$\langle\Omega|\left(\bar{\Psi}(x)\Psi(x)\right)|\Omega\rangle = m\frac{\Lambda^2}{2g} \neq 0$$

the basic field $\Psi(x)$ which characterizes the local interaction and the local propagation structure is not adequate for a description of the asymptotic region and hence for the description of the physical particles. Near the mass shell of these particles it is more appropriate to use a decomposition of the field into its chiral-active and its chiral-frozen components

(5) $$\Psi(x) = e^{-\frac{i}{2}\gamma_5\varphi(x)}\,\psi(x) = \mathbb{S}(x)\,\psi(x)$$

which is analogous to the decomposition of the scalar non-hermitian Higgs field $\phi(x)$ in the Higgs model into its azimuthal and radial components. Under the chiral transformation(3) the 'azimuthal' field $\varphi(x)$ shall transform like

(6) $$\varphi(x) \longmapsto \varphi(x) + \alpha$$

whereas the 'radial' field is assumed to be frozen

(7) $$\psi(x) \longmapsto \psi(x)$$

The $\varphi(x)$ then acquires the meaning of the Goldstone field which generates massless particles from the ground state. The

(8) $$\mathbb{S}(x) = e^{-\frac{i}{2}\gamma_5\varphi(x)} = \begin{pmatrix} S(x) & 0 \\ 0 & S^*(x) \end{pmatrix}$$

is the chiral dressing operator which is constructed from the chiral 'spurions'

(9) $$S(x) = e^{-\frac{i}{2}\varphi(x)}$$

The asymmetry condition (4) can be decomposed into a chiral symmetric radial condition

(10) $$\langle\Omega|\left(\bar{\psi}(x)\psi(x)\right)|\Omega\rangle = m\frac{\Lambda^2}{2g} \neq 0$$

and the symmetry breaking azimuthal condition

(11) $$\langle \Omega | \, s^2(x) \, | \Omega \rangle = \langle \Omega | \, e^{-i\varphi(x)} \, | \Omega \rangle = 1$$

which essentially expresses the absence of any Goldstone modes in the groundstate $\langle \Omega | \varphi(x) | \Omega \rangle = 0$.

For a phenomenological description it is now profitable to rearrange the Lagrangian by extracting all dependence on $\varphi(x)$ and $\partial_\mu \varphi(x)$ from the interaction term using the chiral invairant decomposition[26]

(12) $$\bar{\psi}_\alpha(x) \psi_\beta(x) = m \frac{M^2}{8g} s^2_{\beta\alpha}(x) + a (\gamma_5 \gamma^\mu)_{\beta\alpha} \partial_\mu \varphi(x) + :\bar{\Psi}_\alpha \Psi_\beta:(x)$$

(a = const.). Instead of simply using $\psi(x)$ and $\varphi(x)$ as new field variables one shows[26] that the Lagrangian can be optimally linearized by only partially freezing the chiral degrees of freedom in the propagation of the spinor field. The Lagrangian can be effectively written as

(13) $$\mathcal{L} = \alpha_0 \frac{i}{2} \bar{\psi} \gamma^\mu \overleftrightarrow{\partial}_\mu \psi + \alpha_1 \left[\frac{i}{2} \bar{\Psi} \gamma^\mu \overleftrightarrow{\partial}_\mu \Psi - m' \bar{\Psi} \Psi \right] + A (\partial_\mu \varphi)(\partial^\mu \varphi) +$$

+ residual interaction term

where the residual interaction term now does not contain any φ-coupling anymore (at least in lowest order). The α_0, α_1, are certain constants depending on a in (12) and the coupling parameters in (2). A condition

(14) $$\alpha_0 + \alpha_1 = 1$$

secures that there is no double counting.

For deriving (13) only the chiral symmetric condition (10) but not the asymmetry condition (11) was used. The asymmetry, therefore, can only enter via φ-dependence in higher orders of the residual interaction in (13). The bilinear part of (13) constitutes the phenomenological free Lagrangian which is chiral invariant. It contains effectively two quasi-particles, one massless connected with the chiral-active ψ, and one massive connected with the chiral frozen Ψ. The ψ and the Ψ, however, are only in this approximation dynamically independent fields.

