RROPAGATION OF SOUND AND ULTRASOUND IN NON-HOMOGENEOUS MEDIA*

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Summary This paper describes numerical techniques which may be used to analyse (acoustic) wave motion in non-homogeneous media. The methods specifically considered are based on (1) one dimensional direct and inverse scattering theory (2) the parabolic approximation (3) the geometrical approximation (4) direct Fourier synthesis of the wave field.

1. INTRODUCTION

Imaging processes in physics generally involve an interaction mechanism between waves and matter. It is clearly important to be able to analyse this interaction in order to conceive, develop or improve imaging techniques. Such studies may now be carried in many complicated situations by direct numerical simulation of the wave propagation process. The aim of the present paper is to describe some typical numerical techniques and the underlying approximations which are currently used to deal with acoustic wave motion. It should be stressed that the same (or very similar) techniques are applied in other fields of physics like optics and electromagnetics. In that sense the methods described have a fair degree of generality. We shall not attempt to present, in this limited space, the standard material which may be found in classical books dealing with wave motion like those of WHITHAM[1], LIGHTHILL[2], MORSE & INGARD[3], MIKLOWITZ[4], ACHENBACH[5], OFFICER[6], EWING, JARDETSKY & PRESS[7], BREKHOVSKIKH [8], KLINE & KAY[9], FELSEN & MARCUVITZ[10], MARCUSE[11]. These texts cover in great detail analytical methods and important results. However, only CLAERBOUT'S monography[12] on geophysical data precessing seems to include a detailed discussions of numerical methods for wave propagation. Thus our aim will be to illustrate such numerical treatments and to this purpose we shall briefly consider four problems : (1) wave scattering in one dimension (direct and inverse), (2) multidimensional wave scattering by weak inhomogeneities (3) three dimensional wave refraction in inhomogeneous media, (4) source radiation in a layered medium. The methods applied to these problems will be respectively (1) one dimensional scattering theory and the forward scattering approximation (FSA), (2) the parabolic equation method (PEM), (3) the geometrical approximation (4) direct Fourier synthesis (DFS) of the wave field. In selecting these topics and techniques we do not intend to cover the whole field of numerical wave propagation but only aim to discuss some powerful tools.

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The choice is also a reflection of the author's own research involvment in the subject for the past few years [13] [14] [15] [16] [17]. No attempt has been made to cite the relevant litterature dealing with the problems treated. Numerous references may be found in the classical textbooks [1] to [12] or in the author's publications.

2. WAVE SCATTERING IN ONE DIMENSION

Plane wave propagation in one dimensional inhomogeneous media has been studied extensively. The problem arises in numerous practical situations, it is a good benchmark for approximation techniques and a prototype for more complicated multidimensional inhomogeneous wave propagation. Scattering theory in one dimension is well described by LAX & PHILIPPS [18] or more concisely by ROSEAU [19] and in numerous textbooks on quantum mechanics. Our objectives will be here (1) to describe numerical techniques for solving the direct problem (2) to show that the forward scattering approximation is well suited to the calculation of the reflection response of the medium (3) to show that the inverse scattering problem (i.e. the problem of deducing the structure of the medium from its reflection response) may be handled with a simple algorithm only involving FFT operations and numerical integration of ordinary differential equations.

Formulation and analysis

We consider a one dimensional inhomogeneous medium in which the sound speed c and density ρ_0 only depend on the z coordinate (figure 1). A monochromatic plane wave propagating in the homogeneous region 1 ($z \leq 0$) penetrates in the inhomogeneous region 3 ($0 < z < \delta$) where it is reflected and transmitted. At $z = \delta$ the medium becomes homogeneous and the wave reaching $z = \delta$ propagates in region 2 ($z \geq \delta$) with no further changes of its amplitude and direction. If the xOz plane is chosen to contain the initial wave vector then all wave vectors belong to that plane because the medium is stratified. Furthermore the wavenumber on the transverse x direction is conserved ($k_\perp = k_0 \sin\theta_0$, where θ_0 designates the initial incidence angle).

