X-RAY SOURCES

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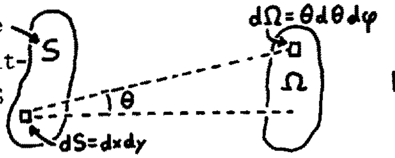
Introduction

We speak of "conventional" continuous sources (X-ray tubes) and synchrotron sources of "hard" X-rays (= that pass through Be window, $\lambda < 3$ or 4 Å) or "soft" X-rays.

"incoherent" source (phase of field emitted by different dS uncorrelated



Power emitted

$$P=\int b(x,y,\theta,\varphi,\nu)\,dS\,d\Omega\,d\nu \simeq bS\Omega\Delta\nu$$

We call b "spectral brilliance", $b\Delta\nu$ "brilliance", $bS\Delta\nu$ "brightness", etc... (remark: other authors call "brightness" what we call "brilliance", etc....)
ΩS area of (4-dim.) phase space, independent of distance and focusing (Liouville).
Spectral and angular distribution might be linked: then in general S, Ω, ν not independent. Which are the most important parameters depends on the particular experiment (consider also time structure and polarization).
For an observer at distance R, irradiated area $\sim \Omega R^2 + S$
Practical unit conversions: $h\nu$ (keV)=12.4/λ (Å) ; γ = 1957 E (GeV)
number of phot/sec = $5\cdot 10^{14}$ power (Watts)·wavelength (Å).

X-ray tubes [8]

Characteristic X-rays: narrow-band spectrum ($\Delta\nu/\nu \sim 0.5\cdot 10^{-3}$) due to rearrangement of atomic electrons after K-level ionization by the electron incident on the anode. K_α lines: L → K transitions (strongest); K_β lines: M → K.
(energy a little bit lower than K absorption edge)
Empirical formula:
(E energy of electron,
E_k energy of K level; $E > E_k$
Z atomic number of anode)

$$\frac{\text{n.photons}}{\text{n.inc.electrons}} = 3\cdot 10^{-4}\left(\frac{E-E_k}{1\ \text{KeV}}\right)^{1.63} e^{-0.095Z}$$

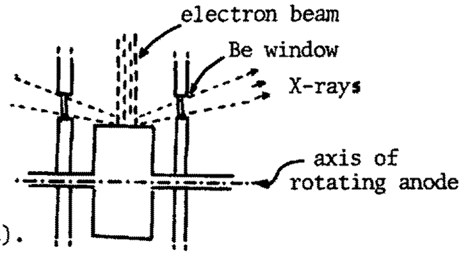
To find photons per unit $d\Omega$ at a given "take-off" angle α from anode surface, multiply by $1/4\pi$ and by a factor ($\lessgtr 1$) higher for higher α and Z and for lower E (=0.5 for $\alpha = 6°$, 50 KeV electrons on Cu). (Lambert's law is not valid).

Bremsstrahlung: radiation due to short and intense accelerations of the incident electrons due to microscopic e.m. fields of atoms of anode: random shoot pulses give a white spectrum up to an energy E (electron K.E.). In a thick target, electrons gradually (on the average) slowed down: spectrum goes linearly to zero at $\nu = E/h$).

For $E \ll 500$ keV ($\gamma \ll 1$) isotropic radiation; for $\gamma \gg 1$ radiation peaked forward, within cone of aperture $\sim 1/\gamma$.
Semi-empirical formula for total Bremsstrahlung energy:

$$\frac{\text{X-ray energy}}{e^- \text{ beam energy}} = kZE \qquad \text{with } k \simeq 10^{-9}\ \text{eV}^{-1}$$

Power limitations are due to the heat produced on the anode by the electron beam[6]. For stationary target: max.power that anode can withstand is proportional to thermal conductivity, to melting temperature minus room temp., and to linear dimensions of spot. (Then power per unit area (then brilliance) is inversely proportional to linear dimensions of anode spot). For 0.1 mm x 1 mm spot, ~ 200 W. For rotating anode it is proportional to square root of velocity of anode surface. For 1 cm ϕ anode, 2500 r.p.m. it can be 6 to 12 times higher. (up to 10000 r.p.m. used).
Spot usually segment ax10a (1x10 mm; or 0.1x1 mm), and it is viewed at a "take-off" angle 6° (0.1 rad), so that it appears of dimensions axa. The Be windows (usually 2) at a distance of a few cm, transmit a solid angle of the order of 0.1 rad.
Anodes most commonly used (good thermomechanical characteristics):
Cu (K_α: λ=1.54 Å; a Ni absorber can filter out K_β and Bremsstrahlung)←
Mo (K_α: λ=0.71 Å, a Zr absorber can filter out).
Al (K_α: λ=8.3 Å)



