SUPERGRAVITY IN THE PHYSICS OF PARTICLES AND FIELDS

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ABSTRACT

Supergravity, the gauge theory of supersymmetry, is reviewed. The main aspects of its Lagrangian formulation and the prospects for a unified theory of fundamental interactions are discussed.

1. INTRODUCTION

Supergravity is a new gauge theory whose gauge principle is a fermionic local invariance[1]. It is the theory of local supersymmetry[2] and therefore supergravity actions are invariant with respect to transformations of the fields which involve a set of arbitrary x dependent self-conjugate spinors $\varepsilon^{\alpha,i}(x)$ $(\alpha = 1 \ldots 4,$ $i = 1 \ldots N)$.

This gauge principle necessarily requires the introduction of new gauge fields $\psi_\mu^{\alpha,i}(x)$ of Rarita-Schwinger type which describe massless quanta of helicity $\pm 3/2$. We will refer to these quanta with the name of gravitinos.

For the first time it is possible to have gauge fields which obey Fermi statistics and therefore a more intimate connection between fermions and the geometry of space-time.

For reasons that will be explained later, supergravity theories with more than one gravitino necessarily unify gravitation with other particle interactions. There are at least two achievements in the framework of local supersymmetry. First it provides the first known examples of finite quantum corrections of elementary particle interactions due to graviton exchanges. Secondly, it is an example of a grand unification scheme in which the graviton, gauge vector bosons, quark and/or leptons and Higgs mesons sit in the same irreducible representation of the underlying symmetry group.

From a more fundamental point of view, supergravity theories arise from the gauging of a Graded Lie Algebra (GLA). This new algebraic structure, which may be regarded as an extension of an ordinary Lie algebra contains (even) Bose-type generators which constitute its Lie algebra part as well as (odd) Fermi-type generators. Even generators obey the commutation relations of an ordinary Lie algebra while odd generators obey anticommutation relations.

GLA's have been extensively studied in the literature and a complete classification of simple GLA's has been given[3]. This classification is analogous to the corresponding Cartan classification of simple Lie algebras.

Because of the connection between spin and statistics, GLA's are of interest in physics whenever their Lie algebra part contains the space-time symmetry group and the odd generators transform as spinors with respect to the homogeneous Lorentz group. GLA's containing the space-time symmetry group are usually called supersymmetries.

There are several types of supersymmetries according to the chosen underlying space-time symmetry structure[4]. Moreover, there is the additional freedom of the internal symmetry group with respect to which the odd spinorial generators can transform.

Lagrangian field theories for global supersymmetry have been investigated for GLA's containing the Poincaré group or the conformal group as space-time symmetry[2].

Supergravity theories, i.e., locally supersymmetric Lagrangians have been constructed in the case of a gauged Poincaré and de Sitter graded algebra[1] and partial results for the gauged conformal graded algebra are also known[5].

Supersymmetries with more than one spinorial generator are called extended supersymmetries.

Extended supergravity theories, i.e., the gauge theories for extended supersymmetry algebras, seem the most exciting development of local supersymmetry. In these theories the graviton and the gravitinos join the fundamental particles of conventional gauge theories in a single irreducible representation of extended supersymmetry.

This review is organized as follows: In Section 2 we give a brief summary on graded Lie algebras and their relevant representations. In Section 3 we review globally supersymmetric Lagrangians and their invariance properties. In Section 4 we consider the basic supergravity Lagrangian, its construction and its relation to the Cartan theory of gravitation. In Section 5 we will be faced with the problem of matter coupling in supergravity, i.e., the promotion to local invariance of globally supersymmetric theories. Section 6 will be devoted to extended supergravity theories and to the problem of gauging a non-Abelian internal symmetry.

2. GRADED LIE ALGEBRAS

Graded Lie algebras (GLA) are generalizations of ordinary Lie algebras. They unify commutation and anticommutation relations in the same algebraic structure. We will consider only Z_2 graded Lie algebras although more refined definitions of grading are possible.

Let us consider a linear D+d dimensional vector space in which the following commutation relations hold:

$$
\begin{aligned}
[A_m, A_n] &= C^{p}_{nm} A_p \qquad\qquad m = 1, \ldots D \\
& \qquad\qquad\qquad\qquad\quad \alpha = 1, \ldots d \\
[Q_\alpha, A_m] &= T^{\beta}_{m\alpha} Q_\beta \\
\{Q_\alpha, Q_\beta\} &= \Gamma^{p}_{\alpha\beta} A_p
\end{aligned}
\tag{2.1}
$$

The even generators A_m form a D dimensional Lie algebra. The odd generators Q_α form a d dimensional representation of the Lie algebra. They satisfy the following Jacobi identities

$$
\begin{aligned}
&[A_m, \{Q_\alpha, Q_\beta\}] + \{[Q_\alpha, A_m], Q_\beta\} + \{[Q_\beta, A_m], Q_\alpha\} = 0 \\
&[Q_\alpha, \{Q_\beta, Q_\gamma\}] + [Q_\gamma, \{Q_\alpha, Q_\beta\}] + [Q_\beta, \{Q_\gamma, Q_\alpha\}] = 0
\end{aligned}
\tag{2.2}
$$

The set of odd generators Q_α are called the grading representation of the Lie algebra spanned by the A_m. There are two additional sets of Jacobi identities involving a triple of A_m and a Q_α and two A_m which are trivial, being simple consequences of the Lie algebra part of the graded Lie algebra. The set of matrices $(C^{P}_{mn}, T^{\beta}_{m}, \Gamma^{p}_{\alpha\beta})$ are the structure constants of the D+d dimensional graded Lie algebra.

