USE OF CODED APERTURES IN GAMMA-IMAGING TECHNIQUES

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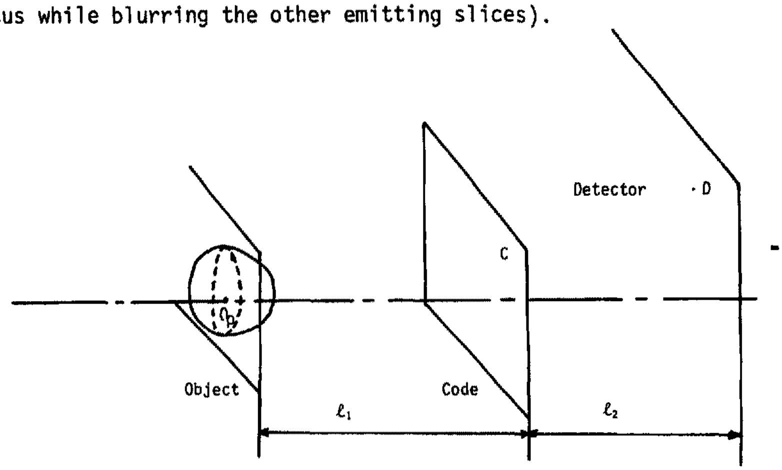
Recently some methods have been developped in the field of nuclear medicine and radiography. Transaxial tomography, multiple pinhole and coded apertures techniques are these main imaging processes which appeared in the last few years. Data processing, by numerical or analogic means, of one or several basic images is a common factor to all "these tomographic methods by reconstruction". Here we present some principles and fundamental considerations on the latest method : Coded Aperture Imaging.

I. Presentation of Coded Aperture Imaging

Coded Aperture Imaging (CAI) is a general imaging method which allows to synthetize a lens in wavelength domains where such an element cannot exist. CAI implies a two step imaging operation.

- In the first step or encoding step, a coded aperture is placed between the object and the detector. The plane containing the aperture is parallel to the detector and generally its transparency is binary. At this stage only, a shadow casting operation is performed. Each point of the object gives an image onto the detector which is the conical projection of the code from this point. The size of the projected pattern depends upon the distance ℓ_1 from the object point to the aperture plane, but also upon the distance ℓ_2 from the aperture to the detector. These different magnifications lead to a virtual tomographic capability (i.e the ability to reconstruct one object plane in focus while blurring the other emitting slices).



Coded aperture imaging ; encoding step

- The second step or decoding step allows to recover the informations on the three dimensional (3-D) emitting distribution from the coded pattern I(x,y). For a plane object, defined by a luminance function O(x,y), and for a code C(x,y), I(x,y) can be expressed as a convolution product defined by

$$I(x,y) = O(-x \frac{\ell_1}{\ell_2}, -y \frac{\ell_1}{\ell_2}) * C(\frac{\ell_1}{\ell_1+\ell_2} x, \frac{\ell_1}{\ell_1+\ell_2} y) \quad (1)$$

where * denotes the bidimensionnal convolution.
In the following the magnification factors $-\frac{\ell_1}{\ell_2}$ and $\frac{\ell 1}{\ell_1+\ell_2}$ are implied because we will first study the reconstruction of a plane object and next the ability of the preceeding processing th successfully restore the 3-D information, so that we can write :

$$I(x,y) = O(x,y) * C(x,y) \quad (2)$$

taking the bidimensionnal Fourier transform of (2), we obtain :

$$\tilde{I}(\nu,\mu) = \tilde{O}(\nu,\mu) . \tilde{C}(\nu,\mu) \quad (3)$$

The decoding of a planar object would consist of an inverse filtering (i.e obtained by dividing spectrum of the coded image by the spectrum of the code).
But such an exact deconvolution is very sensitive to noise, because, for real apertures, the functions $\tilde{C}(\nu,\mu)$ have bands of very small values. Dividing by $\tilde{C}(\nu,\mu)$ at these points will amplify mainly the noise and will damage the decoded result. For this reason we perform an approached deconvolution using a pseudo-Wiener filter.
Classical imaging can be related to this type of processing. Such an analogy gives us a very interesting approach when we will consider the overall modulation transfert function, the impulse response and the tomographic capability.

