

## THE PLANE MIXING LAYER

### FLOW VISUALIZATION RESULTS AND THREE DIMENSIONAL EFFECTS

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The turbulent mixing layer between two streams of different velocities continues to play a central role in research aimed at improved understanding of turbulent shear flows in general. At present, not all researchers are in agreement as to what various experiments imply about the structure of mixing layers at high Reynolds number. The views which are held differ on the question as to how and to what extent three dimensionality develops in these flows and whether the characteristic spanwise organized large vortex structures (rollers) continue to be a dominant feature. The traditional view, as extended to the contemporary scene, is that ultimately (i.e., sufficiently far downstream or at sufficiently high Reynolds number) the flow will be completely disorganized. The view put forward by "eddy chasers" is that such vortex structures are primary elements, characteristic of the underlying mean vorticity field, which is particularly simple for the mixing layer, and that, as long as the velocity difference is maintained, there is a mechanism to regenerate these primary structures by what, for convenience, may be called a Kelvin-Helmholtz instability. The heart of the controversy then is whether, or to what extent, secondary and higher instabilities will ultimately break down, completely disorganize or prevent formation of organized primary structures. In a plane mixing layer, the primary structures would, ideally, be two dimensional, containing the basic single component of vorticity while secondary and higher modes of instability would introduce three dimensionality and the other two components of vorticity into the flow. An interesting question follows: to what extent do such secondary instabilities change the properties (e.g., the growth rate; the Reynolds stress) that the mixing layer would have in ideal two dimensional development? In this paper we examine several aspects of this question and discuss some recent relevant experiments.

#### The Ideal of a Two Dimensional Flow

To lend perspective to the problem and provide a gauge against which to evaluate experiments it would be nice to know how an ideal two dimensional mixing layer would develop. A picture of this could in principle be obtained from a computation of the two dimensional Navier-Stokes equations for suitable initial and boundary conditions.

Then growth rates and Reynolds stresses as well as details of the structure could be compared with measured ones in real flows. An exact calculation of such a flow, for boundary conditions appropriate to the experiments, is not available. Ashurst's (1979) approximate calculation by the method of discrete vortices is so far the most ambitious attempt to calculate a spatially developing mixing layer. The qualitative similarities between the computed two dimensional flow and the experimental ones are striking but inconclusive. It is impossible to say whether the differences, especially at the higher Reynolds numbers which were simulated, are due to limitations of the computation (for example, definitions of initial conditions and of Reynolds number are problematic) or to differences between an ideal two dimensional flow and the real one. Nevertheless, such calculations are useful and provide instructive insights. Of significance for the present discussion is that the large clumps of vortices which evolve and interact with each other and which show similarities to the experimentally observed vortex structures exhibit some statistical features qualitatively similar to those in real flows. In particular, scales and lifetimes of the vortex structures passing a given spatial location are found to be broadly distributed (dispersed). To this extent the flow develops a disorganized or turbulent character without intervention of any three dimensional effects.

#### The Initial Region and the Mixing Transition

To further explore the question of the role of three dimensionality in mixing layers it is instructive to compare the well known measurements by Bradshaw (1966) of Reynolds stress in the initial development region of a mixing layer and recent studies by Konrad (1976), Breidenthal (1978) and Jimenez, Martinez-Val and Rebollo (1979) of the development of mixedness in that region. The two results are schematically compared in Fig. 1. The important result found by Bradshaw is that when the free shear layer originates from an initially laminar boundary layer the development of Reynolds stress  $(\overline{u'v'})$  is very different from that which occurs when the boundary layer is initially turbulent. In the laminar case the Reynolds stress was found to overshoot the final, asymptotic value while in the turbulent case it increased monotonically to approximately the same value. In a more recent investigation of the final state of development of a mixing layer, Foss (1977) concluded that "the flow fields from both the laminar and the turbulent initial conditions are essentially identical". Thus it seems fair to assume that the state downstream of the point marked B in Fig. 1 is the same in both cases and to describe it as "fully developed turbulent". (Previously to Bradshaw's work the point labelled A had been called "the end of transition").