Synchrotron radiation

is radiation from ultrarelativistic electrons in magnetic fields (in practice, macroscopic electric fields are weaker than magnetic fields: for $\gamma \gg 1$, a 300 V/cm electric field is equivalent to 1 G magn. field, and static magn. fields of 50 KG can be produced).
For $\gamma \gg 1$, radiation sharply peaked forward ($\theta \lesssim 1/\gamma$ around direction of electron velocity). The "amplitude" $U(t) = \left(\frac{dP(t)}{d\Omega}\right)^{\frac{1}{2}}$ can be calculated by Liénard's formula (= Green's function of Maxwell's equations) and can be written $U(t) = C_o \gamma^3 B f$, where $f = (1+\gamma^2\theta^2)^{-3}\left[(1-\gamma^2\theta^2) + 4\gamma^2\theta^2\sin^2\varphi\right]^{\frac{1}{2}}$ angular distribution (=1 for θ=0), and f and magnetic field B must be evaluated at the time t' when the field has been emitted: $t = t' + R(t')/c$ (R distance of observer), and $C_o = 1.74\cdot 10^{-7}\ \text{Kg}^{-1/2}\,\text{m}\,\text{sec}^{1/2}\,\text{Cb}$
To calculate spectrum:

$$\left(\frac{dP(t)}{d\Omega}\right)^{\frac{1}{2}} = U(t) \xrightarrow{\text{Fourier Trans.}} U(\nu) \xrightarrow{\text{mod.sq.}} |U(\nu)|^2 = \frac{dP}{d\Omega\, d\nu}$$

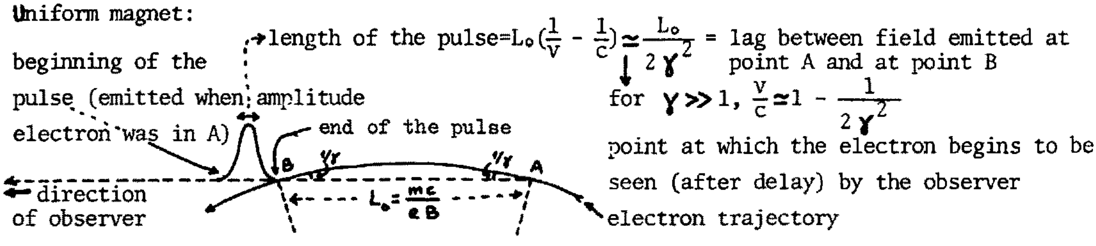
(in general it depends both on $f(\theta(t'))$ and on $B(t')$).

The total power emitted by an electron beam in a magnet is

$P = 4.22 \cdot IBE^3$ Watts per milliradian of orbit (B in Tesla, E in GeV, I in Ampères)

and most of it is within an angle $\pm 1/\gamma$ from the orbit plane.

Uniform magnet:



Undulator: periodic field (period λ_o): same raisonment but observer continuously "illuminated"

1) Uniform magnet. The observer is swept by the e.m. beam (of angular distribution $f(\theta)$) and sees a pulse of duration $\tau_c \sim \frac{m}{eB\gamma^2}$ and then a continuous spectrum extending up to a "critical frequency" $\nu_c \sim \frac{3}{4\pi\tau_c} = \frac{3eB\gamma^2}{4\pi m}$; while the electron moves on an arc of trajectory of length $L_o \sim \frac{mc}{eB}$ (if B is uniform over the distance L_o).
The number of photons emitted per second, per 0.1% bandwidth, per milliradian of orbit, integrated over the vertical (out of orbit) distribution (of width $\sim 2/\gamma$) can be written[2] (I in Ampères, E in GeV, B in Tesla):

$$\frac{dN}{dt}\,(0.1\%,\ \text{mrad},\ \textstyle\int \text{vert}) = 1.6 \cdot 10^{13}\ IE\, g(\lambda/\lambda_c), \qquad \lambda_c = \frac{18.64}{BE^2}$$

where $g(\lambda/\lambda_c)$ is a function plotted in fig.1, which has a maximum of 1 at $\lambda/\lambda_c \sim 3$ and decreases exponentially as $e^{-\lambda_c/\lambda}$ for $\lambda < \lambda_c$ ($\sim \frac{1}{25}$ at $\lambda_c/\lambda = 5$).
For example, if E = 1 GeV, B = 1 T, $L_o \simeq 1.5$ mm, $\lambda_c = c/\nu_c = 18.6$ Å.
In the plane of the orbit the radiation is linearly polarized in that plane, while out of that plane it is elliptically polarized.
The divergence of the photon beam will be $\sim(\sigma'^2 + 1/\gamma^2)^{\frac{1}{2}}$ where σ' is the divergence of the electron beam. If $\sigma' \ll 1/\gamma$, the spectrum depends on the angle from the plane of the orbit (the hardest components being near $\theta = 0$).