We assume that the acoustic wave motion is described by the following set of linear equations

$$\begin{aligned} &\frac{\partial \rho}{\partial t} + \rho_0 \nabla \cdot \underline{v} = 0 \\ &\rho_0 \frac{\partial \underline{v}}{\partial t} + \nabla p = 0 \\ &p = c^2 \rho \end{aligned} \qquad (1)$$

where p, $\underline{v}(u, v, w)$, ρ represent the acoustic pressure, velocity and density perturbations and where ρ_0 and c designate the local density and sound speed. In accord with the above discussion and assumptions the wave field may be cast in the form

$$\begin{bmatrix} p \\ u \\ v \\ w \end{bmatrix} = \begin{bmatrix} p(z) \\ u(z) \\ 0 \\ w(z) \end{bmatrix} e^{ik_\perp x - i\omega t} \qquad (2)$$

which when substituted in (1) yields

$$\frac{d}{dz}\begin{bmatrix} p \\ w \end{bmatrix} = \begin{bmatrix} 0 & i\rho_0\omega \\ ik_n Y_n & 0 \end{bmatrix} \begin{bmatrix} p \\ w \end{bmatrix} \qquad (3)$$

where $k = \omega/c$, $k_n = (k^2 - k_\perp^2)^{1/2}$ is the local longitudinal wave number and $Y_n = \frac{1}{\rho_0 c}\frac{k_n}{k}$ is a longitudinal admittance (for a plane wave in a homogeneous medium Y_n would be equal to the ratio between w and p). The set of equations (3) may be integrated numerically from $z = \delta$ by providing suitable initial conditions in that section. This is accomplished by giving an arbitrary value to $p(\delta)$ for instance $p(\delta) = 1$ and noting that the only wave to exist in that section is plane and propagates outwards so that $w(\delta) = Y_n(\delta)\, p(\delta)$. A similar procedure is applied in the calculation of the reflection and transmission coefficients : the field in region 1 may be written as a superposition of an incoming and on outgoing wave

$$\begin{aligned} p(z) &= A \exp ik_{n1} z + B \exp -ik_{n1} z \\ w(z) &= Y_{n1} (A \exp ik_{n1} z - B \exp -ik_{n1} z) \end{aligned} \qquad , z < 0 \qquad (4)$$

Combining these two expressions at $z = 0$ yields A and B and the transmission and reflection coefficients :

$$R(0) = B/A = [p(0) - w(0)/Y_n(0)]/[p(0) + w(0)/Y_n(0)] \qquad (5)$$

$$T(0) = 1/A = 2/[p(0) + w(0)/Y_n(0)] \qquad (6)$$

As an illustration let us consider the case of plane waves at normal incidence ($k_\perp = 0$) impinging on a medium of constant density exhibiting a linear variation of refraction index N (here N $= Y = \rho_0 c^0 / \rho_0 c$) figure 1). The reflection coefficient modulus descreases with oscillations when the reduced frequency $\omega_* = \omega\delta/c^0$ increases (figure 2a). At $\omega_* = 0$ ($\lambda = \infty$) $|R(0)| = |1 - Y_2/Y_1| / [1 + Y_2/Y_1]$, which is the value it would take for a medium with a steplike refraction index. The phase of R increases monotonically with ω_* (figure 2b) but its variation is not linear and this behavior corresponds to the dispersion in time delay of the reflected waves. Other examples may be found in [16].

<u>Local wave decomposition</u>

A very useful concept in the analysis of wave motion is that of the local wave representation. Instead of describing the global evolution of p and w , the field is written as a sum of two local waves. In accord with the usual conventions in geophysics the axis points downwards and we designate by U and D the up-and downgoing local waves. These two waves may be defined <u>a priori</u> (other definitions are possible) by the following expressions and their reciprocal

$$\begin{aligned} U &= \tfrac{1}{2}(p - w/Y_n) \\ D &= \tfrac{1}{2}(p + w/Y_n) \end{aligned} \qquad (7) \qquad \begin{aligned} p &= U + D \\ w &= Y_n (D - U) \end{aligned} \qquad (8)$$

The definitions are such that for a homogeneous medium U and D coincide with up and downgoing plane waves having k_n as longitudinal wavenumber.

Substitution of (8) into (3) yields a coupled set of differential equations for U and D :

$$\frac{d}{dz}\begin{bmatrix} U \\ D \end{bmatrix} = \begin{bmatrix} -ik_n & 0 \\ 0 & ik_n \end{bmatrix}\begin{bmatrix} U \\ D \end{bmatrix} - \frac{1}{2Y_n}\frac{dY_n}{dz}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} U \\ D \end{bmatrix} \qquad (9)$$

Note that in a homogeneous medium the two equations decouple and that $U = U(0)\exp -ik_n z$ $D = D(0)\exp ik_n z$. Now the form of system (9) suggests a forward scattering approximation which consists of (1) neglecting U in the calculation of D (2) using D obtained in the first step to calculate U. In practice this reduces to replacing system (9) by

$$\frac{d}{dz}\begin{bmatrix} \tilde{U} \\ \tilde{D} \end{bmatrix} = \begin{bmatrix} -ik_n & 0 \\ 0 & ik_n \end{bmatrix}\begin{bmatrix} \tilde{U} \\ \tilde{D} \end{bmatrix} - \frac{1}{2Y_n}\frac{dY_n}{dz}\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} \tilde{U} \\ \tilde{D} \end{bmatrix} \qquad (10)$$

