OPE CONSTRAINTS ON THE D-STATE OF THE DEUTERON

S. Klarsfeld[†], J. Martorell[††] and D.W.L. Sprung[†††]

[†] Division de Physique Théorique, IPN, 91406 Orsay, France

[††] Departamento de Física Teórica, Universidad Autónoma de Madrid
Canto Blanco, Madrid 34, Spain

[†††] Institut für Theoretische Physik, Universitat Tübingen,
D-7400 Tübingen 1, West Germany

One of the most significant characteristics of the NN force is its strong tensor component. The important contribution of this term to the saturation properties of nuclear matter is well-known. Experimentally, where this tensor force manifests itself more clearly is in the mixing of D and S components in the deuteron ground state, and there has been considerable interest in studying this mixing and the structure of the D state as a means of increasing our knowledge of the properties of the tensor interaction. Theoretically, it is predicted that its long range part is due only to OPE (in boson exchange models the term of next longest range comes from ρ exchange), and this is undoubtedly the best understood contribution to the NN interaction.

Two quantities are currently used to characterize the admixture of D to S state: p_D, the percentage of D state, and η, the D to S ratio of asymptotic normalizations. We write the wave function as it is customary |1|:

$$|\Psi^M\rangle = \frac{1}{r\sqrt{4\pi}}\{u(r) + \frac{w(r)}{\sqrt{8}} S_{12}\} |\chi_1^M\rangle \tag{1}$$

where u, w are the radial S and D components, S_{12} is the tensor operator and χ_1^M the triplet spin state. The normalization is

$$\int_0^\infty (u^2 + w^2)\, dr = 1 \tag{2}$$

and p_D is given by

$$p_D = \int_0^\infty w^2\, dr \tag{3}$$

while η is defined from the asymptotic forms of u and w when $r \to \infty$:

$$\begin{aligned} u(r) &\to \tilde{u}_A(r) = N e^{-\alpha r} \\ w(r) &\to \eta\tilde{w}_A(r) = \eta N e^{-\alpha r} (1 + \frac{3}{\alpha r} + \frac{3}{\alpha^2 r^2}) \end{aligned} \tag{4}$$

where $\alpha = (MB/\hbar^2)^{1/2}$, B is the binding energy, M the nucleon mass, and N the asymptotic S-state normalization.

While it has been recently argued that p_D is not a measurable quantity |2-3|, η and N can be extracted from scattering data. They are related to a known quantity, the effective range ρ, by |4|:

$$N^2(1+\eta^2) = \frac{2\alpha}{1-\alpha\rho} \tag{5}$$

and η can be extracted from measurements of tensor polarizations either in subcoulomb (d,p) stripping |5| or in p-d elastic scattering |6|. Both methods have succeeded very recently in giving very accurate values for η|7-9| as is shown in fig. 1.

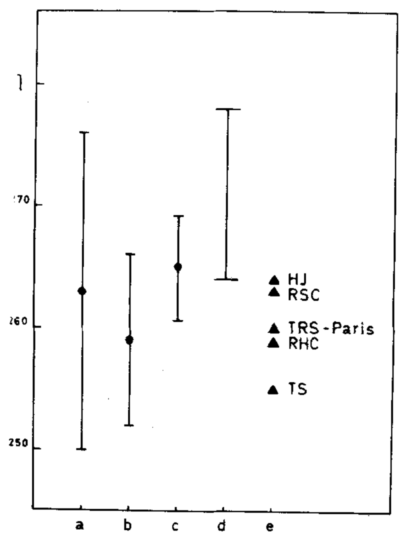

Fig. 1. Values of the D to S ratio of asymptotic normalizations: Experimental values: a) ref. 7, b) ref. 8, c) ref. 9. Theoretical values: d) ref. 12 (see also this text, eq. 14), e) NN potentials: H-J: Hamada-Johnston, RSC: Reid Soft Core, TSR: de Tourreil-Sprung-Rouben, RHC: Reid Hard Core, TS: de Tourreil-Sprung, and Paris potential (values taken from ref. 1 and for the Paris potential private communication from this group).

