COHERENT APPROACH TO NEUTRON BEAM POLARIZATION

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I. INTRODUCTION

The classical approach to polarized neutron beams consists of considering the probabilities of finding a neutron in the "up" or "down" spin state with respect to the magnetic field direction, often referred to as quantization direction. For a spin wave function

$$|\chi\rangle = \alpha|\uparrow\rangle + \beta|\downarrow\rangle \, , \qquad |\alpha^2| + |\beta^2| = 1 \tag{1}$$

(which is often written in a matrix form $|\chi\rangle = \binom{\alpha}{\beta}$) these probabilities are:

$$p_\uparrow = |\alpha|^2 \qquad p_\downarrow = |\beta|^2$$

and the polarization is defined as

$$P = \overline{p_\uparrow} - \overline{p_\downarrow} = \overline{|\alpha|^2} - \overline{|\beta|^2}$$

where the bar means the ensemble average for the neutron beam. To make clear why this approach can be called "incoherent" let us recall the general notions of "interference" and "coherence".

We can consider the solution $|\psi\rangle$ of the equation of motion for an experimental situation as a superposition of a set of (quasi stationary) solutions

$$|\psi\rangle = \alpha_1|\psi_1\rangle + \alpha_2|\psi_2\rangle + \dots$$

The expectation value of an arbitrary physical quantity $\hat{A}$ is then given as (the hat ^stands for operators)

$$\langle\psi|\hat{A}|\psi\rangle = |\alpha_1|^2\langle\psi_1|\hat{A}|\psi_1\rangle + |\alpha_2|^2\langle\psi_2|\hat{A}|\psi_2\rangle + \dots + \alpha_1^*\alpha_2\langle\psi_1|\hat{A}|\psi_2\rangle + \alpha_2^*\alpha_1\langle\psi_2|\hat{A}|\psi_1\rangle \dots$$

The measured value of A in a repeated experiment, i.e. over a thermodynamical ensemble of particle states determined by the experimental conditions (e.g. a beam of particles coming from a given source) reads:

$$\overline{A} = \overline{|\alpha_1|^2}\langle\psi_1|\hat{A}|\psi_1\rangle + \overline{|\alpha_2|^2}\langle\psi_2|\hat{A}|\psi_2\rangle + \dots + \overline{\alpha_1^*\alpha_2}\langle\psi_1|\hat{A}|\psi_2\rangle + \overline{\alpha_2^*\alpha_1}\langle\psi_2|\hat{A}|\psi_1\rangle + \dots$$

We will, rather subjectively, talk of interference if

a) we can think of the solutions ψ_1, ψ_2....... as known, easy to materialise and to visualise entities (e.g. by closing one or the other slit in the case of Young interference);

b) we measure a quantity $\hat{A}$ for which some of the $\langle\psi_1|\hat{A}|\psi_2\rangle$ type matrix elements are not zero.

In this case the expectation value of $\hat{A}$ contains not only the weighted sum of the familiar values for ψ_1, ψ_2..... (terms with $|\alpha_1|^2$, $|\alpha_2|^2$,...) but also the cross-terms, which did not appear for ψ_1, ψ_2..... alone(terms with $\alpha_1^*\alpha_2$, etc.). These cross-terms are called interference.

The interference effects can only be observed in a real experiment i.e. for the pertinent ensemble average, if in addition the "coherence" condition is met: i.e. some of the $\overline{\alpha_i^*\alpha_j}$ ensemble averages are not zero. The degree of partial coherence can be obviously given by the expression

$$P = \frac{|\overline{\alpha_i^* \alpha_j}|}{\overline{|\alpha_i|}\,\overline{|\alpha_j|}} .$$

In view of these the above definition of beam polarization is obviously incoherent: it tacitly assumes that $\overline{\alpha^*\beta} = 0$. The coherent approach is concerned with the possible effects of the interference terms between "up" and "down" spin states. Let us remember that the three spin components for spin ½ particles are related to the Pauli matrices

$$\hat{\sigma}_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \hat{\sigma}_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

and thus the expectation values of the spin components are

$$\begin{aligned} \langle \chi | \hat{S}_x | \chi \rangle &= \tfrac{1}{2}\hbar(\alpha^*\beta + \beta^*\alpha) \\ \langle \chi | \hat{S}_y | \chi \rangle &= \tfrac{i}{2}\hbar(\beta^*\alpha - \alpha^*\beta) \\ \langle \chi | \hat{S}_z | \chi \rangle &= \tfrac{1}{2}\hbar(|\alpha|^2 - |\beta|^2) \end{aligned} \tag{2}$$

