FIELD THEORETICAL EXTENSIONS OF MANY-BODY THEORIES

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I. INTRODUCTION

It is well-known that the one-boson-exchange assumption has produced a quantitatively successful model for the nucleon-nucleon interaction[1]. If one accepts this significance of mesons for the two-nucleon system, it is clear that the dynamical presence of the mesons should also be taken into account for the many-nucleon system. A correct calculation of the nuclear properties should, therefore, be based upon the treatment of the field-theoretical Hamiltonian which underlies the construction of the one-boson-exchange potentials[2].

It is the aim of this talk to discuss some of the consequences and problems connected with the introduction of such a field theoretical Hamiltonian for the description of nuclei. We shall be interested in how standard many-body techniques can be generalized and what are the modifications of the many-particle structure. We shall mainly display these structures for a simplified field-theoretical model Hamiltonian, namely the (non-static) Lee model[3,4]. The reason for discussing this simple example is that the treatment of a "realistic" field-theoretical Hamiltonian describing the coupling of nucleons and mesons is too complex - a consequence of the well-known difficulties with attempts to solve a field theory of strong interacting particles: in fact, even if one includes in the meson-nucleon interaction form factors which make the Hamiltonian mathematically well-defined and - suitably generalized - standard many-body techniques applicable, there are many more abstacles in finding approximate solutions than in the standard case: firstly, the many-body theory becomes much more complex; for example, the expansion of the wave function has to include terms not only with n-particle, n-hole, but also with n-particle, n-hole, k-meson $(k = 1,2,3,\dots)$ intermediate states.

Secondly, there is no simple possibility to <u>fix</u> the Hamiltonian by fitting parameters (introduced into an ansatz for the Hamiltonian) to the empirical few-body data (as it is done in the standard many-body theory). The reason is that the one- and the two- (even the zero-) body problems are not solvable. In standard field-theory, the second difficulty can only be solved in perturbation theory leading to the well-known renormalization procedure. This is successful in QED (weak coupling case) but for the strong interacting case this does not help to fix the Hamiltonian.

These complexities connected with a "realistic" field theoretical Hamiltonian lead to the idea of investigating simplified cases. In this connection, the Lee-model plays a distinguished role since here the second difficulty does not arise: the few-body problem is solvable within this frame. Identifying the elementary particles V,N and Θ , introduced by Lee, with proton, neutron, negative (or positive) charged pion, the model can be viewed as a simplified version of a pion-nucleon Hamiltonian[4)]. By studying the many-particle structure of this model one may, therefore, hope to learn something about the modifications of a many-particle structure when mesonic degrees of freedom are taken into account.

We have organized this talk in the following way: in sect. 2, we shall introduce the Lee-model and demonstrate the "renormalization" i. e. the way how the functions defining the Hamiltonian are related to the solutions of the few-body problem. In sect. 3, we describe the many-body techniques which we use to approximate the ground-state energy, whereas sect. 4 and 5 are devoted to a discussion of the structures of "nuclear" matter within an extended Brückner theory and to the problem of boson-condensation for "neutron" matter.

II. DEFINITION OF THE LEE MODEL AND RENORMALIZATION

We introduce fermion operators V_α, N_β for the elementary particles V (neutron) and N (proton) and a boson operator b_k for the Θ-particle (π^-), α,β,k are short-hand notations for all quantum numbers needed to specify the single particle states. The Lee model is then defined by

$$H = H_o^o + W$$

$$H_o^o = \sum E_\alpha^o V_\alpha^+ V_\alpha + \sum E_\beta N_\beta^+ N_\beta + \sum \omega_k b_k^+ b_k$$

$$W = \sum W_{\alpha\beta k}^o V_\alpha^+ N_\beta b_k + \text{h.c.} \quad .$$

The simplification of the field theory defined by H originates from the fact that H does not only commute with the baryon number operator

$$Q_1 = \sum V^+_\alpha V_\alpha + \sum N^+_\beta N_\beta$$

but also with the "charge" operator $Q_1 - Q_2$ where

$$Q_2 = \sum V^+_\alpha V_\alpha + \sum b^+_k b_k \quad .$$

Defining sectors (q_1, q_2) by the eigenvalues q_1, q_2 of Q_1, Q_2, we fix the parameters entering into the definition of H by looking at the structure of the most simple sectors - this defining the relation between unrenormalized and renormalized quantities:

