Quark Bag Excitations, SL(3,R) Spectrum Generating Group and Vector States in e^+e^- Annihilation*

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Recently a new model for the structure of hadrons has been proposed. A strongly interacting particle is a finite region of space-time, called a "bag", to which fields are confined, in a Lorentz invariant way, by endowing the finite region with a constant energy per unit volume B[1]. Strong interactions are described by fractionally charged color quarks interacting with an octet of colored massless gauge vector gluons and the action integral is

$$W \doteq \int_{t_1}^{t_2} dt \int_R d^3x \ (\mathcal{L}(\text{quarks, gluons}) - B)$$

where the spatial region of integration extends ove a closed, finite part of space (the bag). The lowest-lying hadronic states correspond to the spherically symmetric bag, while the excited states are to be obtained through the bag surface shape quantization.

In this work we consider a spheroidal bag and make use of the symmetry approach to determine the allowed values of the internal orbital angular momentum, and the parity of excited states. Let us go to the bag fixed frame. First of all the dynamics is rotationally invariant, giving rise to the bag internal orbital angular momentum L. Furthermore, spheroidal bag is invariant with respect to the rotations about the bag symmetry axis as well, with the corresponding good quantum number K. Thus, one can describe internal orbital states by the D^L_{KM}-functions. Spheroidal bag is invariant under a discrete symmetry transformation R, which is a rotation of π about an axis perpendicular to the symmetry axis; to be specific we choose a rotation $R=R_2(\pi)$ with respect to the second axie. Now, the spheroidal bag wave function is of the form

$$\chi_K(q)D^L_{K,M}(\alpha\beta\gamma) + (-)^{L+K}\chi_{-K}(q)D^L_{-K,M}(\alpha\beta\gamma),$$

where α, β and γ are the Euler angles and q are the remaining coordinates. The state with K=0 can be labeled by the eigenvalue r

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of R, where $r=(-)^L$ and therefore the allowed values of the internal orbital angular momentum are:

$L = 0, 2, 4, \ldots$ for $K = 0, r = 1$,

$L = 1, 3, 5, \ldots$ for $K = 0, r = -1$.

When $K\neq 0$, there is only a constraint $L\geq K$, i. e.,

$L = K, K+1, K+2, \ldots$ for $K \neq 0$.

The bag dynamics is invariant with respect to the space reflection, and the allowed excited states can be labeled by the parity quantum number as well. In Particle Physics language we say that different L states of fixed K belong to the same Regge trajectory. Note that for K=0 one has the $\Delta L=2$ rule, and the states of same parity. The parity of meson states, i.e. of the quark-antiquark states is given by $(-)^{L+1}$.

It is easy to see, that the last term of the action integral W is invariant with respect to the SL(3,R) group of transformations. SL(3,R) is the group of rotations and constant volume deformations of the three-dimensional space. The ladder UIRs of SL(3,R) have the angular momentum content of the K=0 trajectories, while the representations of principal series of UIRs contain the K=0, K=2, K=4, ... trajectories[2)]. One can also obtain, by making use of SL(3,R), a linear m^2 vs. J spectrum,[3)] or a linear m^2 vs. L spectrum[4)]. Thus, we see that SL(3,R) is a spectrum generating group of a class of excited states in the bag theory of hadronic matter. It provide us with an algebraic method to determine the bag excited states, without solving the entire bag dynamics.

Several heavy vector mesons, $J^{PC}=1^{--}$, have recently been found in e^+e^- annihilation in the energy range 4÷5 GeV.[5)] These new meson states do not fit into the picture of SU(4) quark-antiquark states, i.e., into the corresponding SU(8)×O(3) model where the quark spin as well as the orbital excitations are taken into account. We postulate that the hadron spectroscopy is given by the SU(8)×SL(3,R) spectrum generating group, which contain as the symmetry subgroup SU(8)×O(3). The ladder UIRs of SL(3,R) and the SU(6) invariant two-body quark-quark interaction describe adequately the spectrum of ordinary mesons and baryons.[4)] It is straightforward to extend the SU(6) results to the SU(8) ones with the same success. Now, if we take instead of the ladder UIR a representation belonging to the principal series, there is an additional L=2 state of the K=2 trajectory. This new L=2 state coupled with the S=1 state of the total quark-antiquark spin, yields an additional SU(4) multiplet of vector mesons. We assoiate the neutral members of this multiplet with the new heavy mesons in the 4 GeV region.[6] After making SU(4) assignment of new states, we are in position to make a definite predictions about the mass differences, leptonic and total hadronic widths and so forth. The agreement with the experiment is very good.[6] We also evaluate, by making use of new duality for e^+e^- annihilation, the contribution of the new heavy vector mesons to the ratio $R=\sigma(e^+e^- \to \text{hadrons})/\sigma(e^+e^- \to \mu^+\mu^-)$, and obtain the total asymptotic value $R \approx 5$, in excellent agreement with experiment.

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