Notice furthermore that introducing the angles $0<\vartheta<\pi$ and φ by the definitions

$$\cos\frac{\vartheta}{2} = |\alpha| \qquad e^{i\varphi} = \frac{\alpha}{|\alpha|} \Big/ \frac{\beta}{|\beta|} \tag{3}$$

eqs. (2) become

$$\begin{aligned} \langle \hat{S}_x \rangle &= \tfrac{1}{2}\hbar\left(\sin\tfrac{\vartheta}{2}\cos\tfrac{\vartheta}{2}e^{i\varphi} + \sin\tfrac{\vartheta}{2}\cos\tfrac{\vartheta}{2}e^{-i\varphi}\right) = \tfrac{1}{2}\hbar\sin\vartheta\cos\varphi \\ \langle \hat{S}_y \rangle &= \tfrac{1}{2}\hbar\sin\vartheta\sin\varphi \\ \langle \hat{S}_z \rangle &= \tfrac{1}{2}\hbar\left(\cos^2\tfrac{\vartheta}{2} - \sin^2\tfrac{\vartheta}{2}\right) = \tfrac{1}{2}\hbar\cos\vartheta \end{aligned} \tag{4}$$

Thus there is a unique correspondence between a spin $\frac{1}{2}$ wave function $|\chi\rangle = |\chi(\vartheta,\varphi)\rangle$ and the classical notion of a spin direction unit vector $\vec{S}$, defined by the polar coordinates ϑ and φ.

Eqs. (2) show that the interference term $\alpha^{*}\beta$ appears in the expectation value of the x and y spin components. Thus the coherent approach to neutron spin polarization simply means the study of the spin polarization in three dimensions (in particular the observation of Larmor precessions) as opposed to the usual one dimensional "up" and "down" approach.

This new concept of polarised neutron work was initiated around 1970 in three laboratories(1)(2)(3) on very different grounds, and using different techniques. Of course, it did not grow out of anything like a methodical search for something more general. Rather it was suggested, as it is often the case with "novelties", by simple minded semiclassical analogies taken from other fields of science, which in this case was mostly the nuclear magnetic resonance(4).

In this lecture, however, I will follow the methodical way advocated by Levy-Lebond in his talk. Simplified analogies, classical or not, can be helpful both in producing new ideas and in popularization of science. But only the rigorous quantum mechanical treatment can show their validity and limitations.

In the next section a coherent, fully quantum mechanical theory of the spin $\frac{1}{2}$ particle beam polarization is presented, to my knowledge, for the first time. Section III. is devoted to a short description of the basic experimental concepts and to a summary of the applications of the new technique of three dimensional "coherent" neutron spin polarization analysis.

II. QUANTUM THEORY OF POLARIZATION FOR SPIN $\frac{1}{2}$ PARTICLE BEAMS

1.) Polarization of a single particle

The notion of spin polarization is obvious for (the non-existant abstraction of) a point-like classical particle with an arrow-like spin. Its tempting simplicity is in strong contrast to the quantum mechanical reality, especially in two respects.

Firstly, the quantum mechanical particle wave function has to describe both the spatial and the spin behaviour; in other words a particle is not point-like. The general form of wave function is:

$$|\psi\rangle = \varphi_+(\vec{r},t)\,|\uparrow\rangle + \varphi_-(\vec{r},t)\,|\downarrow\rangle \tag{5}$$

with the normalization

$$\int\left(|\varphi_+(\vec{r},t)|^2 + |\varphi_-(\vec{r},t)|^2\right)d^3\vec{r} = 1$$

Secondly, when we talk of spin polarization as an observable quantity, we have to define exactly with what kind of a measurement we are concerned with, i.e. define the corresponding operator which properly takes into account the spin and space variables. In other words the spin is not an arrow with an obvious direction.