The previous Jacobi identities imply quadratic algebraic relations for the structure constants entirely analogous to the algebraic relations satisfied by the structure constants of an ordinary Lie algebra. Using anticommuting parameters which are elements of a graded manifold, a graded Lie algebra can be exponentiated giving rise to a graded Lie group. A theory of graded Lie groups and of their representations has been formulated in a consistent mathematical framework in Ref. 3).

Because of the connection between spin and statistics graded Lie algebras are interesting in physics whenever their Lie algebra part contains the space-time symmetry group and the odd generators belong to a spinor representation of the homogeneous Lorentz group. We will consider first graded versions of the Poincaré algebra IO(3,1). Next we will consider the grading of the conformal and de Sitter algebras respectively O(4,2) and O(3,2) which are also interesting space-time symmetry groups.

Let us consider a set of N spinor charges Q^i_α $\alpha = 1 \ldots 4$, $i = 1 \ldots N$ which transform according to the fundamental self-conjugate representation of the Lorentz group:

$$i\,[M_{\mu\nu}, Q^i_\alpha] = \sigma_{\mu\nu\,\alpha}{}^{\beta}\, Q^i_\beta \tag{2.3}$$

These charges, for any N, are the odd elements of a graded Poincaré algebra. The even generators are the Poincaré generators themselves and possible central charges Z^{ij}, Z'^{ij}.

The relevant commutation relations are[6)]

$$\begin{aligned}
&[P_\mu, Q^i_\alpha] = 0\\
&\{Q^i_\alpha, \bar{Q}^j_\beta\} = -2\gamma^\mu_{\alpha\beta} P_\mu \delta^{ij} + \delta_{\alpha\beta} Z^{ij} + \gamma_{5\alpha\beta} Z'^{ij}\\
&[Z^{ij}, Q^\ell_\alpha] = [Z'^{ij}, Q^\ell_\alpha] = 0\\
&[Z^{ij}, Z^{\ell m}] = [Z'^{ij}, Z'^{\ell m}] = [Z^{ij}, Z'^{\ell m}] = 0
\end{aligned} \tag{2.4}$$

The odd generators are self-conjugate (Majorana) spinors $Q_\alpha = Q^*_\alpha$ and the matrices Z^{ij}, Z'^{ij} real and antisymmetric. In (2.4) we have used a Majorana (real) representation for the matrices with metric (-+++). Moreover $\gamma_5 = \gamma_0\gamma_1\gamma_2\gamma_3$, $\sigma_{\mu\nu} = \frac{1}{4}[\gamma_\mu, \gamma_\nu]$, $\bar{Q} = Q\gamma^0$.

The algebra given by (2.4) is called extended supersymmetry for $N > 1$ and simple supersymmetry for $N = 1$. In the latter case $Z = Z' = 0$ and we recover the Wess-Zumino original supersymmetry algebra.

All irreducible representations of the graded Lie algebra given by Eq. (2.4) acting on one-particle states of given momentum have been classified[2)4)7)]. They always decompose into a finite direct sum of irreducible Poincaré representations all degenerate in mass and with spin shifted by one-half unity. Among them, particularly interesting are zero mass representations[7)]. In fact they are suitable for describing unified gauge theories in which gauge particles of different spin, like the graviton and the photon sit in the same irreducible representation of the graded Poincaré algebra. The multiplicity and helicity content of the one-particle states of a massless irreducible multiplet can easily be obtained by means of the Wigner method of induced representations. Starting from a singlet state of helicity $|\lambda|$ and successively applying those spinorial charges Q_α^i $(i = 1, \ldots, N)$ which act as $\frac{1}{2}$ unit helicity lowering operators one gets states of helicities $|\lambda|-\frac{1}{2}$ ... up to $|\lambda|-N/2$. The multiplicity of a state of helicity $|\lambda|-K/2$ is $N!/K!(N-K)!$ To get acceptable representations one must add the CPT conjugate states of reversed helicities. These are obtained by starting with a singlet state of helicity $-|\lambda|$ and applying those spinorial charges Q_β^i $(i = 1 \ldots N)$ which act as helicity raising operators. If $-|\lambda| = |\lambda|-N/2$ i.e., $|\lambda| = N/4$, the representation is self-conjugate.

In the construction of gauge local quantum field theories one is interested in those multiplets which correspond to gauge fields and matter fields of fundamental particle interactions. This limits to considering representations with $|\lambda|_{max} = 2$. This is because the highest spin particle consistent with a gauge principle and with a corresponding local Lagrangian formulation is known to be the graviton. Needless to say the corresponding gauge group is the group of general co-ordinate transformations of general relativity.

With these restrictions it is easy to make a list of all possible physically meaningful representations of global extended supersymmetry. Let us call, by convention, scalar, vector, spinor and gauge multiplets those multiplets having $|\lambda_{max}| = \frac{1}{2}$, 1, 3/2, and 2, respectively. Then the scalar multiplet, and therefore an explicit mass term, exists up to $N = 2$. The vector multiplet, which can gauge an internal symmetry commuting with the spinor charges, exists up to $N = 4$. The spinor multiplet up to $N = 6$ and finally the gauge multiplet and therefore supergravity up to $N = 8$. Because of the anticommuting properties of the spinor charges, particle states of helicity $|\lambda_{max}| - K/2$ transform according to the K rank antisymmetric tensor representation of the $SO(N)$ group.