Breidenthal (1978) studied the development of "mixedness" by measuring the amount of chemical product formed in the mixing layer between two streams of water

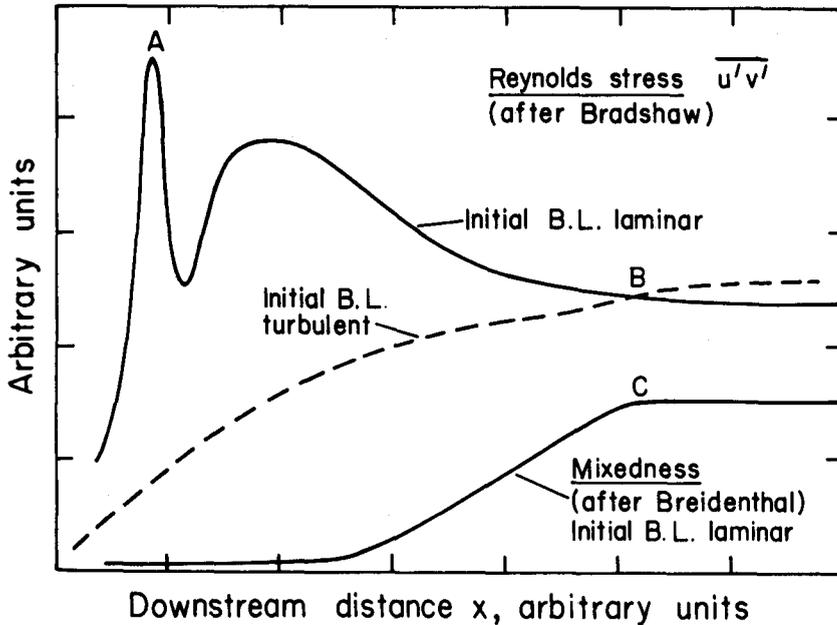


Fig. 1. Development of Reynolds stress and mixedness

carrying appropriate reactants. His result, for initially laminar boundary layers, is also shown in Fig. 1. From low values the mixedness increases, by a factor of more than 10, through a region which he called the "mixing transition" and reaches an asymptotic value at some downstream distance which, in Fig. 1, is indicated by C.

The juxtaposition of these curves in Fig. 1 is schematic and subjective and presupposes that the downstream distances to B and C are the same, in terms of a single parameter such as  $x/\theta_0$ , where  $\theta_0$  is the initial momentum thickness, or  $xU_1/\nu$ , where  $\nu$  is the kinematic viscosity. Actually, as argued by Bradshaw and by Breidenthal, both parameters or a corresponding pair are needed to uniquely define the development. The development is further dependent on other parameters such as the ratio of free stream velocities,  $U_2/U_1$ . In the examples from which Fig. 1 was constructed, point B for Bradshaw's initially laminar condition (with  $U_2 = 0$ ) corresponds to  $x/\theta_0$  from 500 to 1000 and to  $U_1x/\nu$  about  $7 \times 10^5$  while for Breidenthal's experiment (with  $U_2/U_1 = 0.38$ ) point C corresponds to  $x/\theta_0 = 650$  and  $U_1x/\nu = 1.3 \times 10^5$ . Clearly there is need for a definitive experiment in which Bradshaw's and Breidenthal's measurements are made on the same flow. Nevertheless there is considerable indirect evidence that points B and C are the same. For example, Jimenez et al. (1979) found a range of downstream locations (approximately

$3 < 10^{-4} \frac{U_1 x}{\nu} < 6$ , with  $U_1/U_2 = 0.5$ ) in which the dependence on frequency of the power spectrum of velocity fluctuations in the inertial subrange changed from  $-3$  to  $-5/3$  power, suggestive of a change from two to three dimensional turbulent structure of the small scales. In the same transition range the amplitude of velocity fluctuations decreased, as at the end of Bradshaw's transition range. Those results are linked in turn to the measurements of Konrad (1976) and Breidenthal (1978) who associated the increase of mixedness with the development of small scale, three dimensional motions,