This set differs from (9) by a single coefficient (the third coefficient in the second square matrix). A direct numerical check of the FSA is thus straight-forward. Figures 3a & b show the reflection coefficient modulus calculated in this approximation for the medium of figure 1 . They only slighthy differ from the exact values of figures 2a & b. Many other cases treated in this way lead to the same conclusion that the FSA is an excellent tool for solving reflection problems. What makes this result important is that in the FSA it is also possible to give an analytic expression for the reflection coefficient :

$$\tilde{R}(\omega) = \tilde{U}(0)/\tilde{D}(0) = -\int_0^b \frac{1}{2Y_{\parallel}} \frac{dY_{\parallel}}{dz'} \exp\left(i\int_0^{z'} 2k_{\parallel}\, dy\right) dz' \qquad (11)$$

The reflection coefficient appears as a nonlinear Fourier transform of the relative variation of longitudinal admittance.

Inverse scattering in one dimension

There is an extensive literature on this problem, a detailed presentation is given by CHADAN & SABATIER [20]. Our approach [16] is to devise an approximate solution from expression (11). The method consists of inverting the nonlinear Fourier transform and integrating the result as well as an additional equation for the position to obtain $Y_{\parallel}(z)$. The sound speed structure may then be determined if the density is known or similarly the density may be computed if the sound speed is known. If both density and sound speed structures are not known they may be determined from reflection responses obtained in two directions [16]. Figures 4a, b, c show reconstructed and original admittance profiles for three typical media.

For these calculations the waves were normally incident ($k_{\perp}=0$, $k_{\parallel}=k$) and the density was assumed constant. The reconstructed profiles closely follow the exact profiles except for oscillations associated to the limited bandwidth of the incident waves.

3. THE PARABOLIC EQUATION METHOD (PEM)

The parabolic approximation may be introduced like the FSA of the previous section by performing a local wave decomposition of the wave field. Let us assume for simplicity that the pressure field is governed by the nonhomogeneous Helmholtz equation

$$\nabla^2 p + k^2 p = 0 \qquad (12)$$

where $k = \omega/c(\underline{x}) = k_0 N(\underline{x})$

The refraction index $N = c_0/c(\underline{x})$ varies weakly in the domain of propagation. Now, consider waves travelling around a particular axis (for instance the z axis), then up-and downgoing waves may be defined by the following expressions

$$U = \frac{1}{2}\left(p + \frac{i}{k_0}\frac{\partial p}{\partial z}\right) \qquad D = \frac{1}{2}\left(p - \frac{i}{k_0}\frac{\partial p}{\partial z}\right) \qquad (13)$$

$$p = U + D \qquad \frac{\partial p}{\partial z} = ik_0(D-U) \qquad (14)$$

Other definitions are possible, those selected are such that in a homogeneous medium, U and D exactly coincide with up-and downgoing plane waves propagating along the z-axis. Equations for U and D may be conveniently derived by first writing (12) in the form :

$$\frac{\partial}{\partial z}\begin{bmatrix} p \\ \frac{\partial p}{\partial z} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\nabla_{\perp}^2 - k^2 & 0 \end{bmatrix}\begin{bmatrix} p \\ \frac{\partial p}{\partial z} \end{bmatrix} \qquad (15)$$

After some algebra one obtains

$$\frac{\partial D}{\partial z} = \frac{i}{2k_0}(k^2 + k_0^2 + \nabla_{\perp}^2)D + \frac{i}{2k_0}(k^2 - k_0^2 + \nabla_{\perp}^2)U \qquad (16)$$

$$\frac{\partial U}{\partial z} = \frac{i}{2k_0}(k_0^2 - k^2 - \nabla_{\perp}^2)D - \frac{i}{2k_0}(k^2 + k_0^2 + \nabla_{\perp}^2)U \qquad (17)$$

To decouple this set of equations and in analogy with the treatment of the previous section, we may neglect U in the first equation describing D and then use D obtained in this first step to calculate U (the process may be iterated). Now, in this approximation the equation for D takes the form

$$\frac{\partial D}{\partial z} = \frac{i}{2k_0}\left(k^2+k_0^2+\nabla_\perp^2\right) D \tag{18}$$

It is more meaningful to cast D in the form of a nonhomogeneous plane wave $D = \Psi(x,y,z)\, e^{ik_0 z}$. The amplitude function Ψ is then solution of the following parabolic equation

$$2ik_0\frac{\partial \Psi}{\partial z} + \nabla_\perp^2\Psi + k_0^2(N^2-1)\Psi = 0 \tag{19}$$

The parabolic approximation consists of solving this equation instead of (12)* The initially elliptic problem is thus replaced by a Cauchy problem which may be solved with little difficulty by stable accurate and fast finite difference techniques. Further mathematical details are available in references [15] [21] and the literature cited there in (see also BUXTON [22] in the present volume).