So far, we have considered the graded version of the Poincaré algebra. Gradings of the $O(4,2)$ conformal and $O(3,2)$ de Sitter algebras exist as well. In the former case the grading is obtained as follows[5)6)]: for any given N there are $8N$ odd generators Q_α^i, S_α^i $(\alpha = 1, \ldots, 4;\ i = 1, \ldots, N)$. They behave as $2N$ self-conjugate

Lorentz $SL(2,C)$ 4-spinors or more precisely as N self-conjugate conformal $SU(2,2)$ 8-spinors. The even part is given by the 15 generators $M_{\mu\nu}$, P_ν, D, K_ν of the conformal algebra with the addition of the N^2 generators of $U(N)$ *). This group acts in a natural way on the $4N$ chiral 2-spinors given by the left-handed and right-handed projections of the 4-spinors: Q^i_α and S^i_α. The latter case, namely the graded de Sitter algebra can be obtained in a straightforward way as a graded subalgebra of the previous conformal algebra. If one defines the odd generators to be

$$R^i_\alpha = \frac{1}{2}(Q^i_\alpha + S^i_\alpha)$$

then the even generators are given by the 10 generators $M_{\mu\nu}$, $L_\mu = \frac{1}{2}(P_\mu - K_\mu)$ of $O(3,2) \subset O(4,2)$ and the $\frac{1}{2}N(N-1)$ generators of the orthogonal group $SO(N) \subset U(N)$. To summarize we have that the main difference in the graded version of the space-time symmetry groups is in the internal symmetry part. In the Poincaré case this part is given by an Abelian set, namely the central charges [see (2.4)]. In the conformal and de Sitter case respectively by the $U(N)$ and $O(N)$ groups.

*) For $N = 4$ the $U(1)$ factor of $U(N) = U(1) \otimes SU(N)$ becomes a central charge and there are two possibilities, namely to have $SU(4)$ or $U(4)$ as internal symmetry part.

3. GLOBAL SUPERSYMMETRY

Globally supersymmetric field theories are those Lagrangian field theories which are invariant under the action of a graded Poincaré algebra[2]. They give rise to actions invariant under infinitesimal transformations of the fields which involve constant anticommuting spinors.

We shall give an example of a Lagrangian invariant under simple supersymmetry, i.e., with only one spinor charge but local Lagrangian exists up to a quadruplet of spinor charges.

These Lagrangians have the additional property of being invariant, for zero mass, under the bigger graded conformal algebra considered at the end of the previous section.

Although the simplest supersymmetric model field theory is the self-interaction of a scalar multiplet[8] we will consider, as a most interesting example, the supersymmetric extension of the Yang-Mills Lagrangian[9].

Suppose that the particle fields of spin $(1,\frac{1}{2})$ belong to the adjoint representation of a simple compact Lie group. Then the following Lagrangian

$$\mathcal{L} = \mathrm{Tr}\left(-\frac{1}{4} G_{\mu\nu} G^{\mu\nu} - \frac{i}{2} \bar{\lambda}\, \gamma\cdot D \lambda\right) \tag{3.1}$$

is both gauge invariant and supersymmetric. Here, as usual,

$$\begin{aligned} G_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu + ig\,[A_\mu, A_\nu] \\ D_\mu \lambda &= \partial_\mu \lambda + ig\,[A_\mu, \lambda] \end{aligned} \tag{3.2}$$

are the field strength and covariant derivatives of the connection vector field A_μ and of the self-conjugate 4-spinor λ. The Lagrangian (3.1) changes as a total derivative under the following field variations

$$\begin{aligned} \delta A_\mu &= [\bar{\epsilon} Q, A_\mu] = i\bar{\epsilon}\gamma_\mu \lambda \\ \delta \lambda &= [\bar{\epsilon} Q, \lambda] = -G_{\mu\nu}\sigma^{\mu\nu}\epsilon \end{aligned} \tag{3.3}$$

being ε a constant anticommuting Majorana spinor. The spinor charge Q_α is given by a space integral of a local conserved vector spinor current $J_{\mu\alpha}$. The latter can be derived via the usual Noether procedure to be

$$J^{\mu} = -\frac{1}{2} \mathrm{Tr} \left(G_{\nu\rho} \gamma^{\nu} \gamma^{\rho} \gamma^{\mu} \lambda \right) \tag{3.4}$$

The Lagrangian (3.1) has an additional supersymmetry due to conformal invariance. It is, in fact, invariant under a graded conformal algebra with the chiral U(1) as internal symmetry part. The second spinor charge S_{α} is the space-integral of a second conserved spinor current

$$I^{\mu} = - \gamma \cdot x \, J^{\mu} \tag{3.5}$$

The additional chiral invariance is

$$\delta_{\eta} A_{\mu} = 0 \qquad \delta_{\eta} \lambda = \eta \gamma_5 \lambda \tag{3.6}$$

whose Noether current is

$$J_{\mu}^{5} = \frac{i}{2} \mathrm{Tr} \left(\bar{\lambda} \gamma_5 \gamma_{\mu} \lambda \right) \tag{3.7}$$

The three local currents $J_{\mu\alpha}$, J_{μ}^{5} and $\theta_{\mu\nu}$, the latter being the stress tensor, transform irreducibly in the same supermultiplet[10].

More complicated Lagrangians involving vector multiplets coupled to different kinds of scalar multiplets have been constructed and extensively studied in the literature[2]. However, we will focus our attention on the Yang-Mills interaction of a vector multiplet with n massless scalar multiplets belonging to the adjoint representation of the gauge group.

These theories have the remarkable properties of offering examples, for n = 1 and 3 of local Lagrangians invariant under extended supersymmetry, respectively, with N = 2 and 4 spinor charges. Due to conformal invariance the global internal symmetry of these Lagrangians is U(2) [1] for n = 1 and SU(4) [12] for n = 3 [*].