The sectors (1,0) and (0,1) contain the "trivial" eigenvectors $|\beta\rangle = N^+_\beta|0\rangle$ ($|0\rangle$ is the vacuum state) and $|k\rangle = b^+_k|0\rangle$ with eigenvalues E_β and ω_k. So the functions E_β and ω_k stay unrenormalized, they directly describe the dispersion relation for the single particle energy of the proton and the pion, and they can be fixed by setting $E_\beta = \sqrt{p^2_\beta + M^2}$ and $\omega_k = \sqrt{\vec{k}^2 + \mu^2}$.

The sector (1,1) containing the single bare neutron states $V^+_\alpha|0\rangle = |\alpha\rangle$ and the proton-pion states $|\beta k\rangle = N^+_\beta b^+_k|0\rangle$ is non-trivial, the physical neutron state ψ_α is a bound state in this sector with $H\psi_\alpha = E_\alpha\psi_\alpha$ and can be shown to obey

$$\psi_\alpha = \frac{1}{\sqrt{z_\alpha}} \left(|\alpha\rangle + \sum_{\beta k} \frac{W^{o*}_{\alpha\beta k}}{E_\alpha - E_\beta - \omega_k} |\beta k\rangle\right) \tag{2.1}$$

$$E_\alpha = E^o_\alpha + h_\alpha(E_\alpha)$$

where $h_\alpha(z)$ is the mass operator for the neutron

$$h_\alpha(z) = \sum_{\beta k} |W^o_{\alpha\beta k}|^2 / (z - E_\beta - \omega_k) \quad .$$

E_α is the "physical" or renormalized single particle energy of the neutron for the quantum number α. Therefore, it is convenient to prescribe this function setting $E_\alpha = \sqrt{p^2_\alpha + M^2}$ and to eliminate the "unrenormalized" quantity E^o_α from all equations.

Analogously, we define $W_{\alpha\beta\kappa}$, the "physical" or renormalized matrix element of the interaction W with respect to the eigen-state of the neutron setting

$$W_{\alpha\beta k} = \langle\psi_\alpha|W|\beta k\rangle$$

Using eq. (1), $W_{\alpha\beta k}$ is easily related to $W^o_{\alpha\beta k} = \langle\alpha|W|\beta k\rangle$ via

$$W_{\alpha\beta k} = W^o_{\alpha\beta k}|\sqrt{z_\alpha}$$

where z_α is a normalization factor given by

$$z_\alpha = 1 + \sum_{\beta k} |W^o_{\alpha\beta k}|^2/(E_\alpha - E_\beta - \omega_k)^2 \ . \tag{2.2}$$

Thus it is convenient to fix the interaction part of the Lee model Hamiltonian by prescribing $W_{\alpha\beta k}$, e. g. in the case of neutron, proton and pion with standard coupling we would have

$$W_{\alpha\beta k} = \frac{g(k)}{\sqrt{2\omega_k}} \langle\chi_\alpha|\vec{\sigma}\vec{k}|\chi_\beta\rangle\delta(\vec{p}_\alpha - \vec{p}_\beta - \vec{k})$$

(Here, χ_α, χ_β are 2-spinors, g(k) is a form factor to be used to make H mathematically well-defined, a convenient choice is

$$g(k) = g\,\frac{\Lambda^2-\mu^2}{\Lambda^2+k^2}\ , \quad \Lambda \simeq 1.3\ \text{GeV}\ , \ g \simeq 0.08)\ .$$

