ELECTRON SOURCES

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1 - NUMBER OF MODES IN AN ELECTRON BEAM, COHERENCE

If an electron beam is limited by an aperture of size dS perpendicular to the mean ray and if v is the speed of the electrons, the wave front sweeps over the volume $V = vdSdt$ in time dt. The number g of modes available in the phase volume $d\omega p^2 dpV$ $(E = p^2/2m)$ results from Fermi-Dirac statistics (a factor 2 for the spin) :

$$g = 2\,\frac{p^2 dp d\omega v dS dt}{h^3} = 2\,\frac{d\omega dS}{\lambda^2}\,\frac{dt dE}{h} = \frac{4m}{h^3}\,dS d\omega E dE dt \qquad (1)$$

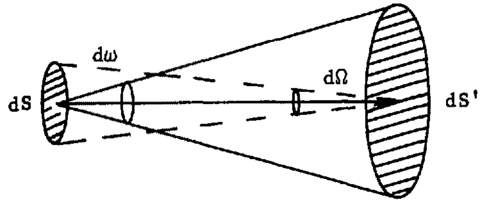


Fig.1 Extent of an electron beam

By analogy with optics[1] we can write the second equality of equation (1) as :

$$g = 2\,\frac{d^2U}{d^2U_1}\,\frac{dt}{\tau} \qquad\qquad d^2U = d\omega dS = d\Omega dS'$$

$d^2U_1 = \lambda^2$ and $\tau = h/dE$ define respectively the extent of the spatial and temporal coherence. Typically $E = 100$ keV in electron microscopy ($\lambda = 0.037$ Å) and $dE \simeq 1$ eV which gives $d^2U_1 = 1.37\ 10^{-3}$ Å^2 and $\tau = 4.14\ 10^{-15}$ s. Total spatial coherence can only be realized in a beam of circular cross section $dS = \pi d_\perp^2/4$ and half aperture α $(d\omega = \pi\alpha^2)$ if $d^2U_1/d^2U \gg 1$, which means :

$$\alpha \ll \frac{2}{\pi}\,\frac{\lambda}{d_\perp} \quad (\alpha < 10^{-4} \text{ rad for } d_\perp = 240 \text{ Å})$$

Associated with τ is the longitudinal coherence length $d_{//} = v\tau = 6800$ Å with $E = 100$ keV and $dE = 1$ eV.

The third equality in equation (1) determines in fact the maximum "theoretical" intensity of the beam (i_{max} = eg/dt) or the theoretical limit in brightness (luminance)

$$B_{max} = \frac{i_{max}}{dSd\omega} = \frac{4em}{h^3} E.dE = 5.2\ 10^9\ VdV\ (A\ cm^{-2}\ sr^{-1})$$

if V is expressed in volts. This is an enormous value ($5.2\ 10^{14}$ with V = 100 kV and dV = 1 V) as compared to the actual best experimental realisations using field emission sources (< 10^{10} A cm^{-2} sr^{-1}). This means that the occupation numbers of the available phase cells are in fact very small (one in 10^5 or so in the extreme experiments) hence as noticed many years ago by D. Gabor(2) the "almost complete identity of light optics and electron optics, in spite of the extreme difference between Einstein-Bose and Fermi-Dirac statistics".

2 - BRIGHTNESS

Whatever the emission process at a metallic cathode (as thermionic, field emission T.F. emission, ...) it is easy to compute the number of electrons G (E_n, E_t) dE_n dE_t emitted by unit area of a plane cathode by unit time whose normal (to the cathode) and transverse energies are in the range dE_n and dE_t respectively. From the distribution G it is possible to compute :

- mean energies $< E_n >$ and $< E_t >$

- density of emitted current $J_c = \iint G\ (E_n, E_t)\ dE_n\ dE_t$

- axial brightness of the beam $B_c = \lim_{\substack{dS \to 0 \\ d\omega \to 0}} \frac{dJc}{d\omega}$ at the cathode

In the limit dS, $d\omega \to 0$, it is easy to see that $d\omega = \frac{\pi dE_t}{E_n}$ and $E_t \to 0$ so that :

$$B_c = \frac{e}{\pi} \int G\ (E_n, 0)\ E_n dE_n = \int B_c\ (E_n)\ dE_n$$

where B_c (E_n) represents the contribution to the cathode brightness of electrons whose normal energies are in the range dE_n.