The latter model gives the highest symmetric interacting theory one can think of in the framework of global supersymmetry. For N spinor charges with N > 4 representations of extended supersymmetry[7] necessarily involve particles with spin J > 1. Spin 3/2 and 2 particles, in order to give rise to consistent interactions, must be coupled to conserved currents so a new gauge principle, local supersymmetry, must be introduced. This new gauge principle is the basic invariance of supergravity theories which therefore can be regarded as gauge theories of graded Lie algebras.

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*) It is important to remark that for n = 3 the Yang-Mills SU(4) extended supersymmetric Lagrangian requires an additional self-coupling between the three scalar multiplets with coupling constant g' = g, g being the gauge coupling.

4. LOCAL SUPERSYMMETRY AND SUPERGRAVITY

Local supersymmetry means invariance of Lagrangian field theories under x dependent supersymmetry transformations. They involve arbitrary anticommuting spinors $\varepsilon(x)$ depending on the space-time point x. This local invariance requires the introduction of a new type of gauge field $\psi_\mu(x)$ which is a vector spinor Rarita-Schwinger field transforming as

$$\delta_\epsilon \psi_\mu(x) = \partial_\mu \epsilon(x) + \ldots. \tag{4.1}$$

under local supersymmetry. The dots stand for terms in the transformation laws not depending on a derivative of ε. The reason for introducing ψ_μ transforming as in (4.1) is due to the following standard argument: take any globally supersymmetric multiplet and make the local variation of the free Lagrangian $\mathcal{L}_R$. Then one gets, up to a four-divergence

$$\delta_{\epsilon(x)} \mathcal{L}_0 = -\partial_\mu \bar{\epsilon}(x) J^\mu \tag{4.2}$$

where J^μ is the supersymmetry Noether current.

One can compensate this new term by adding to $\mathcal{L}_0$ a "minimal coupling" $\mathcal{L}' = \bar{\psi}_\mu J^\mu$ with ψ_μ transforming as in (4.1). The new Lagrangian $\mathcal{L}_0 + \mathcal{L}'$ is now invariant for "weak" fields ψ_μ. To obtain full invariance one must use an iterative procedure but in particular one must take care of the global properties of the new gauge field ψ_μ itself. It is straightforward to write the free Lagrangian of ψ_μ. This is because the massless Rarita-Schwinger*) Lagrangian

$$\mathcal{L}_{3/2} = -\frac{1}{2} \varepsilon^{\mu\nu\rho\sigma} \bar{\psi}_\mu \gamma_5 \gamma_\nu \partial_\rho \psi_\sigma \tag{4.3}$$

*) For the convenience of the reader, in the sequel we will use the textbook (Bjorken, Drell) metric conversion for the γ matrices.

is invariant under (4.1). Equation (4.3) describes a massless field of helicity $\pm 3/2$. However, from the representations of global supersymmetry we know that must add a bosonic partner (4.3) describing a particle field of helicity ± 1 or ± 2.

The graded Lie algebra of the Poincaré group gives us the answer. Observe, for instance, that the fundamental commutation relation

$$\{ Q_\alpha, \bar{Q}_\beta \} = -2 \gamma^\mu_{\alpha\beta} P_\mu \tag{4.4}$$

tells us that local supersymmetry implies local translations; the latter being nothing but the general co-ordinate transformations of general relativity. General covariance therefore answers that the basic pattern of the spin 3/2 particle must be the graviton, the gauge particle of general relativity. The previous arguments show that local supersymmetry implies gravitation, by which we mean the gauge theory of the Poincaré group.

Supergravity is the Lagrangian field theory of local supersymmetry. In its simplest version, in presence of a single supersymmetry charge, the spin 2 graviton and the spin 3/2 gravitino join in an irreducible multiplet. The related local fields are the fundamental ingredients of any supergravity construction.

Let us consider the local vierbein $V_{\mu a}$ and the Rarita-Schwinger $\psi_{\mu\alpha}$ related to the graviton and the gravitino. They are the gauge fields associated to translations P_a and spinor charge Q_α, respectively. We note that in second-order relativity the vierbein connection $\omega_{\mu ab}$, which is the gauge field of the Lorentz generators M_{ab}, is not an independent dynamical variable because the metric postulate

$$\mathcal{D}_\mu V_\nu^{\ a} = 0$$

resolves $\omega_{\mu ab}$ in terms of V^a_ν:

$$\omega_{\mu ab} = \frac{1}{2} \left[V_a^{\ \nu} (\partial_\mu V_{b\nu} - \partial_\nu V_{b\mu}) + V_a^{\ \rho} V_b^{\ \sigma} (\partial_\sigma V_{c\rho}) V_\mu^{\ c} \right] - (a \to b) \tag{4.5}$$

Similarly for the affine connection

$$\Gamma_{\mu\nu}^{\ \ \rho} = \{ \}_{\mu\nu}^{\ \ \rho} = V^{a\rho} (\partial_\mu V_{a\nu} + \omega_{\mu ab} V^b_{\ \nu}) \tag{4.6}$$

$\{\}$ being the Christoffel symbol.

