SPONTANEOUS BREAKDOWN OF SYMMETRY AND

THE GENERALIZED COHERENT STATES

M. Hongoh* and D. Matz

Department of Physics, University of Montreal

Montreal, Québec, H3C 3J7

Part I: Method of invariants and the coherent states associated with the continuous spectrum of noncompact groups[1].

So called SU(2) Van der Waerden invariant

$$W = \prod_i (\eta_j \xi_k - \eta_k \xi_j)^{a_i}, \quad i,j,k = 1,2,3 \tag{1.1}$$

may be extended to the noncompact group SU (1,1) by letting a_i be real or complex. The orthonormal basis functions are [2],

$$\begin{aligned} |\phi\alpha\rangle &= A_\alpha u^{\phi+\epsilon\alpha} d^{\phi-\epsilon\alpha} \\ \langle\phi\alpha'|\phi\alpha\rangle &= \delta(\alpha'-\alpha) . \end{aligned} \tag{1.2}$$

For $f^\alpha(z) \equiv |\phi\,\alpha\rangle$, we introduce the following function,

$$\mathcal{F}(w,z) = \int_{-(i/\epsilon)\infty}^{+(i/\epsilon)\infty} d\alpha\, f^\alpha(w)^* f^\alpha(z) \tag{1.3}$$

$\mathcal{F}(w, z)$ can be viewed as a scalar product of two basis vectors belonging to two distinct Hilbert spaces of IR characterized by 2ϕ. Our aim is to construct coherent states defined over $G/K \sim SU(1,1)/SO(1,1)$ as a certain linear combination of $f^\alpha(z)$. Consider a Hilbert space corresponding to an upper half of the hyperboloid lying along the first axis, and let z (u, d) belong to this Hilbert space. Without loss of generality we can set, $\{|u|^2 + |d|^2\} = 1$. Comparing (1.3) and (1.1) with $a_1 = a_2 = 0$, $a_3 = 2\phi$, we establish the correspondences.

$$(\eta, \xi)_1 \longleftrightarrow (z_1, z_2), \quad (\xi, \eta)_2 \longleftrightarrow (w_1, -w_2)^* . \tag{1.4}$$

* Work presented by M. Hongoh

The factor space may be projected onto a unit disc lying perpendicular to the first axis. Let w_1^*, $w_2^* \to \theta, \rho$ respectively, and further let $\rho \to 1$. We obtain

$$|\theta\rangle = \int_{-(i/\epsilon)\infty}^{+(i/\epsilon)\infty} d\alpha \left[\frac{\Gamma(-\phi+\epsilon\alpha)\Gamma(-\phi-\epsilon\alpha)}{2\pi\,\Gamma(-2\phi)}\right]^{\frac{1}{2}} \theta^{\phi+\epsilon\alpha} |\phi\alpha\rangle \quad (1.5)$$

i) non-orthogonality. Using the complex binomial expansion, we obtain

$$\langle\theta'|\theta\rangle = (1+\theta'^*\theta)^{2\phi} \quad (1.6)$$

where $|\theta'^*\theta| < 1$, $|\arg(-\theta'^*\theta)| < \pi$

In particular, the norm is $\| |\theta\rangle \|^2 = (1+|\theta|^2)^{2\phi}$.

ii) completeness. Let $|\tilde{\theta}\rangle = (1 + |\theta|^2)^{-\phi} |\theta\rangle$, then the completeness condition is

$$\langle f|i\rangle = \int d\alpha \langle f|\phi\alpha\rangle\langle\phi\alpha|i\rangle = \int d\sigma(\theta)\langle f|\tilde{\theta}\rangle\langle\tilde{\theta}|i\rangle \quad (1.7)$$

Writing $d\sigma(\theta) = \sigma(r)\, r dr d\varphi$, we obtain

$$\sigma(r) = \frac{r}{2}\,\delta(r), \quad r \geq 0 \quad (1.8a)$$

or

$$= \frac{r}{\sqrt{2}} \exp(-r^{-2}), \quad r \geq 0 \quad (1.8b)$$

Eqs (1.8), (1.5) and (1.7) completely specify the system of coherent states associated with the continuous spectrum of SU(1.1) algebra.

Part II: Some conjecture on Goldstone bosons

The path-integral formalism[3] is a powerful method used in the analysis of spontaneously broken symmetry. Basic equations are expressed in terms of the Heisenberg fields $\phi_H(x)$, while obserbed quantities correspond to states in the Hilbert space spanned by the asymptotic fields $\phi^{in}(x)$. Using this method, Matsumoto et al[4, 5] studied a spontaneous breakdown of SU(2) symmetry which is relevant to the ferromagnet in solid state physics. Adding an infinitesimal symmetry breaking term that breaks the symmetry in the third axis, they have shown that the generators of transformation for $\phi^{in}(x)$ form the E(2) algebra,

$$[D_1^{in}[f], D_2^{in}[f]] = 0$$

$$[D_2^{in}[f], D_3^{in}] = i D_1^{in}[f] \tag{2.1}$$

$$[D_1^{in}[f], D_3^{in}] = i D_2^{in}[f],$$

while $\phi_H(x)$ transforms according to the SU(2) algebra,

$$[D_i, D_j] = i C_{ijk} D_k \quad . \tag{2.2}$$

It is rather interesting to observe that E(2) can be obtained from SU(2) by group contraction with respect to its continuous subgroup O(2), and in this case O(2) is nothing but a stabilizer of the magnon ground state. Guided by the Wigner-Inönü theorem[6] on contraction we are led to the following conjecture[7].

<u>Conjecture</u>: Let G and K be a symmetry group of the fundamental dynamics, and a stabilizer of the ground state respectively. The symmetry at the observational level is then given by the contraction of G with respect to K, and the number of Goldstone bosons appearing as a consequence of the spontaneous breakdown of symmetry is equal to the number of generators of the invariant abelian subgroup formed by the contracted infinitesimal generators lying perpendicular to the direction of the symmetry breaking, i.e.

$$N_B \doteq N_G - N_K \tag{2.3}$$

It is also interesting to notice that the degenerate magnon ground state is nothing but the SU(2) coherent states ~ SU(2) / O(2), and it is most likely that the degenerate ground state in the general spontaneous breakdown of symmetry is given by the Bloch-type generalized coherent states of Perelomov[8].

REFERENCES

1. M. Hongoh, J. Math. Phys. to appear.

2. A.O. Barut and E.C. Phillips, Commun. Math. Phys. 8 (1968) 52.

3. H. Matsumoto, N.J. Papastamatiou and H. Umezawa, Nucl. Phys. B82 (1974) 45, B68 (1974) 236.

4. H. Matsumoto, H. Umezawa, G. Vitiello and J.K. Wyly, Phys. Rev. D9 (1974) 2806.

5. See G. Vitiello in this colloq.

6. E. Inönü and E.P. Wigner, Proc. Nat. Acad. Sci., 39 (1953) 510.

7. M. Hongoh, D. Matz, H. Matsumoto and H. Umezawa in preparation.

8. A.M. Perelomov, Commun. Math. Phys. 26 (1972) 222.