One more piece of this picture is important. Several methods of flow visualization in the experiments of Konrad and Breidenthal show that the small scale structure downstream of the mixing transition is superimposed on primary vortices which still have good spanwise coherence in the large. An example from Konrad (1976) is given in Fig. 2, which shows simultaneous edge and plan views of a mixing layer in flow with uniform density and  $U_2/U_1 = 0.38$ . Other examples may be found in papers by



Fig. 2 Edge and plan views of a mixing layer  
 $U_1 = 1000$  cm/sec,  $U_2 = 380$  cm/sec,  $\rho_1 =$  density of  $N_2$  at  
 $p = 4$  atm.,  $\rho_2 = \rho_1$  = density of He/Ar mixture.  
 Scale: streamwise dimension of picture = 15 cm.

Bernal et al. (1979) and Breidenthal (1979). This figure is quite significant because it shows the existence of spanwise organized primary vortices well downstream of the mixing transition, which is located at the left hand side of the picture in the region where, in the edge view, there is an obvious qualitative change in the small scale pattern. The pattern of streamwise streaks which, in plan view, is superimposed on the spanwise pattern of primary vortices is important for the discussion in the following section.

On the basis of the above, to some extent circumstantial, evidence the following picture of developments in a mixing layer can be inferred.

(1) In an initially laminar shear layer, the Kelvin-Helmholtz instability produces a pair of two dimensional vortices which merge to form new pairs with twice the initial scale (Freythuth, 1966). These processes, as noted by Bradshaw, are occurring in the region where  $\overline{u'v'}$  has its maximum values. It may be expected that in this region the distribution of scales (vortex spacings) is centered around the initial Kelvin-Helmholtz value, is possibly bimodal with respect to that value, and in any case must be quite different from the broad distribution, with shifting center, which develops further downstream (Brown and Roshko, 1976; Winant and Browand, 1976; Bernal, 1980) after several more pairings have occurred. As mentioned earlier in connection with Ashurst's computation, this redistribution could occur in two dimensional flow and the decrease in shear stress from peak values to the asymptotic value, if connected to this redistribution, would be a correspondingly two dimensional affair. Thus it is not clear whether the small scale, three dimensional motions, which in real flows develop in the same region of redistribution, are incidental or necessary for the decrease in stresses to the asymptotic value.

(2) What is quite clear is that the small scale, three dimensional motions are necessary for the mixing transition. This fact is especially well brought out in Breidenthal's experiments in water, which show that mixedness remains very low (at the initial laminar value) in the region of the first one or two vortex pairings, where shear stresses reach peak values.

(3) It follows that the mixing processes for momentum and for scalars are quite different in the developing region.

(4) Since the final, fully developed turbulent state is the same, whatever the initial conditions, it follows that for initially turbulent boundary layers the primary, spanwise organized vortices must emerge from the initially three dimensional structure. More specifically, the primary structures must develop from a Kelvin-Helmholtz instability of the initial vorticity layer, which in this case consists of "turbular" fluid (the term proposed by Liepmann, 1962). Evidence for the emergence of primary structure from initially turbulent or highly three dimensional conditions may be seen in pictures obtained by Hussein (1979) and by Breidenthal (1980).

### Three Dimensional Structure

The turbulent mixing layer might be viewed as a synthesis of basic structures connected with a hierarchy of instabilities (Corcos, 1979, 1980). As discussed in the preceding, the primary structure would be the spanwise vortex resulting from the Kelvin-Helmholtz instability. The next mode might be either a spanwise instability or a secondary, internal instability or possibly a combination of the two.