Consider now a "cross-over" where at some distance M from the cathode the beam converges due to some lens effects (Figure 2).

The electrons are coming from a zone dS_c, $d\omega_c$ at the cathode. From Liouville's theorem :

$$(dx_1\ dx_2\ dx_3\ dp_1\ dp_2\ dp_3)_M = (dx_1\ dx_2\ dx_3\ dp_1\ dp_2\ dp_3)_C$$

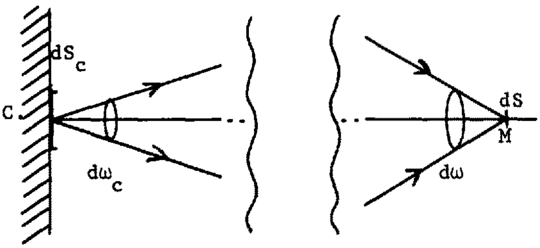


Fig. 2 Relation between cathode C and cross-over M

In an electron gun the beam is accelerated up to a potential V at M (relative to the cathode) so that $(E_n)_M = E_n + eV$ or $p_3\ dp_3$ = constant. We also have $dx_3 \sim p_3 dt$ and $d\omega = dp_1\ dp_2/p_3^2$ so that Liouville's theorem gives :

$$dS_M\ d\omega_M\ (E_n)_M = dS_M d\omega_M\ (E_n + eV) = dS_c\ d\omega_c\ E_n$$

Hence the differential brightness at M :

$$\frac{B_A\ (E_n)}{B_c\ (E_n)} = \frac{(di/dSd\omega)_M}{(di/dSd\omega)_C} = \frac{E_n + eV}{E_n}$$

A result first deduced by Gabor[3]. Finally we get the total brightness B at A[4]

$$B = \int B_A\ (E_n)\ dE_n = \frac{e}{\pi} \int (E_n + eV)\ G\ (E_n,\ 0)\ dE_n \qquad (2)$$

As soon as the beam enters an equipotential space after the gun, V = constant and the brightness of the beam is conserved ($dSd\omega$ = constant).

3 - THERMIONIC EMISSION

Thermionic emission is the most extensively used in electron microscopy. It's characteristics are well known and can be deduced easily from the preceding discussion.

Since the work function ϕ (4.5 volts for W) $\gg$ kT, it is easy to get :

$$G\ (E_n,\ E_t) = \frac{4\pi}{h^3}\ m\ \exp - \frac{(\phi + E_n + E_t)}{kT}$$

$$< E_n > = < E_t > = kT$$

$$J_c = \frac{4\pi em}{h^3}\ (kT)^2\ e^{-\frac{\phi}{kT}} \quad \text{(for W : 1 A cm}^{-2}\text{ at 2650 K)}$$

$$B = \frac{J_c}{\pi} \left(1 + \frac{eV}{kT}\right) \qquad (3)$$

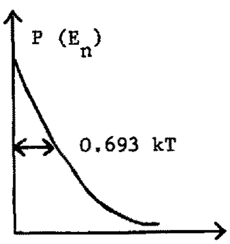
a result first deduced by Langmuir[5].

It is also easy to determine the normal energy distribution $P(E_n)$, $P(E_n)\, dE_n$ giving the fraction of electrons whose normal energy is the range dE_n.

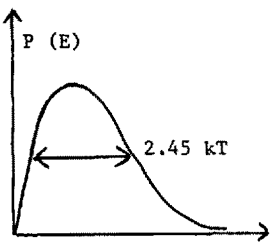
$$P\,(E_n) = \frac{\int G\,(E_n\, E_t)\, dE_t}{\int G\,(E_n\, E_t)\, dE_n\, dE_t} = \frac{1}{kT}\, e^{-\frac{E_n}{kT}}$$