The significance of this definition of the renormalized interaction is seen from the fact that the transition matrix for N-Θ-scattering has a pole at the off-shell energy $z = E_\alpha$ with a residuum given by the matrix element $W_{\alpha\beta k}$ (for details see ref.[4]). The V-N scattering, taking place in the (2.1)-sector, can be shown to be described by a Lippman-Schwinger-equation of the standard type

$$T(z) = U(z) + U(z)\,\frac{1}{z-H_o}\,T(z)$$

$(H_o = \sum E_\alpha V^+_\alpha V_\alpha + \sum E_\beta N^+_\beta N_\beta)$ where the potential U(z) is a "renormalized" one-theta-exchange potential given by[4)]

$$\langle\alpha\beta|U(z)|\alpha'\beta'\rangle = -\sum_k \frac{W_{\alpha\beta' k}W^*_{\alpha'\beta k}}{z-E_\beta-E_{\beta'}-\omega_k}\, r_\alpha(z-E_\beta)\, r_{\alpha'}(z-E_{\beta'}) \tag{2.3}$$

Here, $r_\alpha(z)$ is a dressing factor describing off-shell mass--renormalization effects[4]:

$$r_\alpha(z) = 1 - \sum_{\beta k} (z-E_\alpha) \frac{|W_{\alpha\beta k}|^2}{(E_\alpha - E_\beta - \omega_k)^2 (z-E_\beta-\omega_k)} \tag{2.4}$$

III. FIELD THEORETICAL EXTENSIONS OF MANY-BODY TECHNIQUES

Any standard many-body technique which is based upon second quantization (the HNC-method would not be of that type) is easily generalized to a field theoretical Hamiltonian, since the basic structure is the Wick-rule (which is also valid for mesons) and the construction of diagrammatic methods to characterize a systematic expansion (also here, obvious modifications are possible when including mesonic degrees of freedom).

Within this talk, we shall display a (suitably generalized) exp S-method[5,6] for approximating the ground state for a many particle system interacting via a Lee-model Hamiltonian. We shall describe briefly the basic ideas of the exp S-technique for the case of a general Hamiltonian: we want to treat a many particle system containing fermions (with operators a^+_α, a_α) and bosons (with operators b^+_k, b_k). We exclude antifermions for simplicity. We write for the ground state ψ of the many particle system

$$\psi = e^S \phi \tag{3.1}$$

where $\phi = \pi_{\alpha \leqslant k_F} a^+_\alpha |0\rangle$ is a slater-determinant of fermion states.

With respect to ϕ, we denote by $a(A)$ occupied (unoccupied) single fermion states. S can be expanded in the (unique) form

$$\begin{aligned} S = & \sum S^{(1)}_{aA} a^+_A a_a + \sum \frac{1}{4} S^{(2)}_{aa'AA'} a^+_A a^+_{A'} a_{a'} a_a + \dots \\ & + \sum C^{(1)}_{aAk} a^+_A a_a b^+_k + \sum \frac{1}{4} C^{(2)}_{aa'AA'k} a^+_A a^+_{A'} a_{a'} a_a b^+_k + \dots \\ & + \sum D^{(1)}_{aAkk'} a^+_A a_a b^+_k b^+_{k'} + \dots \\ & + \text{terms with } 3b^+ + \dots \end{aligned} \tag{3.2}$$

An approximation to ψ can be defined by a suitable truncation of this expansion. Considering infinite systems (like nuclear matter or neutron matter), we may disregard the a^+a-term. Standard Brückner--theory takes into account the a^+a^+aa-term. For the field theoretical case, one would have to include in lowest order the a^+ab^+-term - this term is essential in order to get renormalized equations (see sect.5).

A field theoretical extension of standard Brückner-theory including up to one-meson-exchange would consist in truncating the operator S by including the a^+ab^+, a^+a^+aa, $a^+a^+aab^+$-terms (see sect. 4).

For any such ansatz of the wave function ψ (except for $S = 0$) it is not possible to calculate rigorously the expectation value $\langle\psi|H|\psi\rangle/\langle\psi|\psi\rangle$ (H being of the type

$$H = \sum E^{o}_{\alpha}a^{+}_{\alpha}a_{\alpha} + \sum \omega_k b^{+}_{k}b_{k} + \sum (W^{o}_{\alpha\alpha' k}a^{+}_{\alpha}a_{\alpha'}b^{+}_{k} + \text{h.c.}))$$

and to apply a straightforward Ritz-principle for the determination of S. The essential point of the exp S-technique is, therefore, to provide a systematic expansion of $\langle\psi H\psi\rangle/\langle\psi|\psi\rangle$ the truncation of which can be used to determine the operator S by variation. Of course, the validity of this truncation has then in principle to be checked using the resulting S.