Typical example of calculation

We shall here describe one single calculation based on the PEM : the scattering of plane waves by a cylindrical inhomogeneity of circular cross section (figure 5a). The modulus and phase of **D** in three axial sections are represented on figures 5b & c. Their general appearance is that of Fresnel gratings which arise as a result of the interference between a plane wave (the incident wave) and a cylindrical wave (the scattered wave). The interference fringes are correspondingly located on $kx^2/2z$ = Cste parabolas (figure 5d). The fringe pattern described may be observed experimentally (figure 5e). The image, kindly supplied by B. BAERD was obtained with the Bragg diffraction apparatus developed at ONERA . For this classical problem initially solved by RAYLEIGH the PEM and the Born approximation give comparable results. However for more complicated situations in particular for inhomogeneous domains of large extent the PEM is superior to the classical techniques (like the Born approximation, the Method of Smooth Perturbation, the Geometrical Approximation).

4. THE GEOMETRICAL APPROXIMATION

Despite the conclusion of the last section the geometrical approximation is still quite useful for describing the propagation of waves in media whose characteristic scale l and characteristic time τ greatly exceed the wavelength λ and period $T = 2\pi/\omega$. The geometrical solution is constructed in the following three steps : (1) a dispersion relation between the angular frequency ω and the wavevector $\underline{k}$ is obtained (2) the characteristic lines or "rays" of this relation are determined (3) the field amplitude is calculated along the characteristic lines. Descriptions of the geometrical approximation may be found in WHITHAM [1], LIGHTHILL [2] FELSEN & MARCUVITZ [10], KLINE & KAY [9] and details on its numerical implementation are given in [13], [14]. Briefly outlined, the problem is to construct the field $\underline{u}(\underline{x}, t)$ solution of a set of equations of the general form

$$L\left(\frac{\partial}{\partial t}, \nabla; \underline{x}, t\right)\underline{u}(\underline{x},t) = 0 \tag{20}$$

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* Other derivations of this equation are available, for instance TAPPERT [21] .

The solution is sought in the form of a local plane wave

$$\underset{\sim}{u}(\underset{\sim}{x},t) = \underset{\sim}{u}^{o}(\underset{\sim}{x},t)\exp i\Psi(\underset{\sim}{x},t) \tag{21}$$

with $\underset{\sim}{k} = \nabla\Psi$, $\omega = -\partial\Psi/\partial t$

$\underset{\sim}{u}^{o}$ is on amplitude vector and Ψ a phase function. Expession (13) is substituted in (12) and the amplitude vector is then expanded in an asymptotic series $\underset{\sim}{u}^{o} = \underset{\sim}{u}_{0} + \underset{\sim}{u}_{1} + \underset{\sim}{u}_{2}$ ... This procedure yields to the zeroth order a dispersion relation $D(\omega,\underset{\sim}{k};\underset{\sim}{x},t) = 0$ and to the next orders equations for the field amplitudes. In the case of acoustic wave propagation. The dispersion relation has the form

$$\omega^{2} = k^{2}c^{2} \tag{22}$$

and a conservation equation for the wave action $A_{0} = \frac{(\Pi_{0})^{2}}{\rho_{0}c^{2}}\frac{1}{\omega}$ is obtained at the first order :

$$\frac{\partial}{\partial t}A_{0} + \nabla\cdot\underset{\sim}{c}_{g}A_{0} = 0 \quad ; \underset{\sim}{c}_{g} = \frac{\partial\omega}{\partial\underset{\sim}{k}} \tag{23}$$

This equation determines the pressure field amplitude Π_{0} . If the propagation medium is time independant and characterized by the refraction index N $= c_{0}/c$ the rays are given by the following set of equations

$$\frac{d\underset{\sim}{x}}{ds} = \frac{\underset{\sim}{p}}{N^{2}} \qquad , \quad \frac{d\underset{\sim}{p}}{ds} = \frac{\nabla N}{N} \tag{24}$$

where $\underset{\sim}{x}$ designates the current position on the ray and $\underset{\sim}{p} = \underset{\sim}{k}/k_{0}$ is a reduced wavevector. The phase function is then of the form

$$\Psi(\underset{\sim}{x},t) = k_{0}\,S(\underset{\sim}{x}) - \omega t \tag{25}$$

and equation (23) becomes

$$\nabla\cdot\left(\frac{\Pi_{0}^{2}}{\rho_{0}c}\,\underset{\sim}{\nu}\right) = 0 \tag{26}$$

