GAUGE FIELDS AND QUANTUM LIQUIDS

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We report some results on a gauge theory of superfluidity obtained in collaboration with J. Chela-Flores, R. Jánica-de-la-Torre, J. Rodríguez-Gómez, J. Rodríguez-Nuñez and R. Tascôn, all of them at Caracas, Venezuela. The detailed discussion as well as further achievements will appear in Refs. 1)-3).

In Ref. 4) Chela-Flores proposed a gauge theory for superfluid ^{4}He II. The Lagrangian density is

$$L=(-i/2)(\psi\partial_t\psi^*-\psi^*\partial_t\psi)-(1/2M)\left[(\vec{\nabla}+iM\vec{A})\psi^*\right]\cdot\left[(\vec{\nabla}-iM\vec{A})\psi\right]$$
$$-(U/2)|\psi|^4-(Mk/2)(\vec{\nabla}\times\vec{A})\cdot(\vec{\nabla}\times\vec{A}).$$

The Bose condensate is represented by the macrowave function ψ and the depletion by the gauge field $\vec{A}$. The velocity field of the condensate fluid is proportional to the gradient of the phase of ψ , while that of depletion fluid equals $\vec{A}$,[4] up to a constant .[2] The depletion density is an external variable. See also Refs. 5)-8).

Under the hypothesis that the gauge theory of superfluidity is correct, some of our results are the following:

(i) The Hydrodynamic which follows from that gauge theory is such that the density $(\propto|\psi|^2)$ of the condensate fluid is time-invariant[2].

(ii) Mixing of frequencies of the condensate macrowave function is not allowed[2].

(iii) The gauge invariance allows to express the depletion density as an internal variable[2].

(iv) The gauge transformations change the state of the system: The gauge field is a state vector variable[3]. Notice the contrast with electrodynamics, remarkable because L almost equals the Lagrangian density of charged Schrödinger field interacting with electromagnetic field[1]. Result (iv) is deduced in the frame of Dirac's theory of phase-space constrained systems[9)-11)] since L is singular.

(v) The action is invariant under translations and rotations. But the corresponding total linear and angular momenta are gauge dependent[3].

(vi) L depends on the parameters k and U in a non-singular way. However, the physical and formal properties of the system suffer strongly discontinuous changes under certain infinitesimal variations of these parameters[1].

(vii) The system has "memory" of the gauge field even when this field is absent[1].

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