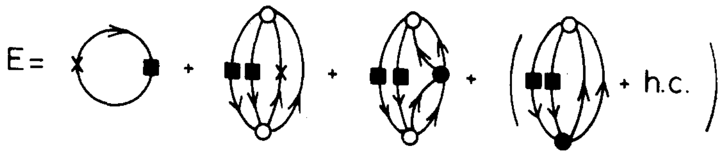
The basic idea for the treatment of $\langle\psi H\psi\rangle/\langle\psi\psi\rangle$ is to expand the exponentials and to calculate $\langle\psi H\psi\rangle$ (respectively $\langle\psi\psi\rangle$) term by term using the Wick-rule to keep track of all possible contributions. Any term of the expansion of $\langle\psi H\psi\rangle$ (and $\langle\psi\psi\rangle$) can then be characterized by a suitable diagram and one can show that the division through $\langle\psi\psi\rangle$ just cancels all contributions from disconnected diagrams ("linked cluster theorem").

In order to illustrate the result, let us write down, for the standard many-body case, the terms in the expansion of $\langle\psi H\psi\rangle/\langle\psi|\psi\rangle$ leading to Brückner theory (including occupation factors)[5]: we have the diagrams of fig. 1 which correspond to the following analytical expression (Note, that now

$$H = \sum E_{\alpha}a^{+}_{\alpha}a_{\alpha} + \sum \frac{1}{4} V_{\alpha_1\alpha_2\alpha'_1\alpha'_2}a^{+}_{\alpha_1}a^{+}_{\alpha_2}a_{\alpha'_2}a_{\alpha'_1}$$

$$S = \sum \frac{1}{4} S_{aa'AA'}a^{+}_{A}a^{+}_{A'}a_{a'}a_{a} \quad ,$$

we disregard the hole-hole interaction for simplicity.)



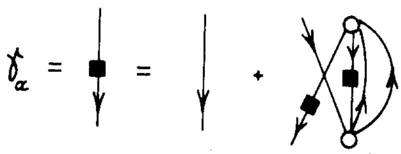


Fig. 1: Diagrams contributing to $\langle\psi H\psi\rangle/\langle\psi\psi\rangle$ leading to standard Brückner theory. Open circles stand for the operator S , closed dots for the N-N-potential, crosses for the kinetic energy.

$$E = \frac{\langle\psi H\psi\rangle}{\langle\psi\psi\rangle} = \sum \gamma_a E_a + \frac{1}{2} \sum_{aa'} \gamma_a \gamma_{a'} \Big(\sum |S_{aa'AA'}|^2 (E_A + E_{A'})$$

$$+ S_{aa'AA'} V_{aa'AA'} + \text{c.c.}$$

$$+ S_{aa'A_1A_2} S_{aa'A_1'A_2'} V_{A_1A_2A_2'A_1'} \Big) \tag{3.3}$$

$$\gamma_a = 1 + \sum_{a'AA'} |S_{aa'AA'}|^2 \gamma_{a'} \gamma_a$$

One can show[5] that the equations of standard Brückner theory are derived by applying the Ritz-variational principle to the expression eq. (3.3) of the energy E for determining the parameters $S_{aa'AA'}$. The system behind the definition of this truncation of cluster expansion of $\langle\psi H\psi\rangle/\langle\psi\psi\rangle$ is the following prescription:

1. Take all terms up to two hole lines .
2. Insert into the bar hole lines "occupation factors" γ_a , where γ_a is defined by an expansion of $\langle\psi a_\alpha^+ a_\alpha \psi\rangle/\langle\psi\psi\rangle$ $(\alpha \leq k_F)$, taken up to the same order.

This yields for $\langle\psi H\psi\rangle/\langle\psi\psi\rangle$ the characteristic two-hole line class of diagrams. (The whole scheme of defining E and the γ_a has, of course, to be chosen such that there is no double counting).