The pure supergravity Lagrangian is[13)14)]

$$\mathcal{L} = -\frac{1}{2K^2} V V^{a\mu} V^{b\nu} R_{\mu\nu ab} - \frac{1}{2} \varepsilon^{\mu\nu\rho\sigma} \bar{\psi}_\mu \gamma_5 \gamma_\nu D_\rho \psi_\sigma$$
$$- \frac{K^2}{32} V \left[(\bar{\psi}^\mu \gamma^\nu \psi^\rho)(\bar{\psi}_\mu \gamma_\nu \psi_\rho + 2 \bar{\psi}_\nu \gamma_\mu \psi_\rho) - 4 (\bar{\psi} \cdot \gamma \psi_\rho)^2 \right] \tag{4.7}$$

with curvature tensor

$$R_{\mu\nu ab} = \partial_\mu \omega_{\nu ab} - \partial_\nu \omega_{\mu ab} + \omega_{\mu a}{}^c \omega_{\nu cb} - \omega_{\nu a}{}^c \omega_{\mu cb} \tag{4.8}$$

and covariant derivative

$$D_\rho \psi_\sigma = \partial_\rho \psi_\sigma - \Gamma_{\rho\sigma}^{\lambda} \psi_\lambda + \frac{1}{2} \omega_{\rho ab} \sigma^{ab} \psi_\sigma \tag{4.9}$$

The Lagrangian (4.7) is invariant up to a four-divergence under the following local supersymmetry variations

$$\delta V_{a\mu} = -i K \bar{\epsilon} \gamma_a \psi_\mu$$
$$\delta \psi_\mu = \frac{2}{K} D_\mu \epsilon - \frac{i}{4} K (2 \bar{\psi}_\mu \gamma_a \psi_b + \bar{\psi}_a \gamma_\mu \psi_b) \sigma^{ab} \epsilon \tag{4.10}$$

The Lagrangian (4.7) was originally constructed starting with the minimally coupled spin 2 and 3/2 Lagrangian and initial variations $\delta V_{a\mu}$ and $\delta\psi_\mu = 2/K\, D_\mu\, \varepsilon$ for the vierbein and Rarita-Schwinger fields[13)]. These variations made the terms which are linear in ψ cancel exactly but a term trilinear in ψ was left. This term was cancelled by adding the four-fermion contact term to the Lagrangian and the bilinear term in $\delta\psi_\mu$, as they stand in Eqs. (4.7) and (4.10).

An alternative simpler derivation of the Lagrangian (4.7) and transformation laws (4.10) was obtained by working in Cartan first-order formuation of general relativity[14)]. Here the vierbein connection $\omega_{\mu ab}$ is treated as an independent dynamical variable. It is, however, an auxiliary field and its equation of motion is algebraic, giving

$$\omega_{\mu ab} = \omega^0_{\mu ab} + K_{\mu ab} \qquad (4.11)$$

where $\omega^0_{\mu ab}$ is given by (4.5) and $K_{\mu ab}$ is the contorsion tensor*). In terms of the spin density of a particle field:

$$S^{\mu\nu\rho} = \delta\mathcal{L}/\delta K_{\rho\nu\mu}$$

the contorsion tensor is

$$K_{ijk} = -(S_{kij} + S_{ijk} + S_{kji} + \delta_{ij} S_{\ell k\ell} - \delta_{ik} S_{\ell j\ell}) \qquad (4.12)$$

In first-order form the Lagrangian (4.7) becomes

$$\mathcal{L} = -\frac{1}{2K^2} V V^{a\mu} V^{b\nu} R_{\mu\nu ab} - \frac{1}{2}\varepsilon^{\mu\nu\rho\sigma}\bar{\psi}_\mu \gamma_5 \gamma_\nu D_\rho \psi_\sigma \qquad (4.13)$$

in terms of the non-minimal curl

$$\varepsilon^{\mu\nu\rho\sigma} D_\rho \psi_\sigma = \varepsilon^{\mu\nu\rho\sigma}(\partial_\rho \psi_\sigma + \frac{1}{2}\omega_{\rho ab}\sigma^{ab}\psi_\sigma)$$

The action (4.13) is invariant up to a four-divergence under the following transformations

$$\begin{aligned} \delta V_{a\mu} &= -iK\bar{\epsilon}\gamma_a \psi_\mu \\ \delta \psi_\mu &= \frac{2}{K} D_\mu \epsilon \\ \delta \omega_{\mu ab} &= B_{\mu ab} + \frac{1}{2} V_{a\mu} B_{\lambda b}{}^{\lambda} - \frac{1}{2} V_{b\mu} B_{\lambda a}{}^{\lambda} \end{aligned} \qquad (4.14)$$

where

$$V B_\mu{}^{\nu\rho} = K\,\varepsilon^{\nu\rho\sigma\tau}\bar{\epsilon}\gamma_5\gamma_\mu D_\sigma \psi_\tau$$

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*) The torsion $C_{\mu\nu\rho} = \frac{1}{2}(\Gamma^\rho_{\mu\nu} - \Gamma^\rho_{\nu\mu})$, the antisymmetric part of the affine connection, is related to the contorsion tensor as follows:

$$K_{\mu\nu\rho} = C_{\nu\rho\mu} + C_{\mu\rho\nu} - C_{\mu\nu\rho}$$

If one makes use of the field equations for the contorsion [see (4.12)]

$$K_{\mu ab} = -\frac{i}{4}\kappa^2(\bar{\psi}_\mu \gamma_a \psi_b - \bar{\psi}_\mu \gamma_b \psi_a + \bar{\psi}_a \gamma_\mu \psi_b) \tag{4.15}$$

and plugs it back into the Lagrangian (4.13) and transformation laws (4.14), one exactly recovers the second-order formulation as given by Eqs. (4.7) and (4.10).

5. MATTER COUPLING IN SUPERGRAVITY

The supergravity Lagrangian previously discussed can be coupled to matter multiplets of global supersymmetry in a locally supersymmetric way.

This is possible if one takes any global supersymmetric field theory and implements local invariance by introducing appropriate interactions with the (2, 3/2) gauge multiplet.