2.) The isospin SU_2 model[25]:

One considers a nonlinear spinor Lagrangian of the form

(15) $$\mathcal{L} = \frac{i}{2} \bar{\psi} \gamma^\mu \overleftrightarrow{\partial}_\mu \psi - M_0 \bar{\psi} \psi - \frac{g}{2M^2} (\bar{\psi} \Gamma^i \tau \psi)(\bar{\psi} \Gamma^i \tau \psi)$$

for an isospinor-spinor field ψ , which is invariant under the SU_2 isospin transformation

(16) $$\vec{I}: \quad \psi(x) \longmapsto e^{-\frac{i}{2}\vec{\beta}\cdot\vec{\tau}}\psi(x)$$

As usual a charge may be defined as

(17) $$Q = \tfrac{1}{2}N_F + I_3$$

with N_F a fermion number connected with an invariance of (15) under phase transformations, which leads to the charge assignment $\psi = (\psi^+, \psi^0)$.

The ground state $|\Omega\rangle$ is assumed to distinguish a direction in isospace (identified with the 3rd axis) as expressed by the asymmetry condition

(18) $$\langle\Omega|\, \bar{\psi}(x)\vec{\tau}\psi(x)\,|\Omega\rangle = c\vec{\delta}_3 \neq 0$$

In combination with an isosymmetric condition

(19) $$\langle\Omega|\left(\bar{\psi}(x)\,\psi(x)\right)|\Omega\rangle = d$$

this establishes a double condensate of $(\bar{\psi}^-\psi^+)$ and $(\bar{\psi}^0\psi^0)$ scalar pairs in the ground state.

For an appropriate description of the asymptotic region we use the decomposition

(20) $$\psi_\alpha(x) = S_\alpha^{\,i}(x)\,\psi_i(x) \qquad (i = 1,2)$$

with $S_\alpha^{\,i}(x) = S_\alpha^{\,i}(\Theta_1(x), \Theta_2(x))$ an isospin dressing operator which is a given functional of the two Goldstone fields $\Theta_1(x)$, $\Theta_2(x)$ connected with the spontaneous breakdown (18) of the isospin rotations around the first and second axis. The $S_\alpha^{\,i}(x)$ has essentially the form of a rotation matrix for rotations around axis' perpendicular to the 3rd axis, the first component of which - we call it isospin spurion - has the form

(21) $$S_\alpha(x) = \begin{bmatrix} \cos\sqrt{\Theta_+\Theta_-} \\ \dfrac{\sin\sqrt{\Theta_+\Theta_-}}{\sqrt{\Theta_+\Theta_-}}\,\Theta_- \end{bmatrix}_\alpha \equiv S_\alpha^{\,1}$$

with $\Theta_\pm(x)$ roughly the two charged Goldstone modes $\frac{1}{2}(\Theta_2 \pm i\,\Theta_1)$. The second component of the dressing operator is the antispurion $\tilde{S}_\alpha$ (G-conjugate)

(22) $$S_\alpha^{\,2} = \tilde{S}_\alpha = (-i\tau_2)_{\alpha\beta}\, S_\beta^*$$

There are two frozen fields ψ_i (i = 1,2) in this non-Abelian case. Their contribution is determined by the two conditions (18,19) which can be decomposed into the two isospin symmetrical conditions

$$\begin{aligned} &\langle\Omega|\bar{\psi}\lambda_3\psi|\Omega\rangle = c \qquad \lambda_3 \hat{=} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}_{ij} \\ &\langle\Omega|\bar{\psi}\psi|\Omega\rangle = d \end{aligned} \tag{23}$$

and the single isospin asymmetry condition

$$\langle\Omega|s^{*}\vec{\tau}s|\Omega\rangle = \vec{\delta}_3 \tag{24}$$

which again essentially expresses the absence of the Goldstone modes in the ground state $\langle\Omega|\Theta_{1,2}(x)|\Omega\rangle = 0$.

Considerations analogous to the chiral case show[26] that the theory can be optimally linearized by only partially freezing the isospin in the propagation of the spinor field, i.e. by using the extended set of fields consisting of the isospinor field ψ with mass M_o and the two frozen fields ψ_i with mass $m_i = M_o + m_o \pm \mu$ where m_o and μ are related to the constants d and c in (23), respectively.

The linearized theory can be shown to be invariant under a modified isospin transformation (T: strong isospin transformation) and a new U_1 transformation connected with a 'spurion number' N_s. The occurrence of this new (structural) symmetry is related to the fact that a U_1 subgroup of the isospin SU_2 group remains unbroken and that this subgroup in this approximation can be independently applied to the ψ fields and the spurions, similar as, for example, in atomic physics in absence of spin-orbit forces the rotations can be independently applied to the orbital motions and the spins. In particular one establishes the relationship

$$I_3 = \tfrac{1}{2}N_s + T_3 \tag{25}$$

between the exactly conserved third component of isospin I_3 and the third component of strong isospin T_3 and spurion number N_s, which both are only conserved in this approximation. In our atomic physics example this should be compared with a relationship like $j_3 = l_3 + s_3$.