IV. A FIELD THEORETICAL EXTENSION OF BRÜCKNER THEORY FOR THE LEE-MODEL

We want to investigate the structure of "nuclear matter" within the Lee-model. This "nuclear matter" is defined by the sector (2n,n) for $n\to\infty$ and contains the "unperturbed" slater determinant

$$\phi = \prod_{\alpha,\beta\le k_F} V_\alpha^+ N_\beta^+ |0\rangle \quad .$$

With respect to ϕ, we denote by a(b) occupied, by A(B) unoccupied V(N) single particle states. Within a Brückner theory with one-meson-exchange the operator S takes the form

$$S = \sum C_{aBk} N_B^+ V_a b_k^+ + \sum S_{abAB} N_B^+ V_A^+ V_a N_b + \frac{1}{2} \sum F_{abBB'k} N_{B'}^+ N_B^+ V_a N_b b_k^+$$

All other terms vanish up to this order because of the symmetries of the Lee-model Hamiltonian H . For the expansion of $\langle\psi H\psi\rangle/\langle\psi\psi\rangle$ ($\psi = \exp S\phi$) we take all contributions analogous to fig. 1 yielding the diagrams of fig. 2. Analytically,

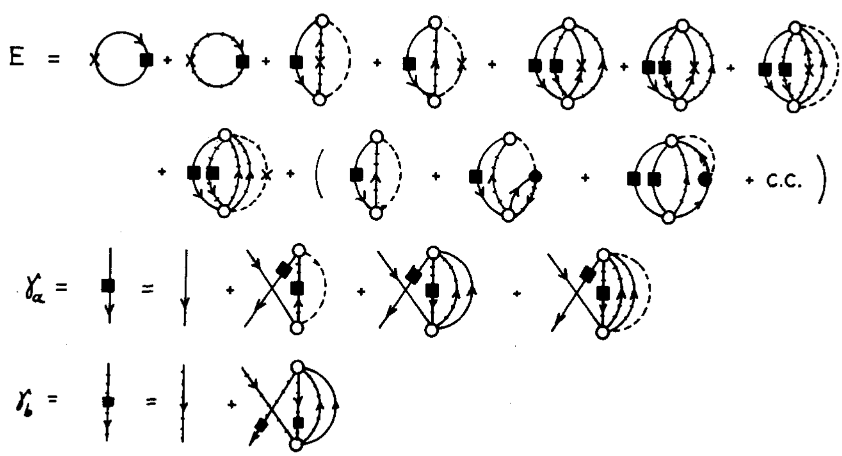


Fig. 2: Diagrams defining the field-theoretical extension of Brückner theory for the Lee-model. Open circles denote the different parts of S , closed dots the Lee-model interaction, crosses stand for the kinetic energy, full lines denote a V-particle contraction, full crossed lines denote that of an N-particle, dashed lines that of a Θ-particle.

we obtain

$$E = \frac{<\psi H\psi>}{<\psi\psi>} = \sum E_a^o\gamma_a + \sum E_b\gamma_a + \sum \gamma_a |C_{aBk}|^2 (E_B+\omega_k)$$

$$+ \sum \gamma_a\gamma_b \Big(|S_{abAB}|^2 (E_A^o+E_B) + |F_{abBB'k}|^2 (E_B+E_{B'}+\omega_k) \Big)$$

$$+ \Big(\sum \gamma_a W_{aBk}^o C_{aBk} + \sum C_{aBk} S_{abAB} W_{Abk}^o \gamma_a\gamma_b +$$

$$\sum \gamma_a\gamma_b F_{abBB'k} S_{abAB'}^* W_{ABk} + \text{h.c.} \Big) \qquad (4.1)$$

$$\gamma_a = \frac{<\psi V_a^+ V_a \psi>}{<\psi\psi>} = 1 - \sum_{Bk} |C_{aBk}|^2 \gamma_a - \sum_{bAB} |S_{abAB}|^2 \gamma_a\gamma_b - \sum_{bBB'k} |F_{abBB'k}|^2 \gamma_a\gamma_b$$

$$\gamma_b = \frac{<\psi N_b^+ N_b \psi>}{<\psi\psi>} = 1 - \sum_{aAB} |S_{abAB}|^2 \gamma_a\gamma_b - \sum_{aBB'k} |F_{abBB'k}|^2 \gamma_a\gamma_b \qquad (4.2)$$

For the variation of E with respect to S it is convenient to treat the quantities $\gamma_a\gamma_b$ as independent variables and to conceive eq. (4.2) as auxiliary condition which is taken into account by Lagrange-parameters $\varepsilon_a, \varepsilon_b$.