By spanwise instability we mean waviness or other deviation of the vortex from a straight cylindrical structure. Instabilities of this kind have been studied by Hama (1963) and have been observed by Chandrsuda et al. (1978). Such disturbances would contribute to loss of spanwise phase coherence in individual vortices and in interaction processes (such as pairings) between them. Examples of the latter (spiral or bifurcated pairings) may be seen in pictures obtained by Chandrsuda et al. of a mixing layer with  $U_2 = 0$ . On the other hand, there is not much evidence of such spanwise instability at finite values of  $U_2/U_1$ , as in the case shown in Fig. 2. It is reasonable to suppose that any spanwise instability will be competing with the primary instability which continually regenerates the primary structure and changes the scale. Thus a spanwise instability may not develop if its rate is slow compared to the primary one. The relative development rates may depend on parameters such as  $U_2/U_1$ , as the cited experiments suggest.

Spanwise imperfections resulting from such instabilities will tend to degrade conventional spanwise correlations. Thus Browand and Troutt (1980) found that in a well developed mixing layer the conventional average correlation coefficient for velocity was down to 0.2 at a spanwise separation of three vorticity thicknesses ( $\Delta z = 3\delta_\omega$ ). On the other hand, by deploying twelve hot wires spanwise they obtained instantaneous "pictures" of velocity correlation which exhibited spanwise well oriented contours, not perfectly two dimensional of course but suggestive of the spanwise organized structures seen in flow pictures.

Whether spanwise instabilities would greatly alter the Reynolds stresses, as compared to those in a two dimensional development, is part of the question we posed earlier. It seems fairly certain that they would not greatly enhance mixedness which, we believe, is increased mainly by the action of what we will call secondary, internal instability that produces strong secondary motions inside the primary structures. In the mixing layer this secondary instability creates pairs of streamwise vortices which are embedded in the primary vortices and in the connecting vortex layers or braids (the term used by Patnaik et al., 1976) which connect the latter.

The first evidence for these streamwise vortices was seen in pictures such as that in Fig. 2 obtained by Konrad (1976) in gas mixing layers, subsequently by Breidenthal (1978) in experiments in water. It was conjectured that the streaks mark the edges of streamwise vortex pairs but direct evidence for this has only recently been obtained by Bernal (1980) who, using a visualization technique

developed by Dimotakis, obtained pictures of the flow through planes normal to the stream direction. Two examples are shown in Fig. 3. In Fig. 3a the visualized plane

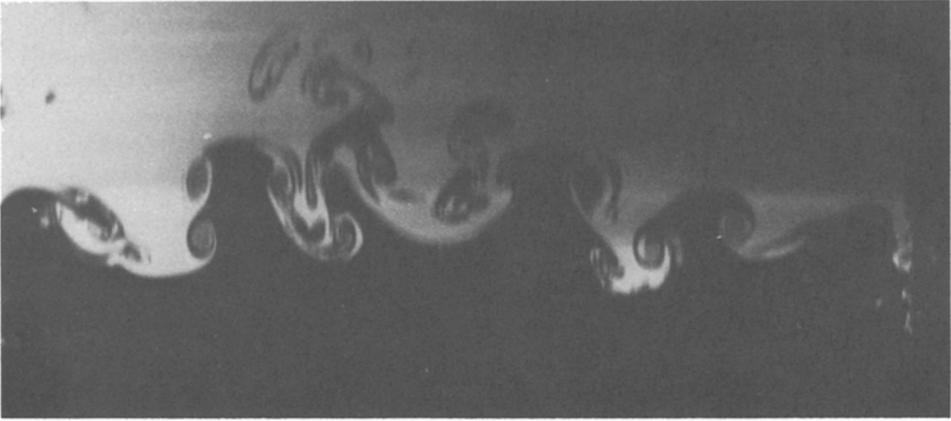


Fig. 3 Cross sectional views normal to flow direction of mixing layer in water.

$U_1 \doteq 32$  cm/sec,  $U_2 \doteq 11$  cm/sec,  $x = 15$  cm.

