EFFECTIVE INTERACTIONS, QUASICONFIGURATIONS AND THE SHELL MODEL

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The ultimate aim of an effective interaction theory should be to link the free nucleon-nucleon interaction and the properties of real nuclei. In such a program one can, at least formally, distinguish two steps; 1) the N-N interaction has to be smoothed by means of a Brueckner procedure to give a G-matrix, and 2) the resulting effective interaction has to be adapted to a model space tractable from a computational and interpretative point of view. We shall be concerned in these notes with the second aspect, the relation between model spaces and effective operators. Nevertheless, we shall start by examining the general problem.

Let us take a system with N-particles interacting via a Hamiltonian H. Suppose one can select a subspace of the full Hilbert space containing the relevant degrees of freedom of the problem, for example, the 0p-0h state if we are interested in the ground state of nuclear matter or the $(sd)^n$ or $(f_{7/2})^n$ configurations if we are studying the low energy spectra of nuclei between A=18 and A=54. We denote this subspace by $\{|I\rangle\}$, the rest of the space by $\{|J\rangle\}$, and one specific state belonging to $\{|I\rangle\}$ or $\{|J\rangle\}$ by $|i\rangle$ and $|j\rangle$ respectively.

On this basis, the matrix to be diagonalized can be written in blocks:

$$\begin{bmatrix} \langle I|H|I'\rangle & \langle I|H|J\rangle \\ \langle J|H|I\rangle & \langle J|H|J'\rangle \end{bmatrix} \qquad (1)$$

obtaining the effective interaction to be used in the model space $|I\rangle$ is then equivalent to finding a transformation from the basis $\{|I\rangle, |J\rangle\}$ to a new one $\{|\bar{I}\rangle, |\bar{J}\rangle\}$, satisfying:

$$\langle \bar{I}|H|\bar{J}\rangle = 0 \qquad (2)$$

The $|\bar{I}\rangle$ and $|\bar{J}\rangle$ blocks being decoupled, the problem reduces to the diagonalization of the matrix $\langle \bar{I}|H|\bar{I}'\rangle$. To be more precise, one also needs to be able to express the new matrix on the old basis, the relation:

$$\langle \bar{I}|H|\bar{I}'\rangle = \langle I|H_{eff}|I'\rangle \qquad (3)$$

defines the effective interaction.

Let us write a general form for the transformed states:

$$\begin{aligned} \langle \bar{i}| &= \langle i| + \langle i|\hat{\underline{V}}|J\rangle\langle J| \\ |\bar{j}\rangle &= |j\rangle - |I\rangle\langle I|\hat{\underline{V}}|j\rangle \end{aligned} \qquad (4)$$

there repeated capitals imply summation; $\hat{\underline{V}}$ is a general many-body operator defined between any two states $|i\rangle$ and $|j\rangle$.

The transformation (4) is a dressing procedure, the bare state $|i\rangle$ gives the label that characterizes the state even if the amplitude it has in $|\bar{i}\rangle$ is very small. In eq.(3) one can see how the dressing and the effective interaction are related. Nevertheless one must keep in mind that the bases used in conjunction with effective interactions are built up of labels, the true states being the heavily dressed configurations that we call quasiconfigurations.

The transformation (4) preserves orthonormality between the $|I\rangle$ and $|J\rangle$ blocks. To determine $\hat{\underline{V}}$ we request perfect decoupling (eq.(2)) leading to

$$\begin{aligned} \varepsilon_{ij}\langle i|\hat{\underline{V}}|j\rangle = \langle i|V|j\rangle + \langle i|\hat{\underline{V}}|J\rangle\langle J|V|j\rangle - \\ - \langle i|V|I\rangle\langle I|\hat{\underline{V}}|j\rangle - \langle i|\hat{\underline{V}}|J\rangle\langle J|V|I\rangle\langle I|\hat{\underline{V}}|j\rangle \ , \end{aligned} \qquad (5)$$

where we split H into H_D+V (H_D is a diagonal part to be discussed later). $\varepsilon_{ij} = \varepsilon_i - \varepsilon_j$ and ε_i, ε_j are eigenvalues of H_D. The decoupling condition (5) is a non-linear integral equation for the operator $\hat{\underline{V}}$. The next step in the solution of the problem depends on the particular case being treated.

-In the case of nuclear matter, eq.(5) can be shown to be equivalent to the e^s equations (see ref.1).

-For finite nuclei, if H is the bare N-N interaction, eq.(5) becomes a set of coupled non-linear integral equations.

-If H is the result of a G-matrix procedure then a perturbative solution of (5) can be envisaged. We shall examine in more detail the last case (ref. 2).