The total energy distribution $P(E)\, dE$ whose total energy is in the range dE :

$$P\,(E) = \frac{E}{(kT)^2}\, e^{-\frac{E}{kT}} \qquad (4)$$



(a) normal energy



(b) total energy

Fig. 3 Energy distributions for thermionic emission

The half width of $P(E)$ is equal to 2.45 kT. In an electron microscope the thermionic cathode is usually made from tungsten wire bent into the shape of a hairpin and emitting only at the tip. From these cathodes heated to 2650 K current densities $J_c = 1$ A cm^{-2} can be drawn from an area of 0.1 mm diameter. The emitted current is controlled by a grid, namely the Wehnelt cylinder surrounding the cathode and biased negatively some - 200, - 300 V relative to the cathode. The electron beam is then accelerated by the anode to a potential V relative to the cathode.

Electrons proceed approximatively parallel to the axis, spreading due to the tangential componant of emission energies. As the Wehnelt has a repulsive action on the emitted electrons, the first part of the gun has a strong converging action. The last part near the anode and its aperture acts as a diverging lens (Figure 4).

Each point of emission gives electron bundles whose peripheral rays leave the cathode surface at grazing angles. The different bundles converge as shown in the figure into a disc of least confusion which is called the cross-over. Each point of the cross-over emits a pencil of rays so that it is the cross-over itself which must be considered as the electron source.

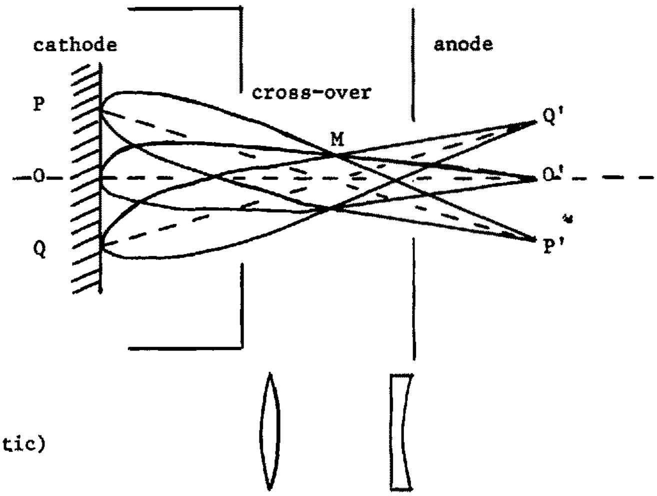


Fig. 4 Thermionic gun (schematic)

It is possible to show[4] that the brightness decreases in the cross-over exponentially with increasing distance r from the axis as :

$$B(r) = B \exp\left(-\frac{r^2}{M^2 a^2}\right)$$

where a is the cathode tip radius, M the gun magnification and B the axial brightness given by (3), $B \sim eVJ_c/\pi kT$ in usual conditions. The radius of the cross-over (Ma) is usually around 25 μm. The maximal axial brightness is around 10^5 A cm^{-2} sr^{-1} at an accelerating voltage V = 100 kV. The average directional beam intensity defined as $di/d\omega$ is around 10^{-2} A sr^{-1} in these conditions.

4 - FIELD AND THERMAL FIELD EMISSION (T.F.)

High electric fields applied to the cathode allow electrons to tunnel directly from energies near the Fermi Level into the vacuum. Typically the field F must be as high as 4.10^7 V cm^{-1} which is only possible with a small tip radius (around 1000 Å). The emissive power J_c is now in the range 10^4 A cm^{-2} and is very sensitive to field fluctuations. Typically a 1 % increase in F produces a 10 % increase of J_c and a 1 % increase in the work function ϕ produces a 15 % decrease of J_c. These fluctuations result from adsorption/desorption processes from residual gases and desorption of the extracting anode under electron bombardement. For this reason the tip is very often heated to a moderate temperature (1300 K) to stabilise these effects and regulate the stability of the emission. We now have[6] :

$$G\,(E_n\,E_t) = \frac{4\pi m}{h^3}\,\frac{\exp\{-c + (E_n - E_f)/d\}}{1 + \exp\{(E_n + E_t - E_f)/kT\}}$$