There are particular nonlinear interactions, as for example in case of Heisenberg's nonlinear spinor theory[19], where not only the spurion number N_s but the number of spurions N_1 and antispurions N_2 are independently conserved. In this case one obtains even a $U_1 \otimes U_1 \otimes SU_2$ as (approximate) symmetry, in addition to the U_1 fermion number transformation. Only a U_1 subgroup of the $U_1 \otimes U_1 \otimes SU_2$ will eventually survive and will be unitarily implementable.

8. Possible relevance of isospin anomalous states for elementary particle physics

The discussion of the isospin-invariant model with an isospin asymmetric ground state showed that the phenomenological Lagrangian contains effectively four local fermi fields consisting of an isodoublet $\psi_{\alpha}(x)$ (under $\vec{I}$ and $\vec{T}$) and two isosinglets (under $\vec{T}$) $\psi_1(x) = s^{*\alpha}\psi_{\alpha}$ and $\psi_2(x) = \tilde{s}^{\alpha}\psi_{\alpha}$, giving rise to four quasi-particles $n = (n^+, n^o)$; λ^+, λ^o (the superscript indicating the charge) with masses M_o; $M_o+m_+ + \mu$, $M_o+m_o-\mu$, respectively.

This strongly suggests[25] to compare these quasi-particles with the four quarks q = (u, d; c, s) of the conventional flavour SU_4 quark model with (n^+, n^o) simulating (u,d) and the isospin-frozen states (λ^+, λ^o) the charmed and strange quarks (c,d). The additonal structural spurion number U_1 symmetry should be identified as

(26) $$N_s = N_1 - N_2 = C + S$$

with S = strangeness and C = charm. This establishes for the charge

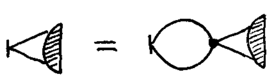
(27) $$Q = \frac{1}{2}N_F + I_3 = \frac{1}{2}(N_F + S + C) + T_3$$

which for C = O agrēes with the old Gell-Mann-Nishijima charge formula.

The comparison also shows that the spurion $s_{\alpha}(x)$ essentially plays the role of the Higgs field $\phi_{\alpha}(x)$ in conventional theories, as e.g. in the Weinberg model[18], but it would not be an independent field in our case.

There is an important difference to the conventional quarks: Our quasi-particles carry integral charges and hence resemble more the constituent particles of the old Sakata theory[27]. This may seem to be a serious difficulty for interpreting the physical fermions.

Let us first consider mass eigenvalue equations for physical bosons. In lowest approximation bosons can be generated by solving the Bethe-Salpeter equation

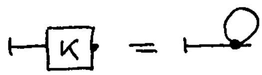


which for every spin-parity assignment J^P ($J \leq 1$, $P = \pm 1$) with the four quasi-particles generates 16 different isospin-strangeness-charm configurations. They appear completely analogous to the 15+1 different $\bar{q}q$ bosons of the SU_4 quark theory. This, of course, is not surprising because the Sakata-nature of our 'quarks' does not matter in these configurations.

Very critical in this respect should be the fermion spectrum because it was their octet structure which suggested the 'eight-fold-way'[28] and ruled out Sakata's original scheme. For fermions one has to solve Bethe-Salpeter equations of the form

where the integral kernel is in

lowest order:



next order:

Considering the lowest order one just establishes the 4 different configurations (n^+, n^o; λ^+, λ^o). But by taking into account the next order of the local selfinteraction with three lines one can demonstrate that now 20 different isospin-strangeness-charm configurations can occur which are analogous to the 20-plet of fermions in the SU_4 quark theory. The 20 different configurations in our case can be subdivided into the 4 quasi-particles which already occur at the first step of the approximation, and 16 new configurations which very likely group into a low-mass octet O with zero central charge, and a high-mass octet O_c with a positive central charge. The octet O carries the quantum numbers of the familiar baryon octet (N, Λ, Σ, Ξ) and the O_c would form a charmed octet.