Thus, at present the long range part of the u,w wave functions appears to be very well-known. Two other experimental quantities, the r.m.s. charge radius and the quadrupole moment are known with good accuracy |10-11|, and they are dominated by the medium range parts of the wave function. It is then interesting to try to correlate all these quantities extracted from experimental data to see if they are consistent among themselves and also with the expected OPE dominance of the long range part of the NN force. What is discussed here is one attempt along these lines |12|, where values for η obtained using all the other experimental data and "theoretical prejudices" are compared with the recently measured values of this quantity.

BOUNDS ON η

To do this, following ref. 13 where similar techniques were used to give lower bounds on p_D, one first constructs u,w functions having the desired asymptotic behaviour, eq. (4), with an arbitrarily chosen value of η. Using the linearity of the coupled Schrödinger equations for S and D states, it is easy to see that the wave functions can be written:

$$u(r) = u_1(r) + \eta u_2(r)$$
$$w(r) = w_1(r) + \eta w_2(r) \qquad (6)$$

where u_1, w_1 and u_2, w_2 are solutions of the same equations satisfying the boundary conditions $u_1 \to \tilde{u}_A$, $w_1 \to 0$; $u_2 \to 0$, $w_2 \to \tilde{w}_A$ when $r \to \infty$. The values of α, N are taken from experiment. In practice since the NN potential is only assumed to be known beyond $r > R \sim \lambda_\pi$, the $u_1 w_1$, $u_2 w_2$ solutions are obtained integrating the coupled Schrödinger equations inwards, from $r \to \infty$ to $r = R$. In this interval u, w are known functions of η, and this suggests dividing their contribution to quantities like the r.m.s. radius and the quadrupole moment into internal ($r < R$) and external ($r > R$) parts. The latter being known functions of η, the internal pieces can then be obtained as:

$$X \equiv X(R,\eta) = \sqrt{50}\, Q - \int_R^\infty dr\, r^2 \left(uw - \frac{w^2}{\sqrt{8}}\right)$$
$$Y \equiv Y(R,\eta) = 4\langle r^2 \rangle - \int_R^\infty dr\, r^2 \left(u^2 + w^2\right) \; . \qquad (7)$$

To prepare the same decomposition for other quantities of interest, it is useful to introduce some notations:

$$U_n = \int_0^R dr\, r^n u^2$$
$$V_n = \int_0^R dr\, r^n uw \qquad (8)$$
$$W_n = \int_0^R dr\, r^n w^2 \; .$$

Then the l.h.s. of eq.(7) can be re-written:

$$X = V_2 - W_2/\sqrt{8}$$
$$Y = U_2 + W_2 \ . \qquad (9)$$

Using Schwarz's inequality:

$$V_2^2 \leq U_2 \, W_2 \ , \qquad (10)$$

and from (9) and (10), the inequality for the unknown quantity W_2 follows:

$$-\frac{9}{8} W_2^2 + (Y - \frac{X}{\sqrt{2}}) \, W_2 - X^2 \geq 0 \ , \qquad (11)$$

which holds only when $W_2 \in [W_{2\,min}, W_{2\,max}]$, with

$$W_{2\,\substack{max\\min}} = \frac{4}{9} (Y - \frac{X}{\sqrt{2}} \pm \sqrt{\Delta}) \ , \ \Delta = Y^2 - \sqrt{2}\, XY - 4X^2 \qquad (12)$$

and this requires Δ positive. Since X, Y depend on η it is found that there is only an interval of values $\eta_1 \leq \eta \leq \eta_2$ for which $\Delta > 0$. As will be seen in the following that this interval is rather small, this gives the desired theoretical estimate of η. Perhaps the most practical way to compute η_1, η_2 is to use the decomposition introduced in ref. 14:

$$\Delta = (Y - 2\sqrt{2}\, X) \, (Y + \sqrt{2}\, X) \ . \qquad (13)$$

Since Y is a positive definite quantity, $\Delta > 0$ is equivalent to $Y + \sqrt{2}\, X > 0$ and $Y - 2\sqrt{2}\, X > 0$; due to the almost linear variation with η of these two functions (this is a consequence of the definitions of X and Y and the smallness of η), it is easy to find η_1, η_2 with very good accuracy. One example is shown in fig. 2.