The simplest situation is that of the nuclear magnetic resonance: the study of the spin states as a function of time, independently of the spatial variables. The corresponding spin component operators are thus given as

$$\hat{S}_i = \frac{1}{2}\hbar\int d^3\vec{r}\,\hat{\sigma}_i \tag{6}$$

To proceed,we will expand the wave function (5) in terms of the free particle eigenfunctions, where,for simplicity,we assume for the moment that the magnetic field B is homogeneous, and parallel to the Z ("up" - "down") axis.(This is convenient, though not necessary):

$$\varphi_\pm(\vec{r},t) = \frac{1}{(\sqrt{2\pi})^3}\int a_\pm(\vec{k})\,e^{i(\vec{k}\vec{r} - \omega_\pm(\vec{k})t)}\,d^3\vec{k} \tag{7}$$

where

$$\hbar\omega_\pm(\vec{k}) = \frac{\hbar^2k^2}{2m} \mp \mu B$$

(m is the mass, μ is the magnetic moment of the neutron). The expectation values of the polarization operators (6) are given as e.g.

$$\langle\hat{S}_x(t)\rangle = \frac{\hbar}{2(2\pi)^3}\int\left\{a_+^*(\vec{k})\,a_-(\vec{k}')\,e^{i[(\vec{k}-\vec{k}')\vec{r} - (\omega_+(\vec{k}) - \omega_-(\vec{k}'))t]} + c.c.\right\}d\vec{k}\,d\vec{k}'\,d\vec{r}$$

where c.c.stands for the complex conjugate. Integrating over $d\vec{r}$ (using δ function normalization[5])

$$\langle\hat{S}_x(t)\rangle = \frac{\hbar}{2}\int a_+^*(\vec{k})\,a_-(\vec{k}')\,e^{i(\omega_+(\vec{k}) - \omega_-(\vec{k}'))t}\,\delta(\vec{k}-\vec{k}')\,d\vec{k}\,d\vec{k}' + c.c.$$

$$= \frac{\hbar}{2}\,e^{-i\frac{2\mu B}{\hbar}t}\int a_+^*(\vec{k})\,a_-(\vec{k})\,d\vec{k} \quad + \text{ c.c.}$$

Similarly

$$\langle\hat{S}_z(t)\rangle = \frac{\hbar}{2}\int\left(|a_+(\vec{k})|^2 - |a_-(\vec{k})|^2\right)d\vec{k}$$

These expressions can be parametrized similarly to eqs. (4):

$$
\begin{aligned}
&< \hat{S}_x(t) > = p \sin\vartheta \cos\varphi(t) \\
&< \hat{S}_y(t) > = p \sin\vartheta \sin\varphi(t) \\
&< \hat{S}_z(t) > = \cos\vartheta
\end{aligned}
\tag{9}
$$

where

$$p = \frac{\left| \int a_+^*(\vec{k})\, a_-(\vec{k})\, d\vec{k} \right|}{\sqrt{\int |a_+(\vec{k})|^2 d\vec{k} \int |a_-(\vec{k})|^2 d\vec{k}}}$$

and

$$\varphi = \varphi_0 - \frac{1}{\hbar} 2\mu B t \quad , \qquad \vartheta, \varphi_0 = \text{const.}$$

We observe that the $<S_x>$, $<S_y>$ interference terms are reduced by the factor p which is essentially the (momentum) overlap of the ↑ and ↓ spatial wave functions. No interference is observable if these wave functions are orthogonal (p=0), quite the same way as no spatial interference is possible between opposite spin states. Furthermore the time dependence of φ corresponds to the well known classical Larmor precession, whose frequency for neutrons is given by the constant

$$\gamma_L / 2\pi = -2\mu / h = 2916.4 \ \mathrm{Hz/Oe}$$

(μ is negative).

In practice $<\hat{S}_i(t)>$ can be conveniently measured for the nuclear spin ensemble in solids or liquids (NMR). On the other hand, for propagating particles,and particularly neutron beams,the space variables always play an essential role, which seriously limits the applicability of this "time only" approach. Nevertheless in special cases such a nuclear magnetic resonance type technique has been successfully used in neutron beam experiments[6][7].

For particle beams what is really measured is the particle flux impinging on the detector. That is why the polarization measured on a beam is in reality the spin flux defined by the operator

$$\hat{S}_i(\vec{r}) = \frac{1}{2} \hbar \hat{\sigma}_i \hat{J}(\vec{r}) \tag{8}$$

where $\hat{J}$ is the familiar current operator[8] $\quad (\nabla_i = \partial / \partial x_i)$

$$J(\vec{r}) = \frac{\hbar}{2mi} \left(\vec{\nabla} \delta(\vec{r}) + \delta(\vec{r}) \vec{\nabla} \right)$$