Hence, despite of the Sakata structure of our quasi-particles fermion octets may result. These octets, however, do not have the qqq-structure of the quark theory but have rather the form

$$O \sim \lambda^o(\bar{\hat{q}}\hat{q}) \qquad \text{and} \qquad O_c \sim \lambda^+(\bar{\hat{q}}\hat{q})$$

with $\hat{q}$ = (n^+, n^o; λ^o) the original Sakata SU_3 triplet. They, therefore, look more like frozen-fermion - boson-octet configurations. Because maximally only two like fermions occur, 'colour' is not necessary for admitting fully symmetric space configurations. 'Colour' would also not be necessary for confining the 'quarks' because the (n^+, n^o; λ^+, λ^o) quantum-number configurations can actually occur as physical particles.

In the framework of our theory SU_3 and SU_4 would never occur as exact symmetries because the masses of the four quasi-particles are, in general, all different. These higher symmetries would formally become exact in the local limit (high energy and high momentum transfer) where mass effects become insignificant. But in this limit we are not allowed anymore to treat the quasi-fields ψ_α and ψ_c as independent fields. Hence the formally established SU_4 for sufficiently high energy and momentum transfer actually collapses into the simple SU_2 of the basic theory. Phenomenologically this would occur for energies where weak and electromagnetic interactions overcome strong interactions.

Because my time is up I cannot go into any more details. They are given elsewhere[(24-26)]. I should stress, however, that we are still far away from a solid theory of elementary particles. There are a great number of unsolved problems of a theoretical as well as a phenomenological nature which still have to be investigated. It is, in particular, clear that our primitive models have to be generalized to include isospin gauge fields either of genuine nature as in the Weinberg model or as 'compounds' as in the Heisenberg theory[(21)]. Only in such models - which are investigated at present - problems like the dynamical stability of the hadron multiplets, the absence of neutral strangeness changing interactions with gauge fields, a dynamical interpretation of the Cabbibo and Weinberg angle become accessible.

In closing let me again emphasize that I am personally quite convinced that hadron physics (perhaps contrary to lepton physics) will not be simply related to the basic dynamics but results, in one way or the other, as some 'many-body' approximation of this dynamics. There should, therefore, exist no principal limitation for introducing higher symmetries for the classification of the spectrum and the interaction processes of hadrons. These groups, however, have only practical significance, they are not invariance groups of the dynamics and hence not really interesting from a fundamental point of view. Still we probably cannot afford to stay away from them because we have to use this complicated phenomenology to extract or to guess the fundamental dynamics and then, by developping appropriate approximations, try to return to the phenomenological level. In going ahead with such a program, I think, one should be courageous and not let oneself be immediately discouraged by some alleged contradictions to 'empirical evidence', because much of this evidence is strongly model dependent and some of it even merely wishful thinking. One should be fully aware that the problems in elementary particle physics are very hard and our imagination all too often biased and limited.

References

(1) For a popular review see e.g. S.L. Glashow: Scientific American 233, 38 (1975).

(2) K. Johnson: Phys.Rev., D6, 1101 (1972);
A. Casher, J. Kogut, and L. Susskind: Phys.Rev., D10, 732 (1974);
J. Kuti: Proceedings of the European Conference on Particle Physics, Budapest, July 1977.

(3) W. Heisenberg: The Physicist's Conception of Nature (ed. by J. Mehra), D. Reidel Publ.Co. 1973, p. 264; Physics Today, March 1976;
H.P. Dürr: Ist ein Elementarteilchen elementar ? Nova Acta Leopoldina, Symposium 1975, to be published.

(4) H.P. Dürr: Properties of matter under unusual conditions. (E. Teller Festschrift). Ed. by H. Mark and S. Fernbach, p. 301. New York: Wiley & Sons 1969.

(5) D. Finkelstein: J. Math.Phys., 7, 1218 (1966);
G. 't Hooft: Nuclear Physics, B79, 276 (1974);
A.M. Polyakov: JETP. Lett., 20, 194 (1974);
S. Coleman: Erice Lectures 1975 to be published;
L. O'Raifeartaigh: Present Colloquium.

(6) H.P. Dürr, W. Heisenberg, H. Yamamoto, and K. Yamazaki: Nuovo Cimento, 38, 1220 (1965).
H.P. Dürr: Proceedings of International Symposium on Mathematical Physics, Warsaw, March 1974, p. 135.
H.P. Dürr: Nuovo Cimento, 27A, 305 (1975).

(7) S.L. Glashow, J. Iliopoulos, and L. Maiani: Phys.Rev., D2, 1285 (1970).

(8) H. Harari: Phys.Lett., 57B, 265 (1975);
Annals of Physics, 94, 391 (1975).

(9) L. Lederman: Proceedings of the European Conference on Particle Physics, Budapest, July 1977.

