TOWARDS A CONSTRUCTIVE APPROACH TO SUPERSYMMETRIC ϕ^3

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As is well known, supersymmetric ϕ^3 is the least divergent of all known field-theoretic models in four dimensions [1],[2]. This is mainly because supersymmetry gives rise to Ward-identities which lead to divergence cancellations already on the level of perturbation theory. In this note, possibilities of a nonperturbative (constructive) approach will be discussed. The basic idea is to put the theory on a finite lattice ($\mathcal{T}$,a) (a=lattice constant) and to construct a "renormalisation-map" R($\mathcal{T}$,a) mapping "bare" parameters into physical normalisation parameters. For A^4_d such a map has been constructed and thoroughly investigated by Schrader [3]. Since, at the moment, there are no correlation inequalities for classical fermion-systems (or any equally powerful methods) the final question as to whether supersymmetric ϕ^3 exists cannot yet be settled. But prospects that it will be among the first candidates for which existence can be proven are promising.

On a Euclidean lattice ($\mathcal{T}$,a) the model has the following "Lagrangian" $(\alpha > 0\, ;\ m, g \in \mathbb{R})$

$$\begin{aligned}\mathcal{L}_n = {} & \frac{\alpha}{2}(\nabla_\mu A_n)^2 + \frac{\alpha}{2}(\nabla_\mu B_n)^2 + \frac{i\alpha}{2}\psi^{(2)}_n \gamma^\mu_E \nabla^\mu \psi^{(1)}_n + \\ & + \frac{\alpha}{2}(F_n^2 + G_n^2) + im\left(F_n A_n + G_n B_n + \frac{1}{2}\psi^{(2)}_n \psi^{(1)}_n\right) + \\ & + ig\left(F_n(A_n^2 - B_n^2) + 2G_n A_n B_n + \psi^{(2)}_n (A_n - \gamma^5 B_n)\psi^{(1)}_n\right) \qquad (1)\end{aligned}$$

Here A,B,F,G are scalar fields and $\psi^{(1)}_n$ is a Euclidean Majorana-spinor: $\psi^{(2)} = \mathcal{C}\psi^{(1)}$ (however $\psi^{(2)} \neq \psi^{(1)+}\gamma^0$, cf. [4]). This Lagrangian is not hermitean but obeys OS-positivity.

∇_μ is the finite difference operator replacing the continuous ∂_μ. It is defined through

$$\nabla_\mu f_n := \frac{1}{(Na)^4} \sum_{k\in\mathfrak{T}} \frac{2\pi i}{Na} k_\mu \, e^{\frac{2\pi i n k}{N}} \tilde{f}(k) \qquad n \in \mathfrak{T}$$

$$\tilde{f}(k) := a^4 \sum_{n\in\mathfrak{T}} e^{-\frac{2\pi i n k}{N}} f_n \qquad k \in \mathfrak{T} \tag{2}$$

Under the Euclidean supersymmetry transformations (which in contrast to the relativistic case are no longer unitary)

$$\begin{aligned} \delta A_n &= \alpha^{(2)} \psi_n^{(1)} & \delta F_n &= i \alpha^{(2)} \gamma_E^\mu \nabla^\mu \psi_n^{(1)} \\ \delta B_n &= \alpha^{(2)} \gamma^5 \psi_n^{(1)} & \delta G_n &= i \alpha^{(2)} \gamma^5 \gamma_E^\mu \nabla^\mu \psi_n^{(1)} \end{aligned}$$

$$\delta \psi_n^{(1)} = - i \nabla^\mu (A_n - \gamma^5 B_n) \gamma_E^\mu \alpha^{(1)} - (F_n + \gamma^5 G_n) \alpha^{(1)} \tag{3}$$

($\alpha^{(1)}$ a Majorana spinor parameter, i.e. $\alpha^{(2)} = \mathcal{C}\alpha^{(1)}$), the action $S = a^4 \sum_{n\in\mathfrak{T}} \mathcal{L}_n$ is not invariant as it stands because ∇_μ does not obey the Leibniz product rule in general: $\nabla_\mu(f_n g_n) \neq \nabla_\mu f_n \cdot g_n + f_n \nabla_\mu g_n$. For an infinitely extended lattice this problem has been solved in [5]. Here, we give another prescription which leads to supersymmetric Schwinger-functions on the finite lattice ($\mathfrak{T}$,a). If the support of f,g in momentum-space is suitably restricted, i.e. supp $\tilde{f},\tilde{g} \subset \mathfrak{T}'$ for a sufficiently small sublattice $\mathfrak{T}'$ of $\mathfrak{T}$ *) the product rule is reinstated: it then holds for products of up to a certain number of such functions depending on the choice of $\mathfrak{T}' \subset \mathfrak{T}$. Also, one can show that

$$d\mu_{\mathfrak{T}',\mathfrak{T}}(A, B, \psi, F, G) := \tag{4}$$

$$:= \prod_{\ell\in\mathfrak{T}\setminus\mathfrak{T}'} \delta(\tilde{A}(\ell))\delta(\tilde{B}(\ell))\,\delta(\tilde{\psi}(\ell))\,\delta(\tilde{F}(\ell))\delta(\tilde{G}(\ell)) \prod_{n\in\mathfrak{T}} dA_n dB_n d\psi_n dF_n dG_n$$

is a measure invariant with respect to (3) and translations, concentrated on the set of functions with momentum-space-support restricted to $\mathfrak{T}' \subset \mathfrak{T}$. Thus, with suitable $\mathfrak{T}'$, the following expression defines supersymmetric Schwinger-functions

*) Of course, $\mathfrak{T}' \neq \phi$.

$$\langle \ldots \rangle_{(\mathcal{T},a)} = \frac{\int \ldots e^{-S}\, d\mu_{\mathcal{T}',\mathcal{T}}}{\int e^{-S}\, d\mu_{\mathcal{T}',\mathcal{T}}} \tag{5}$$

(a perhaps more suggestive way of realising the invariant action might be to write it down in momentum-space where the advantage of our choice of ∇_μ is more obvious).As the normalisation parameters one may take,for instance,

$$y_1 := a^4 \sum_{n\in\mathcal{T}} \langle A_n A_0 \rangle_{conn} \qquad y_2 := i a^4 \sum_{n\in\mathcal{T}} \langle F_n A_0 \rangle_{conn}$$

$$y_3 := a^8 \sum_{m,n\in\mathcal{T}} \langle F_m F_n A_0 \rangle_{conn} \qquad y_i \in \mathbb{R} \tag{6}$$

and the "renormalisation-map" $R(\mathcal{T},a)$ maps $(\alpha, m, g) \longrightarrow (y_1, y_2, y_3)$. (to be sure,the y_i are not identical with the truly physical parameters of the theory).Although (6) gives rise to a rather complicated $R(\mathcal{T},a)$ there are tremendous simplifications due to Ward-identities that follow from supersymmetry.From the present considerations it appears likely that in supersymmetric ϕ^3 there is no upper bound on the physical coupling - in contrast to the coupling constant behaviour of the purely bosonic models considered so far [3], [6].

References

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