A stronger upper bound on η, $\bar{\eta}_2$, can be obtained from the condition $X > 0$, but this requires an additional assumption: that the internal contribution to the quadrupole moment is never negative. Given the r^2 factor in its definition (see eq.(7)) it is clear that this is a sensible condition on X when R is large, but the possible appearance of nodes at small r in the wave functions could give $X < 0$ for too small R. To prevent this R is always chosen to be of the same order or greater than $\lambda\!\!\!^{-}_\pi$.

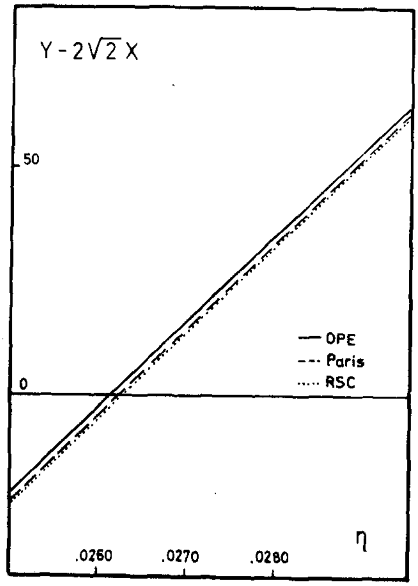

Fig. 2. Plot of $Y - 2\sqrt{2}\ X$ as a function of η, for various potentials and R = 2 fm. The points where the functions change sign are the corresponding η_1.

Although the above discussions already put some limits on the choice of R, the optimal value for this parameter is determined by two more stringent requirements: a) The NN potential must be well-known for $r > R$, in the sense that the external contributions to X and Y can be considered to be well determined. This clearly favours R as large as possible. b) The interval of allowed values of η should be as small as possible. This requires R small as will be shown immediately. To reach a compromise between these two requirements, u, w, of eqs.(6) are constructed using a variety of NN potentials with OPE tails, namely: OPEP; Reid Hard Core, Reid Soft Core; de Tourreil, Sprung; de Tourreil, Rouben, Sprung, and Paris, then $\eta_{min} = \eta_1$, $\eta_{max} = \bar{\eta}_2$ are calculated for different values of R. In Table 1, their dependence on R is shown. The error assigned to each value indicates the spread of values given by the potentials listed above.

R_{fm}	2.0	1.8	1.6
η_{min}	.0261 ± .0001	.0264 ± .0001	.0266 ± .0001
η_{max}	.0292 ± .0001	.0284 ± .0002	.0277 ± .0002

Table 1

Clearly even for R as small as 1.6 fm the values are almost independent of the potential, thus satisfying our requirement a). Requirement b) favours the choice R = 1.6 fm, so that the theoretical estimate for the value of η is finally:

$$\eta = .0271 \pm .0007 , \qquad (14)$$

which, as can be seen in fig. 1, is in good agreement with the present measurements. Thus, the conclusion seems to emerge that there is good consistency between the known long and medium range properties of the deuteron and also our theoretical models for the long range part of the NN force. Some warnings however are in order:

1) a non-relativistic description of the deuteron has been used, and meson exchange currents have not been taken into account. One way to try to include MEC at least approximately is to subtract their contribution from the experimental values of $\langle r^2 \rangle$ and Q, for this an accurate and simultaneous evaluation of the two corrections will be necessary.

2) no attempt has been made to propagate the errors quoted for the experimental values of ρ, B, $\langle r^2 \rangle$, Q, ... to η_{min} and η_{max}. Work in progress in this direction indicates that this increases somewhat, but not substantially, the domain of allowed values of η.

BOUNDS ON p_D

Again, separation of internal and external contributions to p_D combined with use of Schwarz's inequality leads to lower bounds on this quantity. Some time ago Klarsfeld |13| showed that writing:

$$p_D = W_o + Z \qquad (15)$$

with $Z = \int_R^\infty dr\, w^2$, and using that

$$X^2 \leq U_4 W_o \quad \text{when} \quad X \geq 0 \tag{16}$$

leads to

$$p_D \geq Z + \frac{X^2}{U_4} \theta(X) \tag{17}$$

where $\theta(X)$ is the Heaviside function. This is not useful yet, because U_4 is unknown, however from the trivial inequality $U_4 \leq R^2 Y$, one finds:

$$p_D \geq f \equiv Z + \frac{X^2}{R^2 Y} \theta(X) \tag{18}$$

whose r.h.s. is a known function of η, with a minimum falling inside the interval of allowed values of this parameter. The value of f at the minimum gives then the desired bound on p_D.