(10) H.P. Dürr, W. Heisenberg, H. Mitter, S. Schlieder, and K. Yamazaki: Z.Naturforschung, 14a, 441 (1959).

(11) J. Goldstone: Nuovo Cimento, 19, 154 (1961);
J. Goldstone, A. Salam, and S. Weinberg: Phys.Rev., 127, 965 (1962).

(12) Y. Nambu, and G. Jona-Lasinio: Phys.Rev., 122, 345 (1961); 124, 246 (1961).

(13) J. Bardeen, L.N. Cooper, and J.R. Schrieffer: Phys.Rev., 108, 1175 (1957);
A. Abrikosov, L. Gorkov, and I. Dzyaloshinski: Methods of Quantum Field Theory in Statistical Physics, New York, N.Y., 1965.

(14) H. Wagner: Z. Physik, 195, 273 (1966).

(15) J. Wheeler: Private communication 1969.

(16) H.P. Dürr: Quanten und Felder (Heisenberg Festschrift), Vieweg 1971, p. 125.

(17) P.W. Anderson: Phys.Rev., 130, 439 (1963);
P.W. Higgs: Physics Letters, 12, 132 (1964); Phys.Rev., 145, 1156 (1966);
G.S. Guralnik, C.R. Hagen, and T.W. Kibble: Phys.Rev. Lett., 13, 585 (1964).

(18) S.A. Bludman: Nuovo Cimento, 9, 433 (1958);
S.L. Glashow: Nuclear Physics, 22, 579 (1961);
A.Salam and J. Ward: Phys.Lett., 13, 168 (1964);
S. Weinberg: Phys.Rev.Lett., 19, 1264 (1967).

(19) For a discussion of the work before 1966 see: W. Heisenberg: Introduction to the Unified Field Theory of Elementary Particles; Interscience Publ., London 1966.

(20) K. Symanzik: Comm.Math.Phys., 18, 227 (1970); 23, 49 (1971);
C.G. Callan, and K. Symanzik: Phys.Rev., D2, 1541 (1970);
D.J. Gross, and F. Wilczek: Phys.Rev.Lett., 30, 1343 (1973);
H.D. Politzer: Phys.Rev.Lett., 30, 1346 (1973);
S. Coleman, and D.J. Gross: Phys.Rev.Lett., 31, 851 (1973).

(21) H.P. Dürr, and N.J. Winter: Nuovo Cimento, 70A, 467 (1970); 7A, 461 (1972);
R. Budini, and P. Furlan: Nuovo Cimento, 30A, 63 (1975);
K. Akama, and H. Terazawa: Institute for Nuclear Study Report, Tokyo; INS-257, 1976 (unpublished);
H. Terazawa, Y. Chikashige, and K. Akama: Progr.Theor.Phys, 56, 1935 (1976); Phys.Rev., D15, 480 (1977).

(22) A.C. Scott, F.Y.F. Chu, and D.W. McLaughlin: Proc.IEEE., 61, 1443 (1973);
A. Barone, F. Esposito, C.J. Magee, and A.C. Scott: Riv.Nuovo Cimento 1, 227 (1971);
L.D. Faddeev, and L.A. Takhtajau, Dubna Report 1973;
R. Rajaraman: Phys.Reports, 21C, 227 (1975);
R. Jackiw: Rev.Mod.Physics, 49, 681 (1977).

(23) H.P. Dürr, and W. Heisenberg: Z.Naturforschung 16a, 327 (1961);
Nuovo Cimento 37, 1446, 1487 (1965);
H.P. Dürr, and J. Gêhêniau: Nuovo Cimento 28, 132 (1963);

H. Biritz: Nuovo Cimento, 47a, 581 (1966);
H. Saller: Nuovo Cimento: 30A, 541 (1975); 34A, 99 (1976).

(24) H.P. Dürr, and H. Saller: Nuovo Cimento, 39A, 31 (1977).

(35) H.P. Dürr, and H. Saller: Nuovo Cimento to be published (1977).

(26) H.P. Dürr, and H. Saller: In preparation, to be submitted to Nuovo Cimento.

(27) S. Sakata: Progr.Theor.Phys., 16, 686 (1956).

(28) M. Gell-Mann. Cal-Tech. Report CTSL-20 (1961); Phys.Rev. 125, 1067 (1962);
Y. Ne'eman: Nuclear Physics, 26, 222 (1961);
M. Gell-Mann, and Y. Ne'eman: The eightfold way, W.A. Benjamin, New York 1964.