GROUP THEORETICAL EXPANSIONS OF SCATTERING AMPLITUDES FOR PARTICLES WITH SPIN*

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The purpose of this contribution is to report on some new progress achieved in a general research program, the aim of which is to study two body scattering $1+2 \to 3+4$ in terms of two variable expansions of scattering amplitudes. The essence of this approach is that the entire dependence of the amplitudes on the kinematic parameters (e.g. energy and scattering angle) is displayed explicitly in known special functions whereas the dynamics are transferred to the expansion coefficients. A successful separation of kinematics and dynamics along these lines should: (a) Provide a convenient formalism for describing experimental data over large regions of energies and angles. (b) Serve as a tool for the formulation of dynamical theories and models. Most of the earlier work in this direction was restricted to spinless particles (see e.g. [1,2] and the review[3], containing many more references to the original work). Here we generalize to arbitrary spins.

As in the earlier work[3] we shall use the representation theory of the Lorentz group to provide expansions. We shall call the relevant Lorentz group the "Internal Lorentz Group" $\tilde{O}(3.1)$, as opposed to the kinematical Lorentz group $O(3.1)$, providing transformations between different inertial frames of reference. A transformation of this Internal Lorentz Group will transform a state of two particles at rest into a state with momenta $(p_1,p_2) = (m_1 x, m_2 \pi x)$, where x is a unit four vector and π a reflection operator. In order to obtain the usual single variable expansions of helicity amplitudes Jacob and Wick[4] introduced Poincaré irreducible two particle states in a helicity basis

$$|w(a) jn\lambda_1\lambda_2 > = \int d\Omega(\theta,\phi)\ D^{j*}_{n\lambda_1+\lambda_2}(\phi,\theta,0)\ |a\theta\phi\ \lambda_1\lambda_2 > \tag{1}$$

where we have put

$$|a\theta\phi\ \lambda_1\lambda_2 > \equiv |p_1\lambda_1 > \otimes |p_2\lambda_2 > \equiv |x\ \lambda_1\lambda_2 >. \tag{2}$$

($w(a) = (p_1+p_2)^2$ is the invariant energy, $D^j_{n\lambda_1+\lambda_2}$ a Wigner D function).
We further introduce internal Lorentz states

$$|(\rho\nu s) jn > = \sum_{\lambda_1\lambda_2} (s_1\lambda_1 s_2\lambda_2|s\lambda) \int_0^\infty sh^2 a\ da\ d^{\rho\nu}_{js\lambda}(a)\ |w(a) jn\lambda_1\lambda_2 > . \tag{3}$$

Here $d^{\rho\nu}_{js\lambda}$ (a) is an O(3,1) finite transformation reduced matrix element[5], a is related to the invariant energy:

$$\mathrm{ch}^2 a = \frac{w^2-(m_1-m_2)^2}{4m_1 m_2} , \tag{4}$$

s_i and λ_i are the particle spins and helicities, j and n are the total angular momentum and its projection and ρ and ν label representations of O(3,1). Relation (3) can easily be inverted.

Introducing the scattering matrix T and calculating its matrix elements between two particle initial and final states, both expanded in terms of the internal Lorentz states (3), we obtain, after some manipulations, the following complete expansion of the helicity amplitudes:

$$< a\ \theta\phi\ \lambda_3\lambda_4 | T |\ a'00\ \lambda_1\lambda_2 > = \delta(w(a) - w(a')) \tag{5}$$

$$\sum_{ss'} (s_3\lambda_3\ s_4\lambda_4 | s\lambda)\ (s_1\lambda_1\ s_2\lambda_2 | s'\lambda') \int d\rho \sum_{\nu j} T^{\rho\nu}_{jss'\lambda'}\ D^{\rho\nu}_{j\lambda' j\lambda}\ (\Lambda_x) .$$

Here $\Lambda_x = R(\phi,\theta,0)\ B(a)$ is a Lorentz transformation (R is a rotation, B(a) a boost), a satisfies (4); θ is the c.m.s. scattering angle, ϕ an irrelevant azimuthal angle. The Lorentz group D function satisfies

$$D^{\rho\nu}_{j\lambda' j\lambda}(\Lambda_x) = d^{\rho\nu}_{jj\lambda}(a)\ D^{j}_{\lambda'\lambda}(\phi,\theta,0) . \tag{6}$$

The symbols $(s\lambda s'\lambda' | S\Lambda)$ denote O(3) Clebsch-Gordan coefficients. Finally $T^{\rho\nu}_{jss'\lambda'}$ are the "Lorentz amplitudes", carrying the dynamics.

Formula (5) thus provides us with the required result. The energy and scattering angle only enter via known functions. The expansion can be interpreted as the Jacob and Wick O(3) little group expansion, supplemented by an integral expansion of the partial wave helicity amplitudes. Each partial wave automatically has the correct threshold behaviour (in view of the behavior of $d^{\rho\nu}_{jj\lambda}$ (a) for $a \to 0$). The O(3,1) group underlying the expansion is the internal Lorentz group, discussed above, and its introduction is a relativistic analogue of the separating out of the center of inertia motion. In (5) j is the total angular momentum in the initial and final state and it hence satisfies $\max(\lambda_1+\lambda_2,\ \lambda_3+\lambda_4) \le j < \infty$. The O(3,1) label ν is integer or half integer (together with j) and satisfies $-j \le \nu \le j$. The other O(3,1) label ρ is in general a complex number. The integration over ρ in (5) can be restricted to the real axis ($-\infty<\rho<\infty$) if the expanded helicity amplitudes belong to a Hilbert space of square integrable functions. This is a physical assumption, implying e.g. that the corresponding total cross sections decrease for $a \to \infty$. For more general amplitudes, e.g. polynomially increasing ones, the integration path in the ρ plane must be appropriately chosen (and will involve nonunitary representations of O(3,1)).

We have so far restricted ourselves to expansions using a basis corresponding to the reduction $O(3,1) \supset O(3) \supset O(2)$. As in the case of spinless particles[3], other basis can be used. Thus, the reduction $O(3,1) \supset O(2,1) \supset O(2)$ will provide us with Regge pole type expansions in the energy variable, supplemented by an expansion of the reggeized partial wave amplitudes. The $O(3,1) \supset E(2) \supset O(2)$ reduction will lead to a two variable generalization of the eikonal expansions.

For further details we refer to a forthcoming article[6].

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References

[1]. N. Ya. Vilenkin and Ya. A. Smorodinskiĭ, Zh. Eksp. Teor. Fiz. 46, 1793 (1964) [Sov. Phys. JETP 19, 1209 (1964)]; P. Winternitz, Ya. A. Smorodinskiĭ and M.B. Sheftel', Yad. Fiz. 7, 1325 (1968), 8, 833 (1968) [Sov. J. Nucl. Phys. 7, 785 (1968), 8, 485 (1969)]

[2]. J. Bystricky, F. Lehar, J. Patera and P. Winternitz, Phys. Rev. D13, 1276 (1976)

[3]. E. G. Kalnins, J. Patera, R. T. Sharp and P. Winternitz in *Group Theory and its Applications*, edited by E. M. Loebl (Academic, New York, 1975), Vol. 3, p. 369-464).

[4]. M. Jacob and G. C. Wick, Ann. Phys. (N.Y.) 7, 404 (1959)

[5]. W. Rühl, *The Lorentz Group and Harmonic Analysis*, (Benjamin, New York, 1970)

[6]. M. Daumens, M. Perroud and P. Winternitz, Preprint CRM, Montréal, 1977.