The relevant expressions in second order perturbation theory are

a) the decoupling condition

$$\langle\bar{i}|H|\bar{j}\rangle = \langle i|V|j\rangle\left[1+\frac{\varepsilon_j-\varepsilon_i}{\varepsilon_{ij}}+\frac{\langle j|V|j\rangle-\langle i|V|i\rangle}{\varepsilon_{ij}}\right]+ \tag{6}$$

$$+\frac{\langle i|V_N|J\rangle\langle J|V_N|j\rangle}{\varepsilon_{ij}}-\frac{\langle i|V_N|I\rangle\langle I|V_N|j\rangle}{\varepsilon_{IJ}}+0(W^3/\varepsilon^2)\ ,$$

where writing V_N stresses that only strictly non diagonal parts remain. W is a typical non diagonal matrix element.

This decoupling condition shows clearly the importance of the choice of energy denominators. It can be written as

$$\langle\bar{i}|H|\bar{j}\rangle = \frac{W\cdot\Delta}{\varepsilon}+\frac{W^2}{\varepsilon}+0(W^3/\varepsilon^2)\ , \tag{7}$$

where Δ stands for the diagonal terms not included in ε. It is evident that if $\Delta\tilde{\gtrsim}\varepsilon$ no decoupling is obtained.

b) the new matrix in the model space

$$\langle\bar{i}|H|\bar{i}'\rangle = \langle i|H|i'\rangle+\frac{1}{2}\left[\frac{1}{\varepsilon_{iJ}}+\frac{1}{\varepsilon_{i'J}}\right]\cdot \tag{8}$$

$$\cdot\ \langle i|V|J\rangle\langle J|V|i'\rangle+0(W^3/\varepsilon^2)\ .$$

There is a choice of energy denominators that makes $\Delta=0$. It corresponds to taking $|i\rangle$ and $|j\rangle$ such as to make the blocks diagonal, that is:

$$\langle i|H|i'\rangle = \varepsilon_i\delta_{ii'}$$

$$\langle j|H|j'\rangle = \varepsilon_j\delta_{jj'}\ . \tag{9}$$

Nevertheless this choice complicates the extraction of an effective interaction through some kind of expression (3), that is why we have included in our discussion the case in which the energy denominators contain only a part of the diagonal interaction.

Let us apply the notion of dressing (quasiconfigurations) to a model case. Imagine f^n is a model space (think of the sd-shell) suitable for the description of a certain kind of states. An obvious dressing of the f^n state is represented by the process

$$f^n \xrightarrow[\text{Wfhff}]{} f^{n+1}\ h^{-1} \xrightarrow[\text{Wfffh}]{} f^n\ , \tag{10}$$

where h stands for full orbits (core), and the W's are the antisymmetrized two body matrix elements associated to each step. Using the techniques described below, one can easily show that up to second order in perturbation theory this dressing leads to quasiconfigurations f^n satisfying:

$$f_i^n \; H \; f_{i'}^n = f_i^n \; H+\omega_1+\omega_2+\omega_3 \; f_{i'}^n \qquad (11)$$

with

$$\omega_1 = - \frac{1}{\varepsilon} a_f^+ a_f \; W_{fhff} \; W_{fffh}$$

$$\omega_2 = - \frac{1}{\varepsilon} a_f^+ a_f^+ a_f a_f \; W_{fhff} \; W_{fffh}$$

$$\omega_3 = - \frac{1}{\varepsilon} a_f^+ a_f^+ a_f^+ a_f a_f a_f \; W_{fhff} \; W_{fffh}$$

Here we see how many body forces appear in the effective interaction through the dressing procedure. The importance of such terms was already recognized by doing the spectroscopy of the sd-shell. A good description of the region must be independent of the fact of using the ^{16}O as a core and adding particles or the ^{40}Ca and working with holes. This simple fact makes it possible to write closed expressions relating the following experimental quantities; single particle energies and two body centroids, and the energy of the closed shell, the single hole energies and the two hole centroids (see ref.3). To satisfy these sum rules one must call for up to four body forces. In that case, using a realistic interaction (for instance, the Kuo-Brown (4) interaction) and with only three free centroidal parameters left, after fulfilling the sum rules, one gets spectroscopic results of a quality comparable to those obtained by fitting the 63 matrix elements of the two body force. In addition the results shown in Table 1 for