where $\underset{\sim}{\nu} = \underset{\sim}{k}/k$ designates the unit vector in the wavevector direction.
The rays may be obtained by integrating the set of ODE (24). To obtain the field amplitude, one consistent way is to integrate equation (26) on a "wave volume".
One obtains as a result that

$$\frac{\Pi_{0}^{2}}{\rho_{0}c}\,\delta\Sigma = cste \tag{27}$$

along each ray. In this expression $\delta\Sigma$ designates the elementary wavefront section and may be obtained by integrating twelve additional ordinary differential equations [14]. Thus the calculation of the geometrical field reduces to an integration of 18 ODE's. The initial source position $\underset{\sim}{x}_{0}$, initial wavevector angles θ_{0} and α_{0} are to be specified together with 12 other initial conditions. To illustrate this process we only show some typical ray tracings and refer the reader to [14] for amplitude calculations. Figure 6 gives the ray diagram for a source radiating in an ocean 6000 m deep, characterized by a "Munk canonical sound speed profil". The rays are refracted by the medium and reflected by the surface and bottom. The pattern formed is typical of the SOFAR channel. Figures 7a, b, c display three-dimensional ray tracings for a source placed in a subsonic jet. In this case the medium is inhomogeneous in its mean velocity and sound speed. Downstream rays are refracted away from the jet axis while upstream, they curve towards the axis. Further comments on this problem may be found in [13] or [14].

Ray tracings are particularly useful in delineating wave paths and they provide an intuitive picture of the propagation phenomenon. However the geometrical approximation does not account for wave diffraction and in many instances the rays form envelopes (caustic surfaces) and the calculation of the field amplitude requires special techniques which are not easy to implement numerically.

5. DIRECT SOLUTIONS

Up to now emphasis was placed on approximation methods. However in some instances direct numerical solutions may be constructed. One possible and rather general idea is to solve the problem by Fourier transform technique and then synthesize the solution in real space by making use of the fast Fourier transform. This idea is well suited to source radiation problems in bayered media. This kind of problem has been extensively studied in geophysics [6] [7] [8] and electromagnetics [10] but it is difficult fo find a complete graphical representation of the wave field even in the simplest situation. We consider a line source situated at a height $z=-h$ in the presence of a plane interface ($z=0$) separating a region ($z<0$) of density ρ_1 and sound speed c_1 from a region ($z>0$) characterized by ρ_2 and c_2 . The source radiates monochromatic waves of angular frequency ω and its amplitude is 2π . The Fourier transform solution of this problem has the form

$$p_1(x,z) = \int \frac{e^{-\gamma_1|z+h| + i\alpha x}}{\gamma_1} d\alpha - \int \frac{e^{\gamma_1(z-h) + i\alpha x}}{\gamma_1} d\alpha + \int e^{-\gamma_1 h + \gamma_1 z + i\alpha x} \frac{2\rho_2}{\rho_1\gamma_2 + \rho_2\gamma_1} d\alpha \quad , \; z<0 \qquad (27)$$

$$p_2(x,z) = \int e^{-\gamma_1 h - \gamma_2 z + i\alpha x} \frac{2\rho_2}{\rho_1\gamma_2 + \rho_2\gamma_1} d\alpha \quad , \; z>0 \qquad (28)$$

where $\gamma_1 = (\alpha^2 - k_1^2)^{1/2}$, $\gamma_2 = (\alpha^2 - k_2^2)^{1/2}$

Replacing the continuous Fourier transforms by discrete Fourier transforms and taking some precautions in their evaluation, one obtains a direct solution of the problem without much effort and only little computation time. Figures 8a, b show one such solution for $\rho_1 = \rho_2$ and $c_2 = 2c_1$. The total number of points of the FFT used in this calculation was N = 2048 but only the first 128 points are represented. In region 1, the direct, reflected and refracted waves combine to form an interference pattern while a transmitted wave propagates outwards in region 2. Other cases as well as a comparison with approximate solutionsobtained by the stationary phase method will appear in a forthcoming paper [17].

REFERENCES

1. G.B. WHITHAM - Linear and Nonlinear Waves. John Wiley, New York (1974).
2. M.J. LIGHTHILL - Waves in fluids.Cambridge University Press, Cambridge (1978).
3. P.M. MORSE & K.U. INGARD - Theoretical Acoustics. Mc Graw Hill, N.Y. (1968).
4. J. MIKLOWITZ - The theory of Elastic Waves and Waveguides. North Holland, Amsterdam (1978).
5. J.D. ACHENBACH - Wave Propagation in Elastic Solids. North Holland, Amsterdam (1975).
6. C.B. OFFICER - Introduction to Sound Transmission. Mc Graw Hill, N.Y. (1958).
7. W.N. EWING, W.S. JARDETZKY & F. PRESS. Elastic Waves in Layered Media. Mc Graw Hill N.Y. (1957).
8. L.M. BREKHOVSKIKH - Waves in layered media. Academic Press N.Y. (1960).
9. M. KLINE & I.W. KAY - Electromagnetic theory and geometrical optics. John Wiley, N.Y. (1965).