In order to evaluate the polarization described by eq. (8) for a wave packet (in one dimension, for the sake of simplicity) we have to calculate the time integral over the particle passage:

$$<\hat{S}_i(r)> = \frac{1}{2}\hbar\int <\psi(t)|\hat{\sigma}_i \hat{J}(\vec{r})|\psi(t)> dt$$

which gives the following results by a straightforward, but lengthy algebra:

$$<\hat{S}_x(r)> = \frac{1}{2}\hbar\int [a_+^*(k+\delta k)a_-(k)e^{-i\delta k.r} + c.c.]dk$$

$$\dots$$

$$<S_z(r)> = \frac{1}{2}\hbar\int [|a_+(k)|^2 - |a_-(k)|^2]dk$$

Here δk is the wave number shift related to the Zeeman energy $2\mu B$:

$$\delta k = \sqrt{k^2 + \frac{4\mu B m}{\hbar^2}} - k \sim \begin{cases} 2\mu B m/\hbar^2 k, & \text{if } |k|>|\delta k| \\ \sqrt{4\mu B m/\hbar^2}, & \text{if } |k|<|\delta k| \end{cases}$$

For, say, B = 1kOe, $2\mu B \sim 10^{-8}$ eV, and for ordinary thermal neutrons

$$\hbar^2 k^2/2m \sim 10^{-2}\,\text{eV} \quad (|k| \sim 2\,\text{Å}^{-1})$$

so δk can vary between $\sim 10^{-6}$ and 10^{-3} Å^{-1}.

The basic point of our considerations is now the following: if,and only if, $|a_+(k)|$ and $|a_-(k)|$ are "smooth" functions, i.e. slowly varying on the scale of δk:

$$|a_\pm(k+\delta k)| \simeq |a_\pm(k)| \tag{11}$$

then eqs. (10) can be simply interpreted in terms of classical notions. (Note that this relation has to hold in the three dimensional space for any direction of $\vec{k}$. The best known example to the contrary is the classical Stern-Gerlach experiment, where, for the very finely collimated beam, (11) does not hold for k perpendicular to the beam propagation).

Indeed, using assumption (11),we can rewrite eqs. (10) in the following parametrised form (for the case of $|\delta k| \ll k$)

$$\begin{aligned} <\hat{S}_x(r)> &= \frac{1}{2}\hbar\int f(v)\sin\vartheta(v)\cos\varphi(v)\,dv \\ <\hat{S}_y(r)> &= \frac{1}{2}\hbar\int f(v)\sin\vartheta(v)\sin\varphi(v)\,dv \\ <\hat{S}_z(r)> &= \frac{1}{2}\hbar\int f(v)\cos\vartheta(v)\,dv \end{aligned} \tag{12}$$

where $v = \frac{\hbar k}{m}$, and $f(v)$, $\vartheta(v)$, $\varphi(v)$ are defined by the equations

$$f(v) = \frac{\hbar}{m}\left(|a_+(k)|^2 + |a_-(k)|^2\right)$$

$$\cos\frac{\vartheta(v)}{2} = \frac{|a_+(k)|}{\sqrt{|a_+(k)|^2 + |a_-(k)|^2}} \qquad 0 \le \vartheta(v) \le \pi$$

$$\varphi(v) = \frac{\gamma_L B r}{v} + \varphi_0(v) \quad , \quad e^{i\varphi_0(v)} = \frac{a_+^*(k)\, a_-(k)}{|a_+(k)||a_-(k)|}$$

Notice that f(v) can be considered as the classical velocity distribution function corresponding to the wave packet as it would be measured e.g. in a time-of-flight experiment.

Consequently eqs. (12) have the form of averages over a velocity distribution of classical pointlike particles displaying classical Larmor precessions in the field B.

The generalization of these results for time dependent magnetic fields is obtained by differentiation, which gives the classical result

$$\frac{d}{dt}\vec{S}(v) = \gamma_L\left[\vec{S}(v) \times \vec{B}(t)\right]$$

where $\vec{S}(v) = \vec{S}(\vartheta(v), \varphi(v))$ is the classical spin direction unit vector. For spatially inhomogeneous fields the plane waves are no longer stationary solutions of the Hamiltonian. However, the variations of the fields we are interested in occur on a macroscopic scale, i.e. they correspond to wave numbers $< 10^{-6}$ Å^{-1}, as compared to which our wave packets are assumed to be "smooth" and "broad" in the momentum space. Therefore we can consider the solutions over small, but still macroscopic, homogeneous field regions, and join them together to approximate the global result.