Let us consider the locally supersymmetric extension of the Yang-Mills Lagrangian considered in Section 3. The resulting Lagrangian, in first-order form of general relativity, is the sum of three terms[15]

$$\mathcal{L} = \mathcal{L}_{SG} + \mathcal{L}_M + \mathcal{L}_C \tag{5.1}$$

$\mathcal{L}_{SG}$ is the pure supergravity Lagrangian as given by Eq. (4.13). $\mathcal{L}_M$ is the minimally coupled Lagrangian:

$$\mathcal{L}_M = V\, \mathrm{Tr}\left(-\frac{1}{4} g^{\mu\nu} g^{\rho\sigma} G_{\mu\rho} G_{\nu\sigma} + \frac{i}{2} \bar{\lambda} \not{D} \lambda - \frac{i}{4} \kappa\, \bar{\psi}_\mu \gamma^\alpha \gamma^\beta \gamma^\mu \lambda\, G_{\alpha\beta}\right) \tag{5.2}$$

which is the sum of the flat Lagrangian properly covariantized with respect to curved space with an additional coupling of the supersymmetry Noether current (3.2) to the spin gauge field ψ_μ [see, for instance, Eq. (4.2)].

$\mathcal{L}_c$ is an additional contact term

$$\mathcal{L}_c = -\frac{1}{4} V \kappa^2\, \mathrm{Tr}\left(\bar{\psi}_\mu \sigma^{\alpha\beta} \gamma^\mu \lambda\, \bar{\psi}_\alpha \gamma_\beta \lambda\right) \tag{5.3}$$

which must be added to (5.2) to have full invariance.

The Lagrangian (5.1) transforms as a four-divergence when the fields undergo the following supersymmetry transformations

$$
\begin{aligned}
\delta A_\mu &= i\bar{\epsilon}\gamma_\mu\lambda \\
\delta\lambda &= \hat{G}_{\mu\nu}\sigma^{\mu\nu}\epsilon \\
\delta V_{a\mu} &= -i\kappa\bar{\epsilon}\gamma_a\psi_\mu \\
\delta\psi_\mu &= \frac{2}{\kappa}D_\mu\epsilon + \frac{i}{4}\kappa\,\mathrm{Tr}\,\bar{\lambda}\gamma_5\gamma^\rho\lambda\;\gamma_\rho\gamma_\mu\gamma_5\epsilon
\end{aligned}
\tag{5.4}
$$

In Eqs. (5.4) D_μ is the first order covariant derivative including torsion;

$$\hat{D}_\mu A_\nu - \hat{D}_\nu A_\mu + ig[A_\mu, A_\nu] = \hat{G}_{\mu\nu} \tag{5.5}$$

where $\hat{D}_\mu A_\nu$ stands for the supercovariant derivative of A_μ

$$\hat{D}_\mu A_\nu = \partial_\mu A_\nu - \frac{i}{2}\kappa\bar{\psi}_\mu\gamma_\nu\lambda \tag{5.6}$$

Similarly for the matter spinor field λ

$$\hat{D}_\mu\lambda = D_\mu\lambda - \frac{1}{2}\kappa\,\hat{G}_{\alpha\beta}\sigma^{\alpha\beta}\psi_\mu \tag{5.7}$$

Supercovariant derivatives are defined in such a way that $\delta_\varepsilon(\hat{D}_\mu A_\nu)$, $\delta_\varepsilon(\hat{D}_\mu\lambda)$ do not depend on the derivative of the parameter $\varepsilon(x)$. It is interesting to observe that the spin $\frac{1}{2}$ field equation has the very simple form

$$\gamma^\mu\hat{D}_\mu\lambda = 0 \tag{5.8}$$

i.e., the free Dirac equation with first order gravitation, Yang-Mills and super-covariant derivatives.

Local coupling to the scalar multiplet is much more involved in supergravity. This is because the symmetry of the Lagrangian allows any power $(KA)^n, (KB)^n$, K being the gravitational coupling and A, B the scalar and psueoscalar fields of the spin $(\frac{1}{2}, 0^{\pm})$ multiplet.

This is in contrast with the vector multiplet for which the Yang-Mills gauge invariance and power-counting severely restricts the terms present in the Lagrangian. It has been shown[16] that, in the presence of the gravitational coupling only, local supersymmetry fixes the local coupling of the scalar multiplet in terms of an arbitrary function $a(\rho^2)$ of the variable $\rho^2 = K^2/2\ (A^2 + B^2)$. Note that $a(0) = 1$ in order to recover the flat supersymmetry limit. Only if $a = \text{const} = 1$ the resulting Lagrangian is polynomial and in fact reduces to the coupling that was first found in the literature[17].

Among the possible non-polynomial couplings of the scalar multiplet to supergravity there is a preferred one. It corresponds to the choice

$$a(\rho^2) = 1 - \frac{K^2}{2}(A^2 + B^2)$$

For this function the spin $\frac{1}{2}$ matter contact term reduces to pure torsion, i.e.,

$$\mathcal{L}_4^{\chi} = \frac{3}{64} K^2 V (\bar{\chi} \gamma_5 \gamma_\rho \chi)^2$$

χ being the self-conjugate 4-spinor of the scalar multiplet. This value for the function $a(\rho^2)$ is actually selected in extended supergravity theories which contain as subtheory the scalar multiplet coupled to simple supergravity. More general locally supersymmetric Lagrangians have also been constructed. In particular the local coupling of a self-interacting scalar multiplet as well as the minimal extension of locally supersymmetric Q.E.D. have been worked out[16].

6. EXTENDED SUPERGRAVITY

Supergravity theories considered so far involve a single self-conjugate spin 3/2 field ψ_{μ}. They are the gauge theories of a graded Poincaré algebra with a single odd spinor generator Q_{α}. On the other hand, we know that graded versions of the Poincaré algebra exist for any number N of odd generators [see (2.4)] and their representations on one-particle states are also known. One can therefore envisage the construction of gauge theories for this larger class of graded Lie algebras. They are particularly interesting because they allow, in principle, to construct truly unified gauge theories in which the internal symmetries are fused with gravitation in a unique gauge principle. From the particle content of extended supermultiplet it seems also that local gauge theories for extended supergravity can only exist up $N = 8$ spin 3/2 gauge fields[7]. In fact, for $N > 8$ higher spin or more than one spin 2 particles enter in the same multiplet. These particles have no additional gauge principle associated with them, and it seems that a consistent local Lagrangian formulation does not exist in this case.