II. Lens imaging analogy

A lens imaging process (one stage) in coherent monochromatic light can also artificially be decomposed in a two step process which must be related to the preceeding technic. For that purpose we consider the complex amplitude U(x,y) just before the lens. If a(x,y) represents the complex amplitude of the object, U is given by a Fresnel transform

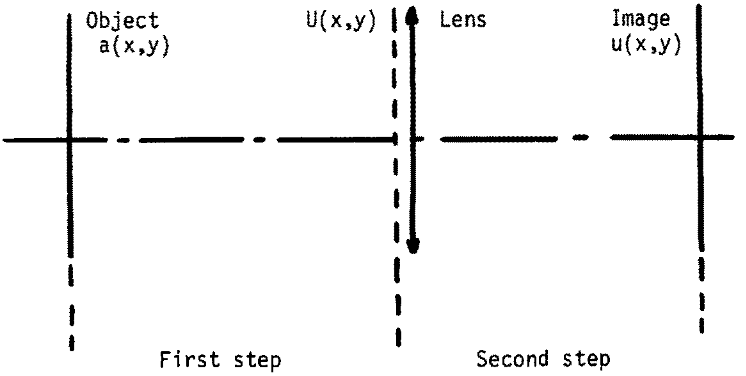
$$U(x,y) = a(x,y) * e^{ik\left(\frac{x^2+y^2}{2d}\right)}$$

Obtained by
- neglecting constant factor $\frac{e^{ikd}}{i\lambda d}$
- assuming $d_1 = d_2 = 2f$ (f focal length)

The second step includes the lens effect and the second Fresnel transform. It is possible to show that this second step is a deconvolution process (if we do not consider the limitation of the diffraction due to the finite extend of the pupil) because it performs a convolution between U(x,y) and $e^{-ik\left(\frac{x^2+y^2}{2d}\right)}$. When the system is diffraction limited the phase factor $e^{ik\left(\frac{x^2+y^2}{2d}\right)}$ which multiplies a(x,y), in the relation between the object and the image u(x,y).

$$u(x,y) = \iint_{-\infty}^{+\infty} h(x-x_0,y-y_0)\ a(x_0,y_0)\ e^{ik\frac{(x_0^2+y_0^2)}{2d}}\ dx_0 dy_0$$

does not change rapidly in the extend of the point spread function {1} and so that it can be neglected. In this case we can conclude that the second step performs a deconvolution with in addition a filtering by the complex amplitude $\tilde{h}(\nu,\mu)$ of the pupil of the lens.



Decomposition of a classical monochromatic imaging process in a two step process

Such an analogy is very useful in practice because it gives the conditions to make images of high quality. But it is not exactly possible to take the same way in coded aperture, because we cannot obtain directly complex or real negative function, so that the synthesis of a $e^{i\alpha(x^2+y^2)}$ coding needs 3 separate encoding steps.
The loss of the dynamic capabilities of single coded images (because no motion of the object, code and detector) makes the multicoding not very used in practice. In the following we present a single recording CAI method using an annular aperture with a pseudo-Wiener deconvolution.

III. Imaging problems in CAI

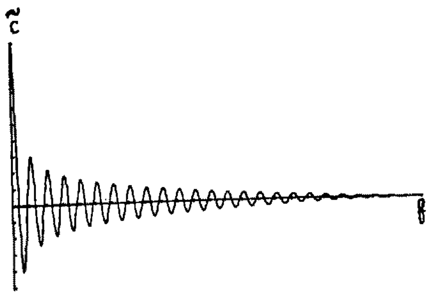
As we explained before it is better to perform an approached inverse filtering than an exact deconvolution. This can be achieved by replacing the inverse filter $\frac{1}{\tilde{C}(\nu,\mu)}$ by :

$$\frac{\tilde{C}}{\tilde{C}^2 + \varepsilon} \tag{4}$$

The aim of this pseudo-Wiener filter (called pseudo because the conditions of application of Wiener filtering are not satisfied for instance when the fluctuations are coming from Poisson noise processes) is to perform a selective rectification of the frequencies of O(x,y) diffeently attenuated by the encoding stage. Yet,this amplification occurs only when frequencies have been encoded in an important means (in the opposite case we will only amplify noise). As we can see, it is the parameter or "noise parameter" whiwh rules the operation. The filter (4) leads to a modulation transfer function (MTF)

$$\text{MTF} = \frac{C^2}{\tilde{C}^2+\varepsilon} \qquad (5)$$

The following figure represents the encoding transfer function for an annular aperture used in nuclear medicine.