That is to say the Fermi-Dirac distribution multiplied by the probability of penetration of the barrier (the numerator). The energy d corresponds as we shall see to the mean transverse energy : $d = \langle E_t \rangle$; d varies varies with the field F as :

$$d(\text{eV}) = 9.76\;10^{-9}\;F\;(\text{volts})/\phi^{1/2}$$

typically $d = 0.25$ eV for a field $F = 5.10^7\ \text{V cm}^{-1}$

$$\langle E_t \rangle = d \tag{5}$$

$$\langle E_n \rangle = E_F - d - \pi\,kT\,\text{cotg}\,\left(\frac{\pi kT}{d}\right)$$

$$J_c = \frac{4\pi em}{h^3}\,d^2\,e^{-c}\,\frac{\frac{\pi kT}{d}}{\sin\left(\frac{\pi kT}{d}\right)} = J_{co}\,\frac{\frac{\pi kT}{d}}{\sin\left(\frac{\pi kT}{d}\right)} \tag{6}$$

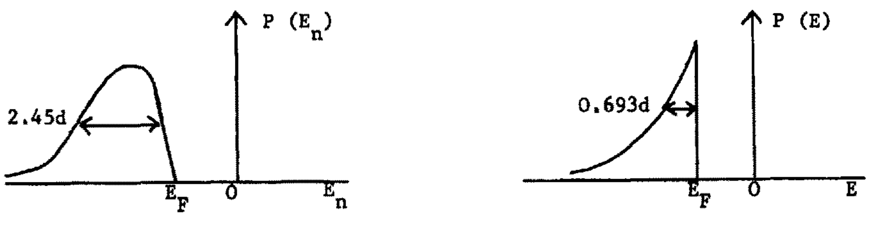
(for W, $J_{co} = 10^4\ \text{A cm}^{-2}$ for $F = 4.10^7\ \text{V cm}^{-1}$ and $10^5\ \text{A cm}^{-2}$ for $F = 4.5\;10^7\ \text{V.cm}^{-1}$)

$$B_c = \frac{J_c}{\pi}\left\{\frac{E_F}{d} - \frac{\pi kT}{d}\,\text{cotg}\,\left(\frac{\pi kT}{d}\right)\right\}$$

$$B = \frac{J_c}{\pi}\,\frac{eV}{d} + B_c \simeq \frac{J_c}{\pi}\,\frac{eV}{d}\ \text{in practical situations.} \tag{7}$$

At 0°K the energy distributions take a simple form :

$$P\,(E_n) = \frac{(E_F - E_n)}{d^2}\,e^{(E_n - E_F)/d} \quad \text{and} \quad P\,(E) = \frac{1}{d}\,e^{(E - E_F)/d} \tag{8}$$



(a) normal energy (b) total energy

Fig. 5 Energy distributions for field emission

Thus d in field emission plays the role of kT in thermionic emission. Total energy distribution P (E) in field emission has the same form and width as the normal energy distribution P (E_n) in thermionic emission[7].

At a finite temperature the total current density emitted with total energy between E and E + dE may be written (J_c and J_{co} as defined in equation (6)) :

$$J_c\ (E)\ dE = P\ (E)\ J_c\ \ dE = \frac{J_{co}}{d}\ dE\ .\ \frac{\exp\ (E - E_F)/\ d}{1 + \exp\ (E - E_F)/\ kT}$$

As compared to the total current density emitted in thermionic emission :

$$J_c^{th}\ (E)\ dE = \frac{4\pi em}{h^3}.E.dE\ \exp - (E + \phi)/\ kT$$

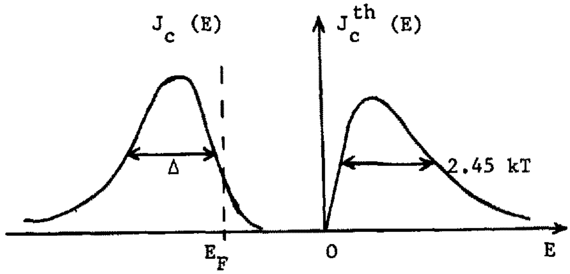


Fig. 6 Total current density at a cathode for field (left) and thermionic emission (right)

Values of the half width Δ/d are given[8] as a function of kT/d. The analytical treatment given here is exact only if kT/d < 0.8 which means T < 1800 K for W with a field F = 4.10^7 V cm^{-1}. The condition is thus always satisfied in practical situations.