In order to discuss the structure of the resulting variational equations, we first quote the formulas of standard Brückner theory, applied to a (static) one-theta-exchange potential V. We have[4)]

$$<\alpha\beta|V|\alpha'\beta'> = - \sum_k W_{\alpha\beta'k} W_{\alpha'\beta k}^* / \omega_k ,$$

there is no NN- or VV-interaction due to one-boson-exchange. The binding energy of nuclear matter is then given by[4)] (Here and below, we drop standard occupation factor refinements)

$$E = \sum E_a + \sum E_b + \sum_{ab} <ab|G|ab>$$

$$= \sum \varepsilon_a + \sum E_b$$

$$\varepsilon_a = E_a + \sum_b <ab|V|ab> + \sum_b <ab|VS|ab>$$

$$\varepsilon_b = E_b + \sum_a <ab|V|ab> + \sum_a <ab|VS|ab>$$

Here, we have replaced the usual G-matrix by the operator $S = \Sigma S_{abAB} V_A^+ N_B^+ N_b V_a$, which is given by the solution of the two-body equation $(H_o = \Sigma E_\alpha V_\alpha^+ V_\alpha + \Sigma E_\beta N_\beta^+ N_\beta)$

$$\sum_{A'B'} \langle AB|\varepsilon_a+\varepsilon_b-H_o-V|A'B'\rangle S_{abA'B'} = \langle AB|V|ab\rangle$$

Application of the variational principle to the "correct" expression of the Lee-model energy, eq. (4.1), yields (for details, see ref.[4]):

$$E = \sum \varepsilon_a + E_b$$

$$\varepsilon_a = E_a + \sum_b \langle ab|U(\varepsilon_a+E_b)|ab\rangle + \sum_b \langle ab|U(\varepsilon_a+E_b)S|ab\rangle$$

$$\varepsilon_b = E_b + \sum_a \langle ab|U(\varepsilon_a+E_b)S|ab\rangle$$

Here, the two-body operator $U(z)$ is taken from the two-body problem, eq. (2.3), and the operator S is given by the three-body equation

$$\sum_{A'B'b'} \langle ABb|\varepsilon_a+\varepsilon_b-H_o-q-U'-U''|A'B'b'\rangle S_{A'B'ab'} = \langle AB|U(\varepsilon_a+E_b|ab\rangle$$

where

$$\langle ABb|U'|A'B'b'\rangle = \langle AB|U(\varepsilon_a+\varepsilon_b)|A'B'\rangle\delta_{bb'}$$

$$q|ABb\rangle = \sum_{b'} \langle Ab'|U(\varepsilon_a+\varepsilon_b-E_B+E_{b'})|Ab'\rangle|ABb\rangle$$

$$\langle ABb|U''|A'B'b'\rangle = \delta_{BB'}\langle Ab|U(\varepsilon_a+E_b+E_{b'}-E_B)|A'b'\rangle$$

Compared to the standard theory, there is a clear similarity, but there are some important modifications:

1. The standard static OBE-potential is replaced by $U(z)$ (yielding U' and U'') which takes into account retardation effects in the medium (see refs.[2,7]) and - via the "dressing" factor $r_\alpha(z)$ - off-shell renormalization effects[4].
2. There is no lowest order contribution to ε_b - the corresponding diagram of perturbation theory would involve three hole lines and is not generated within our order of approximation.

3. The equation for S contains an additional one-body-operator q which takes into account Pauli-corrections to the selfenergy of the V-particle[4].
4. Another new feature is the appearance of a 3-body equation for S due to the additional V-particle-N-hole potential U" . This means that within the here defined two-hole-line expansion there occur terms describing the dressing of pions in the medium. In perturbation theory, these terms correspond to diagrams of the type of fig. 3.