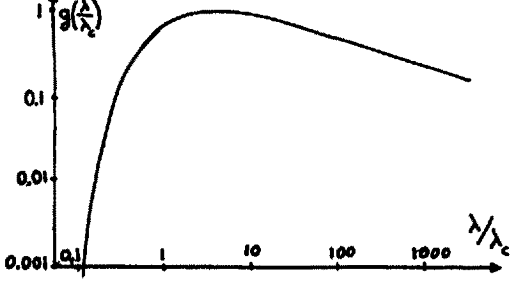


fig. 1: Normalized synchrotron radiation spectrum

2) Undulator. If the magnetic field B is "non uniform" (over a distance L_o) the spectrum depends also on the form at B(Z). With a sinusoidal B(Z) (period λ_o) the radiation emitted could be composed of a narrow ($\nu/\Delta\nu \sim$ number of periods) band at wavelength $\lambda_o/2\gamma^2$ (if $\lambda_o \ll L_o$) or several narrow bands. But the frequency emitted is strongly dependent on the angle of emission, then the possibility of separating narrow bands depends on the angular spread of the electron beam: to get a bandwidth $\Delta\nu$, we must have $\sigma' < (\Delta\nu/\nu)^{\frac{1}{2}}/\gamma$ (then the angular spread must be $\ll 1/\gamma$).
While in a bending magnet the power is spread in the orbit plane, here it is all concentrated within a solid angle $\Omega_o \sim \pi/\gamma^2$.
With suitable B(Z) any polarization could be produced: horizontal, vertical, or circular (with a helical field). In any case, in a given gap h (or diameter of tube), the shortest period λ_o obtainable with appreciable magnetic field is $\sim$ h. For example, with a 2 cm gap, to reach $\lambda = 1$ Å the energy should be at least 5 GeV.
If a pinhole selects a band $\Delta = \Delta\nu/\nu$, spectral brilliance in case $\lambda_o > L_o$ at a peak in spectrum is $\sim N/\Delta$ times higher than from a uniform magnet (same B, N=n. of periods).
For strong field and large angular spread: spectrum $\sim$ "usual": this usually called "wiggler".
Another interesting case (but not for X-rays):
"Edge effect": shift to higher frequencies of spectrum from edge of magnet ($<L_o$). Can be used to see image in visible light of 300 GeV proton beam[7].
Storage rings (= synchrotrons where, particularly by use of a good vacuum (10^{-9} torr) the electron beam can be stored for a time $\sim$10 hours at constant energy, with r.f. cavities compensating at every turn the energy lost by synchr.radiation) are generally used as synchrotron sources because they work at constant energy, with high current and a stable beam.
At present most sources are designed for high energy physics, and not optimized for X-rays. Further developments:
1) "Wigglers": magnets, producing zero net deflection, inserted in straight sections of machine, which have locally a higher magnetic field (up to 6T if superconducting, to 2T if normal) than in bending magnets, possibly multiple-pole to add intensities: they allow a given machine to get lower λ_c (2 GeV machine can get $\lambda_c = 1$ Å).
2) Dedicated machines: with high currents (up to 1 A), low emittance (beam size x divergence), energy 2 GeV for X-rays, and 800 MeV for soft X-rays and vacuum UV.
3) (in the future): undulators, 50 or 100 periods; increase in spectral brilliance over small solid angles.

Pulsed sources

We cannot speak also of pulsed sources, but remark that:
1) Instantaneous power and brilliance from a pulsed X-ray tube can be 2 or 3 orders of magnitude greater than continuous ones.
2) In synchrotrons, instantaneous power and brilliance are 10^2 to 10^3 times higher than the average values (pulse duration $\sim$0.3 to 2 nsec, separation $\sim$0.1 to 1 μsec.)

3) Very interesting soft X-ray pulsed sources (particularly C K_α , $\lambda \simeq 40$ Å) (plasma in a capillary, bombarded by e^- beam (10^{-7} sec) or by pulsed laser (10^{-9} sec)) (see D.Sayre's talk in this workshop).
Can give 10^{15} phot/cm^2 at 20 cm in 10^{-7} sec, 1 p.p.s. with C K_α , then instantaneous spectral brilliance $\sim 7\cdot 10^{18}$ phot/sec/0.1% band/mrad^2/mm^2 ($\sim 10^3$ times less for white spectrum).