This bound can be improved using the less stringent inequality $U_4 \leq R^2 \bar{U}_2$ with:

$$\bar{U}_2 = \frac{4}{9} \left(\frac{5}{4} Y + \frac{X}{\sqrt{2}} + \sqrt{\Delta}\right) \tag{19}$$

and

$$p_D \geq \bar{f} \equiv Z + \frac{X^2}{R^2 \bar{U}_2} \theta(X) \tag{20}$$

as shown in ref. 12. More recently McTavish et al. |14| have found an even better bound: they use the inequality $W_o \geq W_2/R^2$ and the value of $W_{2\,min}$ from eq.(12),so that:

$$p_D \geq g \equiv Z + \frac{W_{2\,min}}{R^2} \; . \tag{21}$$

In practice however it is rather unfortunate that the three lower bounds curves $f(\eta)$, $\bar{f}(\eta)$ and $g(\eta)$ defined in eqs.(18), (20), (21) have almost exactly the same value at the minimum due to the closeness of this point to the one where $X = 0$, at which the three functions are equal to Z. Comparing the results given by the different potentials, as done for η, leads to lower bounds on p_D of 3.50 ± 0.03% (R = 2 fm), 3.96 ± 0.05% (R = 1.8 fm), 4.52 ± 0.08% (R = 1.6 fm) so that it seems safe to conclude that compatibility with deuteron data and OPE requires:

$$p_D \geq 4.5\% \; .$$

This is likely to be a conservative estimate. As discussed in ref.15 it is possible that more accurate measurements of η lead to increased values of these bounds. If just to illustrate this, one takes the extreme view that the one standard deviation error quoted in the experimental values is the interval of allowed values of the asymptotic D to S ratio, then the η measured by Grüebler et al. |8| would lead to bounds of 4.2% (f), 4.5% ($\bar{f}$) and 5.2% (g), with R = 2 fm.

REFERENCES

1 D.W.L. Sprung, in "Few Body Problems in Nuclear and Particle Physics", ed. by J. Slobodrian et al. (Laval Univ. Press, Quebec, 1975) pp. 475-493.

2 R.D. Amado, Phys. Rev. C19, 1473 (1979).

3 J.L. Friar, Phys. Rev. C20, 325 (1979).

4 H.P. Noyes, Ann. Rev. Nucl. Sci. 22, 465 (1972).

5 L.D. Knutson, W. Haeberli, Phys. Rev. Letters 35, 558 (1975).

6 R.D. Amado, M.P. Locher and M. Simonius, Phys. Rev. C17, 403 (1978); R.D. Amado et al., Phys. Letters 79B, 368 (1978).

7 H.E. Conzett et al., Phys. Rev. Letters 43, 572 (1979).

8 W. Grüebler et al., Phys. Letters 92B, 279 (1980).

9 K. Stephenson, W. Haeberli, Phys. Rev. Letters 45, 520 (1980).

10 R.W. Berard et al., Phys. Letters 47B, 355 (1973).

11 R.V. Reid, M.L. Vaida, Phys. Rev. Letters 34, 1064 (1975); D.M. Bishop, L.P. Cheung, Phys. Rev. A20, 381 (1979).

12 S. Klarsfeld, J. Martorell and D.W.L. Sprung, Nucl. Phys. A, to be published.

13 S. Klarsfeld, Orsay Report IPNO/TH 74-5 (1974).

14 J.P. McTavish, M.W. Kermode and A. McKerrell, J. Phys. G. Nucl. Phys. 6, L129 (1980).

15 S. Klarsfeld, J. Martorell and D.W.L. Sprung, J. Phys. G. Nucl. Phys. 6, L171 (1980).