Spontaneous breakdown of the gauge symmetry and observable phase operator

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The aim of this note is to discuss the connection between the spontaneous breakdown of the gauge symmetry of the first kind and the existence of an observable phase operator for a solvable model with non trivial interaction.

Refering to superfluidity, supraconductivity and laser physics, one expects that, if in a model involving Bose particles a second order phase transition occurs together with a spontaneous breakdown of the gauge symmetry, the phase be observable in the condensed thermodynamical equilibrium state ω. More precisely, in the algebraïc formalism of statistical mechanics, we mean that there should exist a unitary operator $\mathcal{U}$ (the exponential of the phase) in the Gelfand-Naïmark-Segal (G.N.S.) representation Π_ω of the C.C.R. algebra Δ associated with ω satisfying the two conditions :

i) $\mathcal{U}$ verifies the usual conjugation relation $[U,N] = \mathcal{U}$ (N ; particle number operator defined as the infinitesimal generator of the gauge transformations of the first kind in the representation Π_ω).

ii) $\mathcal{U}$ belongs to the weak closure $\Pi_\omega(\Delta)''$ of $\Pi_\omega(\Delta)$ ("Observability" condition).

The existence of such a $\mathcal{U}$ has already been shown in two situations corresponding to condensed systems of free bosons [(1), (2)].

As an example of the interacting case we consider the gauge invariant Dicke Hamiltonian :

$$H_{N,V} = \sum_{\underline{K},\sigma} |\underline{K}|\, a^+_{\underline{K},\sigma} a^-_{\underline{K},\sigma} + \varepsilon \sum_{i=1}^{N} S_i^z + \frac{\lambda}{\sqrt{V}} \sum_{i=1}^{N} \left(a^+_{\underline{K}_0,\sigma_0} S_i^- + a^-_{\underline{K}_0,\sigma_0} S_i^+ \right) \quad (1)$$

which describes the interaction of a system of spins $\{S_i\}$ with one mode $\{\underline{K}_0, \sigma_0\}$ of the electromagnetic field. Contrasting with a similar Hamiltonian studied by Hepp and Lieb [(3)], $H_{N,V}$ takes into account all the modes of the photon field; this point is essential for our purpose since a necessary condition for the existence of an $\mathcal{U}$ is the non-quasiequivalence of ω with a Fock state and this excludes the finite mode system [(1)]. (For the sake of definiteness, we state our results for classical spins; the purely quantum case is very similar [(4)]).

The equilibrium photon state at any inverse temperature β is the thermodynamical limit (th.lim : $N,V \to \infty$, $N/V = d$) of Gibbs states :

$$\omega(\delta_{\underline{f}}) = \text{th.lim}\left[\left(\mathrm{Tr}\, e^{-\beta H_{N,V}} \right)^{-1} \mathrm{Tr} \left\{ e^{-\beta H_{N,V}} e^{i A_V(\underline{f}_V)} \right\} \right] \quad (2)$$

In this formula $A_V(f_V)$ is the smeared field operator of the Fock representation in the volume V and $\delta_{\underline{f}}$ is a generating element of the photon algebra [2]. (In a given representation Π_ω, one has $\Pi_\omega(\delta_{\underline{f}}) = e^{iA_\omega(\underline{f})}$). The calculation of ω leads to [4] :

$$\omega(\delta_{\underline{f}}) = \omega_{th}(\delta_{\underline{f}})\, J_0\left(2\,\frac{(2\pi)^{3/2}}{\sqrt{2|\underline{K}_0|}}\;\frac{\lambda d S_0}{|\underline{K}_0|}\left|\tilde{f}_{\underline{\sigma}_0}(\underline{K}_0)\right|\right) \tag{3}$$

where ω_{th} is the thermal equilibrium state of a free photon field and S_0 is solution of the gap equation :

$$\coth\beta D_0 - \frac{1}{\beta D_0} = \frac{|\underline{K}_0|}{2\lambda^2 d}\, D_0 \qquad \left(D_0^2 = \varepsilon^2 + 4\,\frac{\lambda^4 d^2}{|\underline{K}_0|^2}\, S_0^2 \right)$$

Above the critical temperature T_c ($T_c = \beta_c^{-1}$) defined by $\coth\beta_c\varepsilon - \frac{1}{\beta_c\varepsilon} = \frac{|\underline{K}_0|}{2\lambda^2 d}\varepsilon$ we have $S_0 = 0$ and $\omega = \omega_{th}$. The G.N.S. representation associated with this factorial state is well known and there is no observable phase operator in this case.

