ON THE EXTENSION OF VECTOR FIELDS AND THE SUPERLUMINAL TRANSFORMATIONS

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A very simple way of extending [1] the standard Lorentz group Λ

$$x' = \gamma(x-vt);\ t' = \gamma(t-vx)\ ;\ \gamma = (1-v^2)^{-1/2},\ c=1,\ v < 1 \qquad (1)$$

to any value of v is to define the transformations,

$$x' = \Gamma(x-vt);\ t' = \Gamma(t-vx)\ ;\ \Gamma = (v^2-1)^{-1/2},\ v > 1 \qquad (2)$$

Eqs.(1)-(2) constitute a semigroup(closed property) with identity, but only the subfamily given by eqs.(1) form a group.

In spite of the absence of the group property the parameter $v_{1,2}$ of the transformation obtained by composing two transformations (with parameters v_1 and v_2) of the kind (1) or (2) is given by the usual formula,

$$v_{1,2} = (v_1 + v_2)/(1 + v_1 v_2) \qquad (3)$$

These results are not in disagreement with the fact[2] that the group property is automatically fulfilled by any single-connected monoparametric semigroup with identity.In fact the manifold ot the v-space of (1)-(2) is not a single-connected one, and accordingly only the connected component of it, given by eqs (1), is obliged to form a group.

Since the eqs.(1)-(2) violate the principle of relativity(because of the no-fulfilment of the group property for $v > 1$) it would be of great physical value to replace eqs.(2) by other formulae in order to get a new enlarged set of transformations Λ_e forming a group. In order to reach this goal, the vector field $\vec{X}$(on the x,t manifold) representing the infinitesimal form of Λ_e is of little practical value since $\vec{X}$ only carries information on the part of Λ_e connected to the identity. Since this part is always given by eqs.(1) we shall always have,

$$\vec{X} = -t\ \ d/dx\ - x\ \ d/dt \qquad (4)$$

for any possible group Λ_e extending Λ . Therefore, the only way left open(and which apparently has not been considered up to now) in order to extend Λ seems to be the study of the first mo̲noparameter groups([3]) of Λ and Λ_e , since in them is in fact contained intrinsic information which can be absent in the particu̲lar realization of them on the x-t manifold.

The expression of the invariant vector field of the first mono̲parameter group of Λ is inmediately obtained from eq.(3) and is given by:

$$\vec{Y} = (1-v^2)\ d/dv \qquad |v|<1 \qquad (5)$$

Now, the classification of the groups of the line v(like the group defined by eq.(3) is far more simple ([4]) than the classification of the groups of the plane(like the group defined by eqs.(1)) In fact, any group in a single variable([5]) cannot contain more than three essential parameters and must be similar to the projective group of the real line(or to a subroup of it). In particular, the group defined by eq.(5) is the subgroup of

$$v' = (a_1 v + a_2)/(a_3 v + 1) \qquad (6)$$

for $a_2 = a_3$ and $a_1 = 1$.

Therefore, the original problem of extending eqs.(1) has been reduced to the problem of extending the vector field $\vec{Y}$, defined by (6), outside the range $|v| \leq 1$.

The problems concerned with the extension of vector fields are very important in the practical applications([6]) and its study seems to be rather recent. Our extension problem has obviously (if we do not impose concretė restrictions) many solutions.Two, at least, restrictions seem to be necessary. The first one is the possible presence in the enlarged vector field $\vec{Y}_e$ of more than one singularity.We recall that $\vec{Y}$ is singular for $|v| = 1$ (i.e. $\vec{Y}$ ($v=\pm 1$) = $\vec{0}$).Since the problem of extending a vector field is not well defined([6]) unless further information is given about the singularities that the new enlarged vector field must have, it would be interesting trying to get an extension compatible with a whole sequence $v_1, v_2, ..$ of singularities outside $|v| \leq 1$. Each of these singularities represents , from the physical point of view, a characteristic speed of propagation of possible long-range physical fields(of zero rest mass, like the electromagnetic field, whose characteristic speed of propagation gives the first singularity contained in $\vec{Y}$, for $v = \pm 1$), since there are physical reasons justifying the explicit presence in Λ_e of these characteristics speed of propagation([7]).

A second restriction on $\vec{Y}_e$ comes from the fact that $\vec{Y}_e$ must be a vector field that, when integrated, must generate a global group([8]) and, therefore, $\vec{Y}_e$ must be a complete vector field in a parameter representing a velocity.

The open problem of extending $\vec{Y}$ under these two restrictions is much more concrete that the original one of just finding the group extensions of (1). On the other hand, since the number of variables involved has been reduced from two -(x,t)- to one -v- the solution of finding a real group extension of Λ should be now much more easy to find.

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