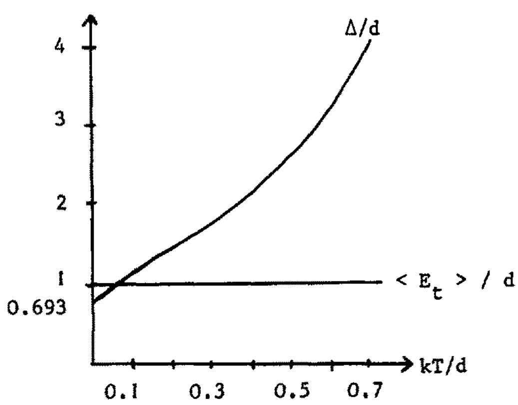


Fig. 7 Half width (Δ) and mean transverse energy (< E_t > of field emission as a function of temperature

Typically with a field $F = 4\ 10^7\ V\ cm^{-1}$, $d = 0.18$ eV ; $\Delta = 0.13$ eV at 0°K ; 0.23 eV at 300 K and 0.72 eV at 1300 K a temperature often used to stabilize the emission. For comparison 2.45 kT = 0.59 eV for half width of thermionic emission at 2750 K. In practice a field emission gun often works with a pointed cathode of small radius a (≃ 1000 Å) at a small distance from an extracting anode held at some positive potential $V_1 \simeq a\ F$ where F is the electric field at the cathode. The cross-over radius is[9]

$$r_c \simeq a\ \left(\frac{< E_t >}{V_1}\right)^{1/2} \simeq a^{1/2}\ \left(\frac{< E_t >}{F}\right)^{1/2} \qquad (9)$$

Typical values are $< E_t > = 0.2$ eV, $F = 4.10^7$ V/cm, a = 1000 Å and $r_c \simeq 22$ Å. Source size is thus approximately four orders of magnitude lower than conventional thermionic one. Similarly axial brightness is approximately four orders of magnitude higher. One difficulty however is the poor directivity of field emission, 10^{-4} $A.sr^{-1}$ at the best as compared to 10^{-2} $A.sr^{-1}$ or so for thermionic emission so confinment processes are often used (oxygen processing[10], build-up[11], ...).

5 - ENERGY BROADENING IN ELECTRON BEAMS

Boersch[12] was the first to observe an increase in the energy spread of an electron beam at high current densities. Simultaneously, the Maxwellian energy distribution changes to an apparent Gaussian distribution. These observations have been verified repeatedly, generally with thermionic guns and recently with a field emission cathode[8]. In this later case, for instance with an emission current of 0.3 µA an energy half width of 0.24 eV was measured. The half width reaches 0.4 eV for an emission current of 4 µA (at room temperature). A theory by Loeffler[13] was based upon an analysis of electron-electron collisions at a cross-over and predicts a broadening :

$$\Delta E \propto \frac{1}{\alpha}\ \left(\frac{I}{r}\right)^{1/2} \qquad (10)$$

where α is the angle of convergence, I the current and r the radius of the beam at the cross-over. However, it seems that such a law is not experimentally satisfied. Recently Crewe[14] proposed for ΔE a double-valued function of α with a maximum effect at some critical angle of convergence. This law has however not been compared to experimental results mainly due to difficulties in defining cross-over parameters in the electron gun. This situation is rather unfortunate since the energy spread of the illuminating beam attenuates the contrast transfer function in electron microscopy. Improvement of resolution limit can thus be reached only at low beam current not exceeding a few µA. Recently Troyon[15] measured the contrast attenuation due to the different envelope of the transfer function and deduced from these measurements the energy spread half width ΔE of a field emission electron microscope. At about 5 µA $\Delta E = 1.4 \pm 0.3$ eV and at about 100 μA $\Delta E = 3 \pm 0.6$ eV, the corresponding brightness being at 75 kV respectively 7×10^{7} A cm^{-2} or $^{-1}$ and 2.5×10^{8} A cm^{-2} or $^{-1}$.

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