The gauge theories of extended supersymmetry have a natural global $SO(N)$ internal symmetry, the N real spinor charges of the graded Lie algebra lying in the vector representation of $O(N)$. The orthogonal group acts as a natural outer isomorphism of the graded Poincaré algebra. Particles of a fixed helicity in a given supermultiplet belong to a completely antisymmetric tensor representation of $SO(N)$.

Let us consider the simplest of these theories, i.e., $SO(2)$ extended supergravity. The gauge multiplet contains the singlet graviton, a doublet of gravitinos and a photon singlet.

The Lagrangian, in second order formalism, is[18]

$$\begin{aligned}\mathcal{L} = & -\frac{1}{2K^{2}} V R - \frac{1}{2}\varepsilon^{\mu\nu\rho\sigma}\bar{\psi}_{\mu}^{i}\gamma_{5}\gamma_{\nu}D_{\rho}\psi_{\sigma}^{i} - \frac{1}{4}V(F_{\mu\nu})^{2} \\ & - \frac{1}{2\sqrt{2}}K\varepsilon^{ij}\bar{\psi}_{\mu}^{i}(VF^{\mu\nu} - i\gamma_{5}\tilde{F}^{\mu\nu})\psi_{\nu}^{j} \\ & - \frac{1}{8}K^{2}\varepsilon^{ij}\varepsilon^{k\ell}\bar{\psi}_{\mu}^{i}\psi_{\nu}^{j}\left[V(\bar{\psi}^{\mu k}\psi^{\nu\ell}) - \frac{i}{2}\varepsilon^{\mu\nu\rho\sigma}\bar{\psi}_{\rho}^{k}\gamma_{5}\psi_{\sigma}^{\ell}\right] \\ & - \frac{1}{32}K^{2}V\left[(\bar{\psi}^{\lambda i}\gamma^{\mu}\psi^{\rho i})(\bar{\psi}_{\lambda}^{j}\gamma_{\mu}\psi_{\rho}^{j} + 2\bar{\psi}_{\mu}^{j}\gamma_{\lambda}\psi_{\rho}^{j}) - 4(\bar{\psi}^{i}\cdot\gamma\psi_{\sigma}^{i})^{2}\right]\end{aligned} \tag{6.1}$$

The action is invariant under local fermionic transformations involving a doublet of Majorana spinors $\varepsilon^i(x)$ with field variations

$$\delta V_{a\mu} = -i\kappa\, \bar\epsilon^i \gamma_a \psi_\mu^i$$

$$\delta A_\mu = -\sqrt{2}\, \varepsilon^{ij}\, \bar\epsilon^i \psi_\mu^j$$

$$\begin{aligned}\delta \psi_\mu^i = {} & \frac{2}{\kappa} D_\mu \epsilon^i - \frac{i}{4}\kappa\,(2\bar\psi_\mu^j \gamma_\rho \psi_\nu^j + \bar\psi_\rho^j \gamma_\mu \psi_\nu^j)\,\sigma^{\rho\nu}\epsilon^i \\ & - \frac{i}{\sqrt{2}}\, \varepsilon^{ij} \sigma^{\rho\nu} \hat F_{\rho\nu} \gamma_\mu \epsilon^j\end{aligned} \qquad (6.2)$$

where

$$\hat F_{\mu\nu} = F_{\mu\nu} + \frac{\kappa}{\sqrt{2}}\, \varepsilon^{ij}\, \bar\psi_\mu^i \psi_\nu^j$$

is the supercovariant field strength.

The Lagrangian (6.1) is globally SO(2) invariant. The gauge vector field A_μ is not the gauge field of this SO(2) group but rather of the central charge which appears in the graded Poincaré algebra [see Eq. (2.4)]. It couples non-minimally to the complex spin 3/2 field

$$\psi_\mu = \frac{1}{\sqrt{2}}\,(\psi_\mu^1 + i\,\psi_\mu^2)$$

via an anomalous magnetic moment type interaction as dictated by the Lagrangian (6.1). One can further show that the Lagrangian (6.1) has an additional SU(2) global invariance under SU(2) rotations of the chiral projections

$$\psi_{\mu\,L,R}^i = \frac{1}{2}\,(1 \pm \gamma_5)\,\psi_\mu^i$$

The equations of motion derived from the above Lagrangian are also U(1) invariant under duality transformations on the field strength $F_{\mu\nu}$ accompanied by a chiral rotation of the spin 3/2 doublet[19].

SO(2) supergravity can be coupled to matter SO(2) supermultiplets in a standard way as discussed in Section 5. Of particular interest is the case of the coupling to a massive scalar multiplet with particle content $2(\frac{1}{2})$, $2(0^{\pm})$ because a non-trivial central charge Z is present in this case[19]. The related Noether current is

$$J_\mu = m\left(A_1 \overleftrightarrow{\partial_\mu} A_2 + B_1 \overleftrightarrow{\partial_\mu} B_2 + i\, \bar{\chi}_1 \gamma_\mu \chi_2 \right) \tag{6.3}$$

and it is coupled minimally to the gauge field A_μ with coupling $g = Km$.

We consider now supergravity theories with $N > 2$. These theories have the new feature of containing in the same irreducible multiplet non-gauge fields of spin $\frac{1}{2}$ and 0.