Thus in conclusion we find that the time-space evolution of the polarization of a "well behaved" spin ½ quantum particle in a magnetic field $\vec{B}(\vec{r},t)$ can be formally obtained as the average over a corresponding ensemble of pointlike classical particles, for which the spin precession is described by the classical equation

$$\frac{d}{dt}\vec{S} = \gamma_L\left[\vec{S}(t) \times \vec{B}(\vec{u}(t), t)\right] \tag{13}$$

where $\vec{u}(t)$ is the classical particle trajectory. The term "well behaved" means that the three dimensional wave packet describing the quantum particle is "smooth and broad" in momentum space on the scale defined by the Zeeman energy $h\gamma_L|B|$, in the sense given in detail above.

It seems that in practice this "well behavedness" can always be assumed, if there is no specific reason to the contrary (e.g. deliberate beam collimation). Especially the observation of Larmor precessions of classical character for neutron beams in fields of order of 100 Oe imply that the incoming "untreated" neutron wave functions can only correspond to wave packets not narrower than about 10^{-5} Å^{-1}: (for average wave vectors of 1 Å^{-1}). This is by no means a trivial statement, since a priori we know little about the initial wave functions of particles coming from a complex source like a reactor. On the other hand, the best known case of not "well behaved" particles, the Stern-Gerlach experiment, not surprisingly, displays "real quantum effects" as opposed to the usual "classical" Larmor spin precessions. It is interesting to note that for very low energy (ultra cold) neutrons[9] and for very monochromatic beams involved in single crystal neutron interferometry[10] it would be particularly easy to produce "non-well-behaved" particle states and, consequently, "non-classical" spin polarization effects.

2.) Polarization of an ensemble of particles

As we have seen above, the complete quantum mechanical analysis of spin $\frac{1}{2}$ polarization for cases of interest in neutron beam scattering results, paradoxically enough, in a classical type picture, as opposed to the superficially quantum mechanical, commonly used "up" - "down" picture. As a consequence, this "classical" description is obviously applicable for an ensemble of particles too, with a more general distribution function $f_{\vec{r},t}(\vec{v},\vartheta,\varphi)$ which is the probability of finding a pointlike classical particle of velocity $\vec{v}$ and spin vector $\vec{S}=\vec{S}(\vartheta,\varphi)$ at position $\vec{r}$ and time t. The evolution of ϑ and φ i.e. $f_{\vec{r},t}$ in space and time is governed by eq. (13) and the vector polarization at a given point $\vec{r}$ and instant t is obviously given as ($d\Omega$ stands for integration over the polar angles)

$$P_x(\vec{r},t)=\int f_{\vec{r},t}(\vec{v},\vartheta,\varphi)\cos\varphi\sin\vartheta\, d\vec{v}\, d\Omega$$

$$P_y(\vec{r},t)=\int f_{\vec{r},t}(\vec{v},\vartheta,\varphi)\sin\varphi\sin\vartheta\, d\vec{v}\, d\Omega$$

$$P_z(\vec{r},t)=\int f_{\vec{r},t}(\vec{v},\vartheta,\varphi)\cos\vartheta\, d\vec{v}\, d\Omega$$

This picture has, however, a fundamental quantum mechanical limitation: while a velocity distribution can be directly measured, a distribution of spin directions vectors $\vec{S}(\vartheta,\varphi)$ can not . To show this let us consider the result of the measurement corresponding to an operator $\hat{O}$ over the ensemble of quantum particles described by the distribution $f(\vartheta,\varphi)$ of the spin states $|\chi(\vartheta,\varphi)\rangle$ (cf.eqs. (1)(2) and (3)).

$$\overline{\langle\hat{O}\rangle}=\int f(\vartheta,\varphi)\langle\chi(\vartheta,\varphi)|\,\hat{O}\,|\chi(\vartheta,\varphi)\rangle\, d\Omega$$

It is readily seen that this becomes:

$$\overline{\langle \hat{O} \rangle} = \langle \uparrow | \hat{O} | \uparrow \rangle \int f(\vartheta,\varphi) \cos^2\frac{\vartheta}{2}\, d\Omega \; +$$
$$+ \langle \downarrow | \hat{O} | \downarrow \rangle \int f(\vartheta,\varphi) \sin^2\frac{\vartheta}{2}\, d\Omega \; +$$
$$+ \left(\langle \downarrow | \hat{O} | \uparrow \rangle \int f(\vartheta,\varphi) \cos\frac{\vartheta}{2} \sin\frac{\vartheta}{2}\, e^{-i\varphi}\, d\Omega \; + \; c.c. \right)$$