Below T_c, S_0 is different from zero. The G.N.S. representation associated with ω is [4]:

- representation space : $\mathcal{H} = \mathcal{H}_F \otimes \mathcal{H}_F \otimes \mathcal{M}$ ($\mathcal{H}_F$: Fock space; $\mathcal{M}$: Hilbert space of square integrable functions on the unit circle);
- cyclic vector : $\Omega = \Omega_F \otimes \Omega_F \otimes \chi_0$ (Ω_F : Fock vacuum; $\chi_0(\alpha) = 1$ $\forall\alpha \in [0, 2\pi]$).
- field operator :

$$A_\omega(\underline{f}) = A_{th}(\underline{f}) \otimes \mathbb{1} + \mathbb{1} \otimes 2\,\frac{(2\pi)^{3/2}}{\sqrt{2|\underline{K}_0|}}\;\frac{\lambda d S_0}{|\underline{K}_0|}\left(\mathrm{Re}\,\tilde{f}_{\underline{\sigma}_0}(\underline{K}_0)\, C + \mathrm{Im}\,\tilde{f}_{\underline{\sigma}_0}(\underline{K}_0)\, S \right)$$

where $A_{th}(\underline{f})$ is the "thermal" field operator defined on $\mathcal{H}_F \otimes \mathcal{H}_F$ which appears above T_c and C and S are defined on $\mathcal{M}$ and such that :

$$C\,\chi(\alpha) = \cos\alpha . \chi(\alpha) \qquad\qquad S\,\chi(\alpha) = \sin\alpha . \chi(\alpha)$$

The state ω is no longer factorial and the center of the Von Neumann algebra $\Pi_\omega(\Delta)''$ is generated by a unitary operator $U = \mathbb{1} \otimes \mathbb{1} \otimes (C + iS)$ which satisfies the above two requirements of a phase operator. Moreover U appears to be an observable at infinity as expected for an intensive macroscopic variable.

One can rewrite formula (3) :

$$e^{\frac{1}{2}(\underline{f},\underline{f})}\,\omega(\delta_{\underline{f}}) = e^{\frac{1}{2}(\underline{f},\underline{f})}\,\omega_{th}(\delta_{\underline{f}})\; e^{\frac{1}{2}(\underline{f},\underline{f})}\int_0^{2\pi}\frac{d\theta}{2\pi}\,\omega^{\theta}_{\underline{K}_0,\underline{\sigma}_0}(\delta_{\underline{f}}) \tag{4}$$

where

$$\omega^{\theta}_{\underline{K}_0,\underline{\sigma}_0}(\delta_{\underline{f}}) = e^{-\frac{1}{2}(\underline{f},\underline{f})}\exp\left\{2i\,\mathrm{Im}\left(\frac{(2\pi)^{3/2}}{\sqrt{2|\underline{K}_0|}}\;\frac{\lambda d S_0}{|\underline{K}_0|}\;\tilde{f}^{*}_{\underline{\sigma}_0}(\underline{K}_0)\, e^{i\theta}\right)\right\}$$

is a generalized coherent state as introduced in ref.(5). In this expression the condensed photon state appears as the coherent superposition (in Glauber's terminology) of a thermal state and of an integral over the gauge group of states $\omega^{\theta}_{\underline{k}_0,\underline{\sigma}_0}$ which can be considered as states with definite phase θ (improper eigenstates of $\mathcal{U}^{(6)}$).

The connection between the existence of an observable phase operator below T_c and the occurence of the spontaneous breakdown of the gauge symmetry (obvious on formula (4)) is emphasized by the following result : in presence of an external field, the equilibrium state gets a definite phase (connected with the argument of the external field) and keeps it when this perturbation is turned off below T_c.

References

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