10. L.B. FELSEN & N. MARCUVITZ - Radiation and scattering of waves. Prentice Hall, Englewood Cliffs (1973).

11. D. MARCUSE - Light transmission optics. Van Nostrand N.Y. (1972).

12. J.F. CLAERBOUT - Fundamentals of geophysical data processing. Mc Graw N.Y.(1976).

13. S.M. CANDEL - Analyse theorique et experimentale de la propagation acoustique en milieu inhomogène et en mouvement. Thèse de Doctorat es Sciences. Univ. de Paris VI (1977). ONERA Publication 1/1977.

14. S.M. CANDEL - Numerical solution of conservation equations arising in linear wave theory : application to aeroacoustics. J. Fluid Mech. 83 (1977), 465-493.

15. S.M. CANDEL - Numerical solution of wave scattering problems in the parabolic approximation. J. Fluid Mech. 90 (1979) 465-507.

16. S.M. CANDEL - F. DEFILLIPI & A. LAUNAY - Determination of the inhomogeneous structure of a medium from its plane wave reflection response. Part 1 and 2. Submitted to J. Sound Vib. (1979).

17. S.M. CANDEL & C. CRANCE. Direct Fourier synthesis of wave fields in layered media. To be published (1979).

18. P.D. LAX & R.S. PHILIPPS - Scattering theory.Academic Press N.Y. (1967).

19. M. ROSEAU - Asymptotic wave theory. North Holland, Amsterdam (1976).

20. K. CHADAN & P.C. SABATIER - Inverse problems in quantum scattering theory. Springer N.Y (1977).

21. F. TAPPERT - The parabolic approximation method in wave propagation and Underwater Acoustics, J.B. KELLER & J.S. PAPADAKIS ed. Springer, Berlin (1977).

22. B.F. BUXTON - The elastic scattering of fast electrons. This volume.

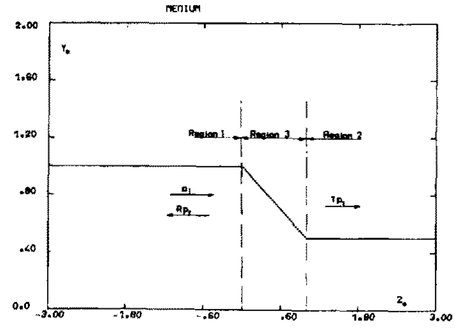
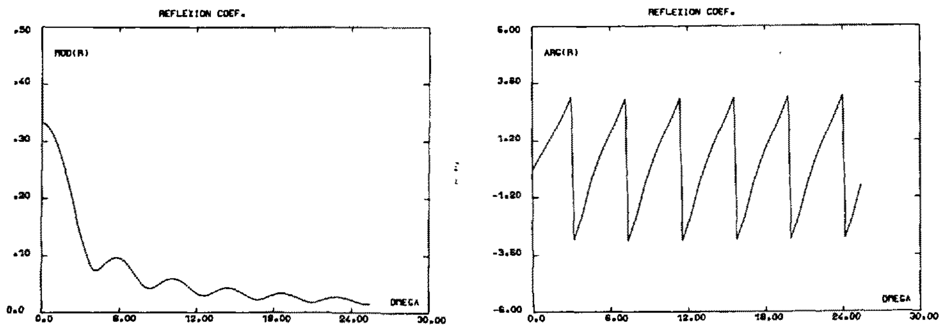


Fig. 1 Geometry of the one dimensional scattering problem.