We see that, whatever $\hat{O}$ may be, the result of the measurement only depends on the four integrals on the right hand side (which we denote in order $\rho_{\uparrow\uparrow}$ $\rho_{\downarrow\downarrow}$ $\rho_{\uparrow\downarrow}$ and $\rho_{\downarrow\uparrow}$ and it is independent of any further details of $f(\vartheta,\varphi)$. Thus from a measurement we can not have more direct information about $f(\vartheta,\varphi)$ than these four integrals; still we can often consider $f(\vartheta,\varphi)$ as known in more detail by calculating its evolution starting from an a priori (more or less) known situation. (E.g. for a beam which has not been in contact with anything magnetized, we can assume $f(\vartheta,\varphi) = 1/4\pi$)

Note for completeness that obviously

$$\rho_{\uparrow\uparrow} = \frac{1}{2} + \frac{1}{2} P_z \qquad \rho_{\downarrow\downarrow} = \frac{1}{2} - \frac{1}{2} P_z$$
$$\rho_{\uparrow\downarrow} = \frac{1}{2}\left(P_x + i P_y\right) \qquad \rho_{\downarrow\uparrow} = \rho_{\uparrow\downarrow}^{*}$$

The matrix $\hat{\rho}$ formed by these four quantities is called the spin $\frac{1}{2}$ density matrix and formally one finds that

$$\hat{\rho} = \frac{1}{2} + \frac{1}{2} \sum_i P_i \hat{\sigma}_i \qquad \text{and} \qquad \overline{\langle \hat{O} \rangle} = \mathrm{Tr}\left(\hat{O}\hat{\rho}\right)$$

III. BASIC EXPERIMENTAL FACTS AND APPLICATIONS

A polarised beam can be produced e.g. by reflection of neutrons from an appropriate, ferromagnetic crystal or ferromagnetic thin film structure which is magnetically polarised to saturation by an applied field. Within the strict framework of the coherent description of polarization, the polarizer should be described by a quantum mechanical operator (so called $\hat{S}$ matrix) (cf. ref.(11)). For most practical purposes however, we can limit ourselves to the traditional approach, within which the polarizer is characterized by the two reflectivities, $R_\uparrow$ and $R_\downarrow$, for neutrons with the spin respectively parallel and antiparallel to the magnetizing field. This implies that for an unpolarized incoming beam the reflected beam polarization is:

$$P_x = P_y = 0 \qquad\qquad P_z = \frac{R_\uparrow - R_\downarrow}{R_\uparrow + R_\downarrow} = P$$

where P is called the polarizer's efficiency. On the other hand, the reflectivity for a polarized incoming beam is obviously:

$$R(P_z) = \frac{1}{2}(R_\uparrow + R_\downarrow) + \frac{1}{2} P_z (R_\uparrow - R_\downarrow) \tag{14}$$

Thus the polarization dependence of the reflectivity permits the measurement of P_z (spin analysis) too.

To produce a beam with a vector polarization P_x, $P_y \neq 0$, one has to turn the polarization (initially P_z) with respect to the field. For this purpose different techniques have been developed using the evolution of the neutron spin direction $\vec{S}$ in inhomogeneous, time independent magnetic fields, as given by eq. (13), which has two simple limiting cases:

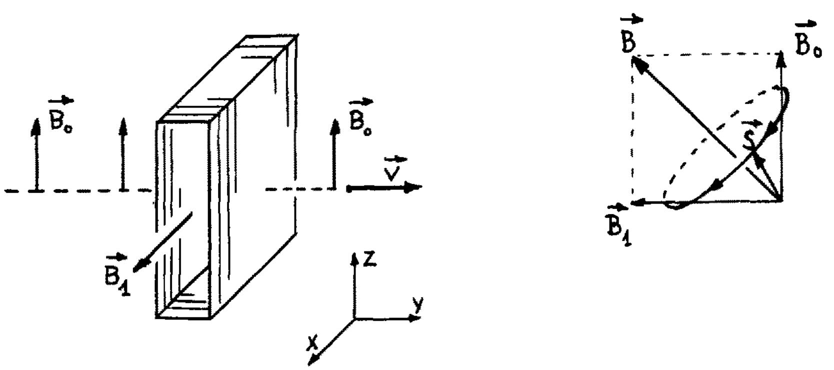
a) Adiabatic case: if the direction of $\vec{B}$ changes slowly, as seen by the neutron, as compared to the Larmor frequency i.e.

$$\gamma_L \, |\vec{B}(\vec{u}(t))| \gg \frac{1}{|\vec{B}(\vec{u}(t))|} \frac{d}{dt} \vec{B}(\vec{u}(t))$$

the angle ϑ between $\vec{B}$ and $\vec{S}$ stays constant i.e. the precession cone of $\vec{S}$ around $\vec{B}$ rigidly follows $\vec{B}$.

b) Majorana field flip: if $\vec{B}$ changes direction very rapidly as compared to $\gamma_L|B|$, $\vec{S}$ has no time to "follow" and the angle between $\vec{S}$ and $\vec{B}$ changes.