- a) Section between primary vortices
- b) Section through a primary vortex

cuts through the vortex sheet (braid) between primary vortices. It may be seen that the vortex sheet is highly distorted by the streamwise vortices, whose cross sections are imaged in the plane of the picture. In Fig. 3b the plane of view is through a primary vortex, whose spanwise cylindrical structure is highly sculptured by streamwise vortex pairs on both sides of the mixing layer. It is not difficult to suppose

that the secondary motions induced by those streamwise vortices will be much more effective in promoting internal mixedness than would the spanwise instabilities discussed above. But again, it is not clear how much they modify the Reynolds stresses connected with the primary structures.

The existence of the secondary vortex pairs poses further, interesting questions. Upstream of the mixing transition region their spanwise spacing is fairly regular and approximately equal to the Kelvin-Helmholtz spacing in the initially laminar shear layer (Breidenthal, 1978; Bernal 1980). If they are an essential part of the structure of a turbulent mixing layer then, it might be argued, their scales should increase as the mixing layer grows downstream. Indeed, there is some evidence for this from spanwise correlation measurements (Jimenez et al., 1979a; Bernal, 1980) and from flow pictures such as the one in Fig. 2. In the plan view of Fig. 2 the streaky pattern in the initial part of the mixing layer upstream of the mixing transition repeats itself, at a larger scale, near the right side of the picture well downstream of the mixing transition. This reorganization to a larger scale is reminiscent of a similar phenomenon observed by Taneda (1959) in vortex streets in the wakes of cylinders.

The intriguing question is how such a change of scale is accomplished. If it is by amalgamation processes, these would be rather more complex than those between the primary vortices because the streamwise, secondary vortices occur in pairs of opposite sign and are deployed on either side of the mixing layer. Some hint of an interaction process is seen in Fig. 3b, where two vortex pairs at the bottom right appear to be rotating in opposite directions. Possibly the streamwise vortices are regenerated locally, forming streamwise elongated loops, which line up by mutual interaction to produce the pattern of extended streamwise streaks, and changing scale only when the primary scale has developed to a sufficiently large value.

### Summary and Conclusion

We have posed the possibility that development of a mixing layer is largely determined by two sets of organized structures: the primary, spanwise vortices and the streamwise, counter rotating vortex pairs. The Reynolds stress and the growth of the layer are controlled mainly by the primary vortices while the secondary set provides internal mixing, and possibly modifies the stress. With increasing Reynolds number, higher order, smaller scale structure will be necessary, and available, for accomplishing internal mixing. It does not seem likely that the higher order structures will be perceived to be organized and it may be sufficient to view them in terms of the classical cascade and to model them appropriately.

Due to loss of phase coherence, the turbulent or random character of the flow appears already in the primary structure; this produces a broad spectrum of scales (wave lengths) about the mean value appropriate to any downstream position  $x$ . It

apparently results from the effects of noise in the initial conditions and in the external flow (Delcourt and Brown, 1979). In contrast, a small, periodic initial or free stream disturbance tends to dominate and make coherent that portion of the shear layer whose scale is commensurate with the imposed wave (Wynanski, Oster and Fiedler, 1979).

The secondary set of vortices, which is superimposed on the primary set, has a curious downstream development. The spanwise spacing tends to remain constant while the primary scale is increasing, through at least one amalgamation, but it ultimately readjusts to a larger scale. The persistence of spacing is probably connected with the streamwise orientation. Whether the readjustment to larger spacing occurs by amalgamation or by regeneration is not yet clear. Possibly those two processes are simply aspects of one and the same instability process. In fact such a complementarity is suggested in the results obtained by Patnaik et al. (1976) for the primary instability.

The motion generated by the system of primary and secondary vortices would evidently be complex even if phases were coherent. Adding to this the loss of phase in the primary system it is clear that these two sets of organized structures would generate a "turbulent" flow. For Reynolds number tending to infinity, a cascade of smaller structures would be needed to accomplish internal mixing of scalars, but it seems likely that the main features of momentum exchange and corresponding growth rate of the layer may be determined by the system consisting of the primary and secondary structures.

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