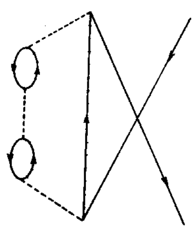


Fig. 3: Diagram of perturbation theory contributing to the V-particle energy ε_a which is generated by the three-body equation (4.1).

V. THE PROBLEM OF BOSON CONDENSATION AND REFINEMENTS OF THE HOLE-LINE EXPANSION

Let us consider the system of V-matter given by the ground-state of the sector (n,n), $n \to \infty$ (corresponding to neutron matter when V,N and Θ are identified with n,p and π^-). The problem of Θ--condensation[8] is then defined through a symmetry breaking in the sense, that the expansion of the ground-state based upon ϕ_p given by

$$\phi_p = e^{cb_p^+}\phi \qquad\qquad \phi = \prod_{\alpha\leq k_F} V_\alpha^+|0>$$

yields more binding energy than that upon ϕ . (Clearly, ϕ_p breaks the symmetry given by the operator Q_2). For a given ansatz of S , the onset of Θ-condensation is determined by calculating for $\psi_p = \exp S\phi_p$ the expectation value $<\psi_p H\psi_p>|<\psi_p\psi_p>$ and looking for that density where - for a suitable momentum p - a minimum of the energy occurs for $c \neq 0$.

Within this talk, we shall discuss only lowest order choices of S , i. e. up to a N^+Vb^+-term. We shall demonstrate the structure of the results for the classical case (lowest order) and the prediction of the one-hole-line and an RPA expansion of $\langle\psi_p H\psi_p\rangle/\psi_p\psi_p\rangle$ for the next order.

i) The "classical" approximation to the Θ-condensate consists in taking the ansatz

$$S = \sum_{a\beta} d_{a\beta} N_\beta^+ V_a \qquad (a \leq k_F) \tag{5.2}$$

The expectation value $E_p = \langle\psi_p H\psi_p\rangle/\langle\psi_p\psi_p\rangle$ is now rigorously calculable: for $c = 0$ we obtain a minimum for $d_{a\beta} = 0$ and $E = \Sigma E_a^o$. This energy is decreased with $c \neq 0$ when k_F is larger than that given by the phase transition condition

$$\omega_p + \sum_{\beta a} \frac{|W^o_{a\beta p}|^2}{E^o_a - E_\beta} = 0 \quad .$$

This is the classical, standard Green's function condition, which is not senseful within our model because of lack renormalization.

ii) The simplest generalization is to take for S the ansatz

$$S = \sum_{a\beta} d_{a\beta} N_\beta^+ V_a + \sum_{a\beta k} c_{a\beta k} N_\beta^+ V_a b_k$$

$$k \neq p \quad \text{for} \quad c \neq 0$$

(The condition $k \neq p$ for $c \neq 0$ is to avoid double counting). For $c = d_{a\beta} = 0$ and $\langle\psi H\psi\rangle/\langle\psi|\psi\rangle$ taken within a one-hole-line expansion (see fig. 4)

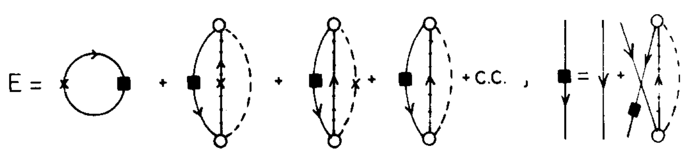


Fig. 4: One-hole-line expansion for the binding energy of normal V-matter.

we obtain

$$E = \sum_a \gamma_a E_a^o + \sum_{a\beta k} C_{a\beta k} W^o_{a\beta k}\gamma_a \ {}^{+c.c.} + \sum_{a\beta k} |C_{a\beta k}|^2 (E_\beta + \omega_k)\gamma_a$$

$$\gamma_a = 1 + \sum_{\beta k} |C_{a\beta k}|^2 \gamma_a$$

yielding after variation

$$E = \sum_a E_a \ , \quad C_{a\beta k} = W^o_{a\beta k}/(E_a - E_\beta - \omega_k) \ , \ \gamma_a = z_a$$

like in the one-body case (eq. (2.1)).