One scheme[(2)] of turning $\vec{S}$ from the field direction Z to e.g. the perpendicular direction X is illustrated in the figure.



The neutron with velocity $\vec{v}$ goes through a rectangular coil, whose field $\vec{B}_1 || X$ confined to its interior adds to the external homogeneous field $\vec{B}_o || Z$. When the neutron enters the coil $\vec{B}(\vec{u}(t))$ jumps from $\vec{B}_o$ to $\vec{B}_o + \vec{B}_1$. Assume that the neutron spin was initially parallel to $\vec{B}_o$, and $|B_o| = |B_1|$. Then inside the coil $\vec{S}$ will start to precess around $\vec{B}$ on a cone of 45°. If the thickness of the coil is such, that the neutron leaves the coil after one half precession, this will happen at the point where $\vec{S}$ is parallel to X. Outside the coil again, $\vec{S}$ will now of course precess around $\vec{B}_o$. More generally, the Larmor rotation inside the coil (or in a field limited the same way) can be described by a rotation matrix transformation[12][13] which e.g. in the present case gives

$$P_x \rightarrow P_z \qquad P_y \rightarrow -P_y \qquad P_z \rightarrow P_x$$

$$\text{i.e.} \quad \vec{P}(P_x, P_y, P_z) \rightarrow \vec{P}'(P_z, -P_y, P_x)$$

This coil device can be used to initiate Larmor precessions by turning the polarization perpendicular to the field, but also to analyse them by turning P_x into P_z which then can be measured directly (cf. eq. (14)).

A detailed review of the applications of the novel technique of three dimensional polarization analysis can not be given here. These new possibilities will only be pointed out.

The change of neutron polarization in a magnetic field provides a quite sensitive microprobe for the study of the field itself. E.g. thermal neutrons (velocity $\simeq$ 2000 m/sec) in 20 kOe field ($\simeq$ saturation moment of iron) make a full, 360° Larmor precession within about 30 μ distance. Thus the change of the neutron beam vector polarization in the course of transmission through a ferromagnet carries a great deal of information about the magnetic domain structure of the sample on the scale of a few μ. The method has been developped and successfully used by Rekvelt[1][12] and the Leningrad group[3].

Another field which can benefit of the use of vector polarization analysis is the study of complicated magnetic crystal structures. It has been shown by Blume[14] that there is in general a tensorial relation between the polarization vector of the incoming and the Bragg reflected neutron beams,which becomes particularly complex e.g. for non-centrosymmetrical or non-collinear structures. The existence of such tensorial effects has been demonstrated by Alperin[15] and a vector polarization analysis instrument for such crystallographic use is being developped at the ILL[16].

Finally we will consider a special use of the Larmor precessions in inelastic neutron scattering[17]: the neutron spin echo,which was introduced in 1972[2]. The general principle of the method[18] will be discussed in a form somewhat simplified by the omission of the momentum dependence of the scattering effects.

In inelastic neutron scattering we are concerned with the change E of the neutron energy in the course of the scattering process, which is related to the atomic dynamics of the scatterer[17]:

$$E = \frac{1}{2} m v_1^2 - \frac{1}{2} m v_0^2$$

where v_0 and v_1 are the incoming and outgoing neutron velocities, respectively. The usual inelastic scattering methods are based on the determination of the average of v_0 and v_1 for the whole beams, and the measured value of E becomes

$$\bar{E} = \frac{1}{2} m(\bar{v}_1)^2 - \frac{1}{2} m(\bar{v}_0)^2$$

This is a difference of two quantities measured separately and the resolution in $\bar{E}$ is limited by both of these measurements, especially by the monochromatization of the incoming beam.

By the use of the Larmor precession technique it became possible to observe the beam average of the energy change of each individual neutron instead, i.e.

$$\bar{E} = \overline{\frac{1}{2} m v_1^2 - \frac{1}{2} m v_0^2}$$

The main interest of such a direct measurement of the difference as compared to the above classical approach is that the resolution is no longer limited by the monochromatization of the (incoming) beam. This permits better resolutions with still acceptable beam intensities.