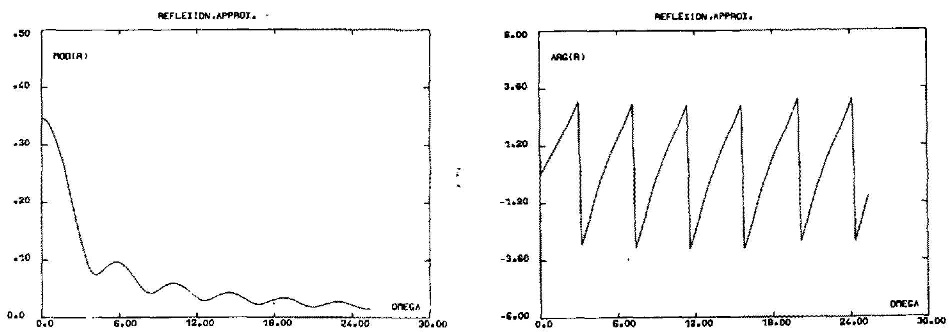
a



b

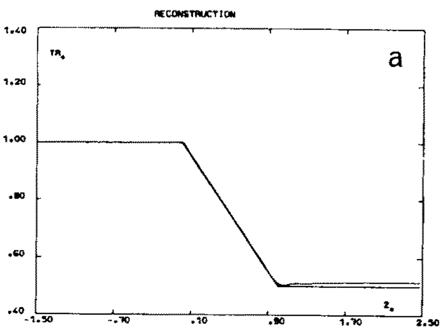
Fig. 2 Propagation of plane waves in a medium characterized by a linear distribution of refraction index. (a) reflection coefficient modulus. (b) Reflection coefficient phase.

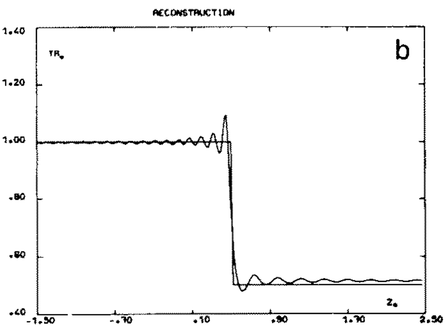
a



b

Fig. 3 Reflection coefficient calculated in the forward scattering approximation for the medium of fig. 1. (a) Reflection coefficient modulus. (b) Reflection coefficient phase.





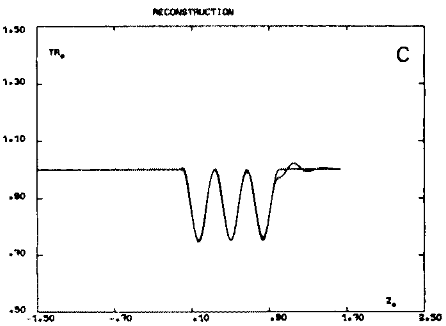


Fig. 4a,b,c Original and reconstructed refraction index profiles for three typical media

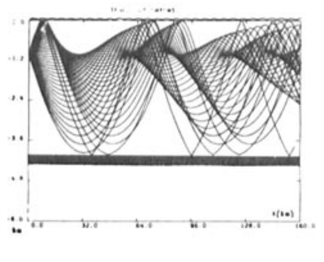


Fig. 6 Ray diagram for a source radiating underwater. Canonical sound speed profile $c(z) = c_0 + c_0 \epsilon \left(e^{-\eta} - 1 + \eta\right), \eta = \frac{2(z - z_A)}{B}$ $z_A = 1000$ m, B = 1000 m, $\epsilon = 0{,}57 \times 10^{-2}$, $c_0 = 1500$ m/s.