Assuming $c \neq 0$ and expanding $\langle\psi_p H\psi_p\rangle/\langle\psi_p\psi_p\rangle$ up to the same order, the onset of a phase-transition is predicted when the condition

$$\omega_p + \sum_{a\beta} |W_{a\beta p}|^2/(E_a - E_\beta) = 0$$

is fulfilled. This is just the classical condition,"ad hoc" renormalized, i. e. modified by replacing unrenormalized by renormalized quantities[8]. Thus the Lee-model calculation supports this standard renormalization prescription.

iii) For a boson condensate, it will probably occur that the "norm" of the operator $S_1 = \Sigma C_{a\beta k} N^+_\beta V_a b^+_k$, given by the "wound integral" $\kappa_1 = \frac{1}{n}\langle\phi S_1^+ S_1 \phi\rangle$, becomes large, and it is expected that this invalidates the one-hole-line expansion of $\langle\psi_p H\psi_p\rangle/\langle\psi_p\psi_p\rangle$. A natural generalization is to include all RPA-bubbles. For $c = d_{a\beta} = 0$, the corresponding truncation of the expectation value expansion is given in fig. 5. The variational equations of the corresponding

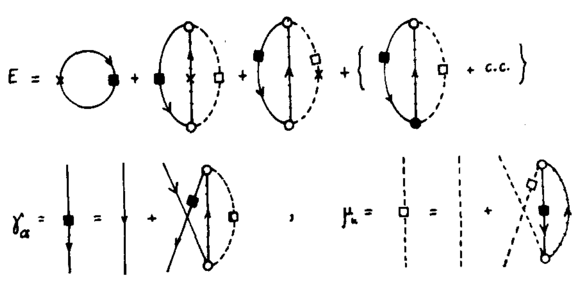


Fig. 5: RPA-expansion of the expection value for $S = \Sigma C_{a\beta k} N^+_\beta V_a b^+_k$.

analytical expressions of the binding energy can be cast into the form

$$E = \sum \varepsilon_a + \sum_k (\mu_k - 1)(\Omega_k - \omega_k)$$

$$\varepsilon_a = E_a + \sum_{\beta k} r_a^2(\varepsilon_a) |W_{a\beta k}|^2 \left(\frac{\mu_k}{\varepsilon_a - E_\beta - \Omega_k} - \frac{1}{\varepsilon_a - E_\beta - \omega_k} \right)$$

$$\Omega_k = \omega_k + \sum_{a\beta} |W_{a\beta k}|^2 \rho_a / (\varepsilon_a - E_\beta - \Omega_k)$$

$$\rho_a^{-1} = 1 + \sum_{\beta k} |W_{a\beta k}|^2 \left(\frac{\mu_k}{(\varepsilon_a - E_\beta - \Omega_k)^2} - \frac{1}{(E_a - E_\beta - \omega_k)^2} \right)$$

$$\mu_k^{-1} = 1 - \sum_{a\beta} |W_{a\beta k}|^2 \rho_a / (\varepsilon_a - E_\beta - \Omega_k)^2$$

A corresponding RPA-expansion of $\langle\psi_p H \psi_p\rangle / \langle\psi_p \psi_p\rangle$ $(c \neq 0)$ yields the phase transition condition

$$\omega_p + \sum_{a\beta} \frac{|W_{a\beta p}|^2 \rho_a}{\tilde{\varepsilon}_a - E_\beta} = 0$$

$$\tilde{\varepsilon}_a = \varepsilon_a + \sum_{k\beta} \mu_k \frac{|W_{a\beta k}|^2 \rho_a}{(\varepsilon_a - E_\beta - \Omega_k)^2} (\omega_k - \Omega_k)$$

Compared to eq. (5.1), the medium now acts in renormalizing both the V- and Θ-particle energies and the interaction. It should be worth while to check the validity of this RPA-expansion for the solvable model introduced in ref.[9].

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