As we have seen above (cf. eqs. (12)) the angle of the Larmor precession in a constant, homogeneous magnetic field B over a distance ℓ can be considered as a measure of the neutron velocity v:

$$\varphi = \gamma_L \frac{B\ell}{v}$$

In a neutron spin echo scattering experiment we make both the incoming neutrons precess before the scattering (field B_0 of length ℓ_0), and the scattered neutrons precess in the opposite sense after the scattering (field B_1 of length ℓ_1). Thus the total Larmor precession angle is:

$$\varphi = \gamma_L \left(\frac{B_0 \ell_0}{v_0} - \frac{B_1 \ell_1}{v_1} \right)$$

Let us take $v_0 = \bar{v}_0 + \delta v_0$ and $v_1 = \bar{v}_1 + \delta v_1$ where $\bar{v}_0$ and $\bar{v}_1$ are the respective average values, and use the approximations

$$\varphi = -\gamma_L \left(\frac{B_0 \ell_0}{\bar{v}_0^2} \delta v_0 - \frac{B_1 \ell_1}{\bar{v}_1^2} \delta v_1 \right) + \bar{\varphi}$$

and

$$E = m \left(\bar{v}_1 \delta v_1 - \bar{v}_0 \delta v_0 \right) + \bar{E}$$

where $\bar{\varphi} = \gamma_L (B_0 \ell_0 / \bar{v}_0 - B_1 \ell_1 / \bar{v}_1)$ and $\bar{E} = \frac{1}{2} m (\bar{v}_1^2 - \bar{v}_0^2)$ are constants. φ will become a practical measure of E if for each combination of the variables δv_0 and δv_1

$$\varphi - \bar{\varphi} = t(E - \bar{E})$$

with a constant t. This is fulfilled if and only if:

$$\frac{\gamma_L}{m} \frac{B_0 \ell_0}{\bar{v}_0^3} = \frac{\gamma_L}{m} \frac{B_1 \ell_1}{\bar{v}_1^3} = t$$

This is the spin echo condition for this case. Now, if the scattering probability to be determined is S(E), the measured P_x polarization will be given as $\left(\int S(E) dE = 1\right)$

$$P_x = \int S(E) \cos(\varphi(E)) dE = \int S(E) \cos(Et) dE$$

i.e. $P_x(t)$ is the Fourier transform of S(E). The Fourier parameter t is proportional to the magnetic field which can be varied in an experiment.

The approximations we have used above (and those used in the more general case) are more or less precise according to the range of δv_0 and δv_1 used (monochromatization). In practice, depending on the experimental details, energy resolutions of 10^{-3} - 10^{-5} can be achieved with modest monochromatizations of 10^{-1} - 10^{-2}. This is precisely the most interesting feature of the neutron spin echo from the point of view of applications. Until now it has been successfully used to study both quasielastic scattering effects, like the diffusion in polymer solutions[19] or the magnetic dynamics of spin glass systems[20], and scattering on elementary excitations, like phonons and rotons in superfluid He liquid[21] with a highly improved energy resolution (up to 50 times).

IV. CONCLUSION

We have seen that the interference effects between the familiar "up" and "down" spin states can be observed in a particle beam by observing all three vector components of the spin polarization. As usual, interference can only be observed if the coherence can be maintained over the whole particle ensemble. In this particular case the decay mechanism of the coherence is particularly conspicuous. In equations (12) we find in the argument of the sine and cosine function the Larmor precession phase angle $\varphi = \frac{\gamma_L B r}{v}$, which can achieve very high values: e.g. for B = 100 Oe field, r = 1 m distance and v = 2000 m/sec (thermal neutron velocity) $\varphi \simeq 900$ radians. Thus both inhomogeneities of the field, and a distribution of the velocity result a scatter of φ, i.e. a dephasing of the precessions. If this becomes of the order of 2π, the coherence is completely lost and $P_x = P_y = 0$. This explains why it is more difficult to work with a three dimensional polarization than to restrict attention to the phase insensitive P_z, as it was the rule earlier. As we have seen above, in the neutron spin echo the neutron velocity differences are explicitly taken care of, thus the only remaining dephasing factor is the inhomogeneity of the magnetic fields, which is an important feature for further applications[22]. This is why the neutron spin echo represents to the highest degree the coherent approach to neutron beam polarization.

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