Note: Figure 5 appears on page 105

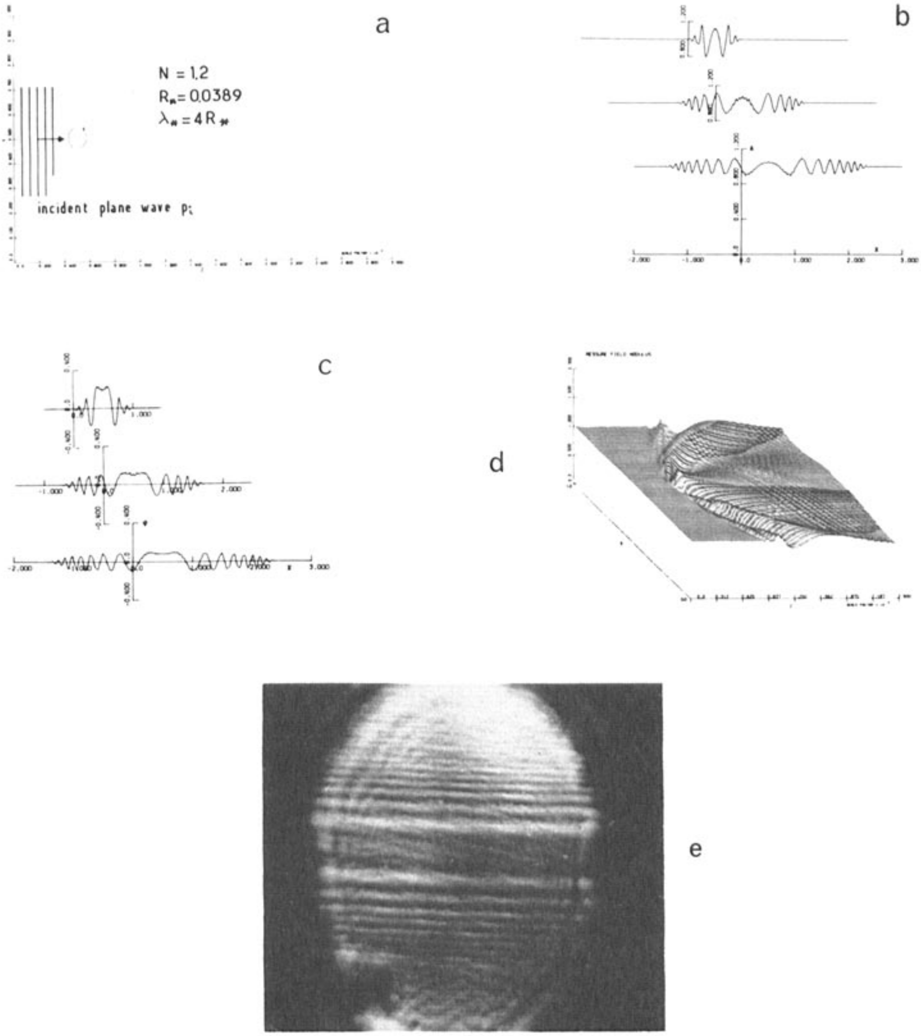


Fig. 5

Scattering of a plane wave by a cylindrical inhomogeneity of uniform refraction index N=1.2. The incident wave propagates in the positive z direction with a wavelength λ_0= 4R. (a) Geometry of the problem. (b) Field modulus in three axial sections. (c) Phase calculated with respect to the incident wave and represented in three axial sections. (d) Field modulus in perspective (fig. extracted from ref. [15]). (e) Fringe pattern observed with a Bragg diffraction apparatus. Ultrasound of wavelength λ_0 = 1 mm impinges on a ϕ 0.4 mm copper wire. The fringes are observed at a 63.8 mm from the wire.

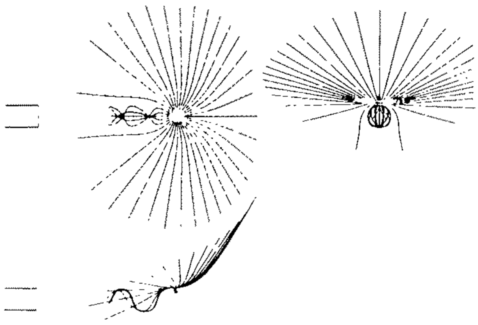


Fig. 7a, b, c Ray tracing for a source situated in a subsonic jet (initial velocity U_0 = 390 m/s, initial temperature T_0 = 870 K). The source is at x_1^0 = 6D, x_2^0 = 0, x_3^0 = 0.6D (fig. extracted from ref. [15] .

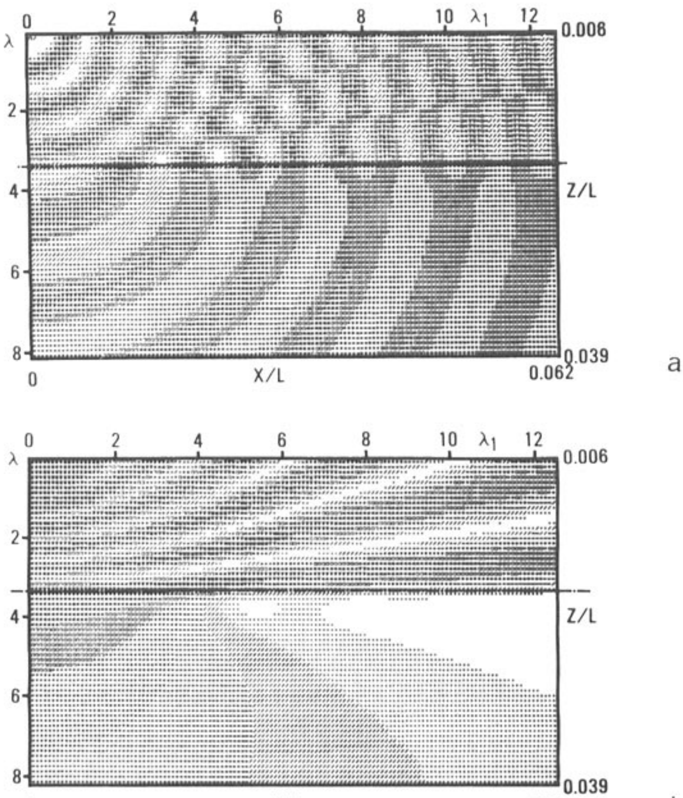


Fig. 8 Source radiation in a layered medium. The solution is obtained by direct Fourier synthesis of the wavefield. (a) Real part of the pressure field. The 10 level grey scale covers the range -1 to +1. (b) Modulus of the pressure field. The 10 level grey scale covers the range 0 to 2. (Figure extracted from ref. [17]).