Table
of orders of magnitude relevant to continuous-operation high power tubes and of some storage rings (examples taken from Europe: one low energy, soft X-rays, one designed for high energy physics, one 2 GeV dedicated, and a proposed high energy dedicated machine). For tubes, Cu K_α is only an example. For synchrotrons, we consider 1 mrad of orbit, but 10 mrad or more can be collected (and possibly focused, but brilliance does not change). The useful wavelength range extends down to $\lambda_c/4$. For intervals $\Delta\nu$, Ω , S smaller than indicated, power scales down linearly; for greater interval, does not increase (or, for undulators, increase with $\Delta\nu$ or Ω but not both, up to $\Omega_0 \sim \pi/\gamma^2$). For undulators, the "integrated flux" is within a pinhole selecting a $\Delta\nu/\nu \sim 7\%$ band (and only $\sim$15% of the power).

Table 1 - X-ray tubes and storage rings: some orders of magnitude of main parameters.

		wavelength range (Å)	$\frac{\Delta\nu}{\nu}$	apparent source(mm)	Ω (sterad)	integrated flux(W)	Spectral brilliance (phot/sec/ /0.1% band/ /mrad^2/mm^2)
sealed-off X-ray tube	CuK	1.54	0.05%	1x1	0.1	0.01	$1.6\cdot 10^8$
2 kW, 50 kV	Bremss.	0.25	white			0.02	10^5
V.H.power rot.-anode	CuK	1.54	0.05%	1x1	0.1	0.27	$4\cdot 10^9$
tube 50 kW 50 kV	Bremss.	0.25	wh.			0.5	$3\cdot 10^6$
μfocus rot.-anode tube	CuK	1.54	0.05%	0.1x0.1	0.1	0.02	$2.8\cdot 10^{10}$
3.5 kW 50 kV	Bremss.	0.25	wh.			0.04	$2\cdot 10^7$
ACO (Orsay) (operating)							
0.54 GeV, 150 mA, B=1.6T		λ_c=40 Å	wh.	0.5x0.6	$2\cdot 10^{-6}$	0.16	$4.6\cdot 10^{12}$
ADONE (Frascati) (operating)							
1.5 GeV 100 mA, B=1T		λ_c=8.3	wh.	1x0.4	$\frac{2}{3}\cdot 10^{-6}$	1.4	$3.5\cdot 10^{13}$
with 1.8T 5-pole wiggler		λ_c=4.6	"	1x0.1		2.5	$6.9\cdot 10^{14}$
SRS (Daresbury) (under constr.)							
2 GeV 300 mA, B=1.2T		λ_c=3.9	"	5x0.2	$\frac{1}{2}\cdot 10^{-6}$	12	$7.6\cdot 10^{13}$
4.5T 1 pole wiggler		λ_c=1.1	"			45	"
ESRF (European S.R.Facility, proposal)							
5 GeV, 500 mA, B=0.7T		λ_c=1	"	1x0.1	$\frac{1}{5}\cdot 10^{-6}$	185	$7.9\cdot 10^{14}$
3T 1 pole wiggler		λ_c=0.23	"			790	"
undulator λ_0=5.6 cm, B=0.2T		λ_1=5	7%	1x0.2	$2\cdot 10^{-9}$	78	$7\cdot 10^{18}$
5 m long (λ_5=5th harmonic)		λ_5=1	7%			26.4	$4.7\cdot 10^{17}$

References

1) E.E.Koch "Synchrotron radiation sources" in: "Interaction of radiation with condensed matter" vol.2, p.225-274 (Int.Centre for Theor.Phys., winter college 1976) IAEA, Vienna 1977.

2) A.Bienenstock, H.Winick "Synchrotron Radiation Research" SSRL Report, Stanford 1978.

3) Proceedings of course on Synchrotron Radiation Research (Alghero 12-24 Sept. 1976) (Ed. by A.N.Mancini and I.F.Quercia), vol.1.
   a) H.Winick: "Introductory lecture, wigglers, and considerations for the design of S.R.facilities", p. 3-61.
   b) D.W.Lynch "Comparison of S.R. with other sources" p. 298-300.
   c) P.Pianetta, I.Lindau "Phase-space analysis applied to X-ray optics" p. 372-387.

4) J.D.Jackson "Classical Electrodynamics" 2nd ed. J.Wiley 1975.
   R.Feynman, R.Leighton, M.Sands "Lectures on physics" Addison-Wesley.

5) R.Coïsson "Some remarks on radiation in non-uniform fields and undulators for X-rays. talk given at Wiggler Meeting, Frascati 29-30/6/78.

6) M.Yoshimatsu, S.Kozaki "High Brilliance X-ray Sources" in "X-ray optics; applications to solids" Ed. H.J.Queisser, p.9-33. Topics in Applied Physics vol.22 (Springer-Verlag 1977).

7) R.Coïsson, Opt.Comm. 22, 135 (1977); R.Bossart et al., submitted to Nucl.Instrum.& Meth. (1979).

8) A.Guinier, Théorie et technique de la radiocristallographie, Dunod,Paris.