$\tilde{C}$ = encoding transfer function

Moreover, the CAI method makes it possible to reconstruct slices situated in planes parallel to the code. An analysis in the Fourier and in the object domains has shown {2} that the defocused impulse responses kept a nearly constant energy. But the extension of these functions is all the more important as defocusing increases. Such a conclusion is related to the defocusing effect on the point spread functions obtained in classical imaging.

IV. Noise problems in CAI

There exists two types of noise in CAI.

- The first is related to detector noise. This type of fluctuations is very important in infra-red domain. The use of multiplex imaging {3} allows a gain in signal to detector noise ratio which can be related to the Fellgett's advantage in Fourier Spectroscopy.
- But X and γ-ray imaging photon noise coming from Poisson process is the main fluctuation.

  Compared with pinhole imaging for instance a large gain in collection efficiency G (maybe 100 or more) is achieved. But it leads to a gain in signal-to-noise ratio only for lemitting point. We can understand this singular result in the following way :

  . if n is the number of photons collected in pinhole imaging and coming from one pixel, the noise is $\sqrt{n}$ and the signal to noise ratio S/B = $\sqrt{n}$.

. In coded aperture the number of photons collected from the same point is Gn, but the noise does not come only from this point. In fact each other emitting point gives, at the reconstruction step, noise in the preceeding pixel. This influence cannot be evaluated in a simple way but it implies a loss in the signal-to-noise ratio compared with $\sqrt{Gn}$ (in fact $\sqrt{Gn}$ is obtained if there is one emitting point). We have shown (4) that in nuclear medicine configurations a gain of 2 or 3 nevertheless remains. Such a factor will not be important in laboratory experiments but it allows a reduction in exposition time from 4 to 9 in "in vivo" medical studies.

V. CAI applied to nuclear medicine

In nuclear medicine the classical images consist of parallel or conical projections of the 3-D distribution of the radiotracor injected to the patient. The superimposition of the activities of the slices is one of the main limitations in the diagnosis. CAI is one way which can be used to extract a certain 3-D information.
We have mainly applied this method in heart imaging. In this field the radioisotope used is 201 Thallium which emits γ rays of 70 keV. From the interpretation of the results we obtained, one can expect that CAI, which provides a contrast enhancement, will improve the sensitivity in detecting homogeneities of the tracor distribution. Also it increases the degree of accuracy in determining 3-D site and extent of the radioactivity defects corresponding to necrotic or ischaemic tissues. On the other hand, if the ECG (electrocardiogram) signal is used to drive the gamma camera, so that photons are registered during the sytole or the diastole, it is possible to obtain tomographic images, that allows more accurate evaluation of dimensions of the carties and of wall thickness. The following pictures show the comparison between classical projections and four reconstructed slices in a patient study made in the Nuclear Medicine Division of Hospital Cochin in Paris.

Conclusion

As we say at the begining, coded aperture imaging is a general method not limited to nuclear medicine. It can be used in several domains such as neutrons, α particles imaging. We begin to develop it in microimaging for laser induced plasmas diagnosis. The first results that we obtained in this field show a resolution of 10 μm and we hope rapidly a gain of 2 to 4 with regard to this number.

References

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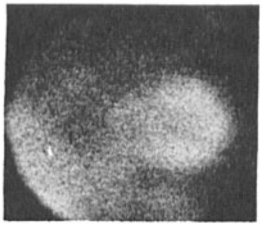

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M^{r} K... Abraham Coded Aperture Imaging
Reconstruction of 4 slices separated by 8mm.