For instance, the SO(3) extended supergravity[20] has a particle spin content 2, 3(1),, 3(3/2), $\frac{1}{2}$. Therefore there is a true matter field of spin $\frac{1}{2}$ in the same multiplet of the graviton.

Supergravity theories with $N > 3$ look more complicated because of an anticipated non-polynomial structure in the scalar fields present in the gauge supermultiplet. The complete Lagrangian and transformation laws of SO(4) supergravity are known[21] and partial results for the SO(8) theory have also been derived[22]. Interestingly enough, all these theories have a global U(N) invariance in which the unitary group acts in a natural way on the chiral projections of self-conjugate four spinors and on the self-dual (anti self dual) combinations of the vector field strengths[19].

An intriguing point is the gauging of the SO(N) internal symmetry in extended supergravity. The SO(N) group can be promoted to a local gauge group[23] provided one adds to the previous Lagrangians a cosmological term and a mass-like term for the spin 3/2 of universal strength

$$V^{-1}\mathcal{L}' = V^{-1}\mathcal{L} - \frac{g}{K}\sqrt{2}\, \bar{\psi}_\mu^{\,i} \sigma^{\mu\nu} \psi_\nu^{\,i} + 6\left(\frac{g}{K^2}\right)^2 \tag{6.4}$$

where g is the SO(N) gauge coupling.

The origin of the gauged SO(N) structure is purely algebraic[24] and can be simply understood by noticing that, with the added terms, $\mathcal{L}'$ is gauge invariant with respect to a graded de Sitter algebra rather than Poincaré algebra. As explained at the end of Section 2 the $O(N)$ generators rather than the central charges appear in the extended de Sitter graded algebra. For vanishing gauge coupling $g = 0$, $O(3,2) \otimes O(N)$ contracts to $IO(3,1) \otimes Z$, Z being the centre.

Apart from the difficulties related to the size of the cosmological term, these theories look as truly unified theories of particle interactions in which an underlying Yang-Mills gauge group $SO(N)$ is present. It is not clear at present if these theories can be used for current particle phenomenology. The main difficulty lies in the fact that the biggest gauge group consistent with local quantum field theory seems to be $SO(8)$. Renormalizability properties, which have not been discussed here, seem to be in favour of a superunified theory in which all fundamental particles lie in the same gauge supermultiplet of extended supergravity. These pure gauge theories have been shown in fact to be one- and maybe two-loop renormalizable[25)26)]. This is a great improvement with respect to the ordinary Einstein theory which always led to divergent quantum corrections in the presence of matter fields. The shortcoming of supergravity theories lies in the fact that local couplings with independent matter supermultiplets are not renormalizable[1)27)]. The consequence of this is a drastic limitation to the structure of the gauge internal symmetry group.

Extended supergravity theories so far constructed have $SO(8)$ as their biggest gauge group of fundamental interactions. Although this group has interesting properties, for instance it can accommodate colour gauge symmetry with four quark flavours[28)], it is too small for incorporating a unified theory of weak, electromagnetic and strong interactions, the latter having $SU(3) \otimes SU(2) \otimes U(1)$ as minimal gauge group. A possible alternative scheme is to have a unified gauge theory based on the gauging of the conformal group[5)]. In the absence of fermions, such an approach gives the Weyl Lagrangian

$$V^{-1}\mathcal{L} = (R_{\mu\nu})^2 - \frac{1}{3} R^2 \tag{6.5}$$

as the basic gauge Lagrangian for pure gravity.

The supersymmetric version of the Lagrangian (6.5) corresponds to the gauging of a graded conformal algebra. In the presence of N odd spinor symmetry charges this leads to the gauging of $U(N)$ rather than $O(N)$ internal symmetry[6)]. Left-handed gravitinos transform according to the vector representation of $U(N)$. We observe further that the gravitational coupling is dimensionless in Weyl theory and it just coincides with the gauge $U(N)$ coupling.

Although this theory seems more promising as far as the internal symmetry sector is concerned, it seems to have problems with ghosts and unitarity due to the higher derivative terms present in the spin 2, $3/2$ sector.

7. CONCLUSIONS

We have presented in this review some basic aspects of globally and locally supersymmetric Lagrangians. Local invariance in the framework of supersymmetry is particularly interesting because it offers, in principle, a unification scheme of fundamental interactions.

We have followed here an approach based on a Lagrangian formulation in terms of particle fields defined in Minkowski space. Other approaches to supergravity exist in the literature. One of them makes essential use of a Goldstone spinor[29]. Goldstone spinors play an important role in supergravity theories, being related to the problem of spontaneous symmetry breaking of the symmetry. This breaking must occur at a certain stage to resolve the mass degeneracy of the particle supermultiplets[30]. Another approach is based on the idea of superspace and of superfields[31]. Superfields are fields defined on superspace, i.e., a space whose points are labelled, in addition to the usual Minkowski co-ordinates, by a set of anticommuting variables.

Techniques based on superfields are important and should play a fundamental role for a better understanding of the geometry underlying supergravity Lagrangian constructions. Other approaches make use of pure geometrical techniques based on differential forms and fibre bundles[32]. Developments of supergravity theories introducing a set of auxiliary fields (Lagrangian multipliers) which resolve the highly non-polynomial character of present supergravity Lagrangians and transformation laws have also been pursued[33].

It may be that different supergravity constructions, based on a deeper understanding of the underlying geometrical structure and with different graded Lie groups can improve the present framework. Certainly, the idea of a unified theory of all fundamental interactions, including gravity, via a renormalizability criterion, seems very appealing. It gives a subtle linking between the internal symmetry structure and the spin of elementary constituents of matter which is ultimately related to the underlying geometrical structure of space and time.

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