Element	A(T)	Exp (MeV)	E_c (MeV)
O	18 (1)	- 12.19	- 12.08
F	18 (0)	- 13.29	- 13.30
O	19 (3/2)	- 16.15	- 16.12
F	19 (1/2)	- 23.72	- 23.71
Ne	20 (2)	- 23.96	- 23.98
Ne	20 (1)	- 30.43	- 30.44
Ne	20 (0)	- 40.69	- 40.46
Ne	21 (3/2)	- 38.59	- 38.57
Ne	21 (1/2)	- 47.46	- 47.07
Na	22 (0)	- 58.52	- 57.56
Ne	22 (1)	- 57.82	- 57.70
Mg	24 (0)	- 87.48	- 87.41
Si	28 (0)	- 136.42	- 136.33
S	32 (0)	- 183.60	- 183.70
Cl	34 (1)	- 203.63	- 203.27
Cl	35 (1/2)	- 216.27	- 216.35
Cl	35 (3/2)	- 210.62	- 209.84
Cl	36 (1)	- 224.86	- 224.58
Ar	36 (0)	- 231.53	- 231.82
Cl	36 (2)	- 220.54	- 220.39
Ar	37 (1/2)	- 240.315	- 240.01
Ar	37 (3/2)	- 235.33	- 235.35
K	38 (0)	- 252.40	- 252.18
Ar	38 (1)	- 252.15	- 251.84
Ca	39 (5/2)	- 259.12±0.5	- 258.62
Ca	39 (1/2)	- 263.02	- 263.02
Ca	39 (3/2)	- 265.52	- 265.52
Ca	40 (0)	- 281.119	- 281.119

the binding energies are obtained.

Another interesting example of the application of the quasiconfiguration approach to the obtainment of an effective interaction is provided by the reduction of the $(1f_{7/2}\ 2p_{3/2}\ 1f_{5/2}\ 2p_{1/2})^n$ model space used to describe the low-lying states of nuclei with $40 \leq A \leq 56$, to a $(1f_{7/2})^n$ model space (ref.2). In second order perturbation theory, the relevant dressings of f^n states are of the type $f^{n-1}r$ and $f^{n-2}r^2$; $r \equiv (2p_{3/2}\ 1f_{5/2}\ 2p_{1/2})$. Then expression (8) can be written explicitly as:

$$\langle \bar{f}^n_i | H | \bar{f}^n_{i'} \rangle = \langle f^n_i | H + \frac{1}{\varepsilon_1} H_{R1} + \frac{1}{\varepsilon_2} H_{R2} | f^n_{i'} \rangle \qquad (12)$$

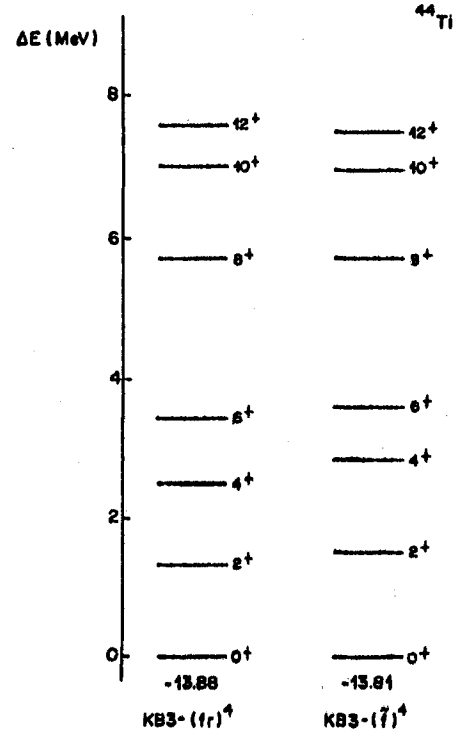
with

$$H_{R1} = -1/2 \sum_{\Gamma\Gamma'} (-1)^{f+r-\Gamma'} \left(\frac{[\Gamma\Gamma']}{[r]}\right)^{1/2} W_1^\Gamma W_1^{\Gamma'} \left[((AA)^\Gamma B)^r (A(BB)^{\Gamma'})^r\right]^0$$

$$H_{R2} = -1/2 \sum_{\Gamma} [\Gamma]^{1/2} (W_2^\Gamma)^2 \left[(AA)^\Gamma (BB)^\Gamma\right]^0$$

$$W_1^\Gamma = \langle ff | H | fr \rangle_\Gamma \qquad W_2^\Gamma = \langle ff | H | rr \rangle_\Gamma$$

Again the effective interaction incorporates three body forces. Spectroscopic calculations have been done using realistic interactions corrected by monopole behavior (see ref.2).



Energy denominators are taken as the differences between the centroidal energies of the configurations f^n and $f^{n-1}r$ or $f^{n-2}r^2$. The agreement between the quasiconfigurations and exact results is impressive A typical example is given in figure 1. It is worth commenting that the wave function resulting from the complete diagonalization contains amplitudes $f^4_{7/2}$ which are as small as 30%. A link cluster mechanism must be present in the shell model matrices to keep such a degree of accuracy in the quasiconfiguration picture. The spectroscopy carried out for all nuclei in the region

shows that realistic interactions do a good job for energy levels and transitions. Binding energies demand a small three body correction coming from the external space; once this correction of ∿10 KeV is included, the agreement becomes comparable to that of energy levels, that is ∿200 KeV.

In conclusion, we have shown in this talk how the quasiconfiguration approach, 1) makes it possible to reduce drastically the shell model spaces, while keeping control on the effective interactions, 2) clarifies the role of energy denominators in the convergence of perturbative expansions, and 3) the shell model spectroscopy can give precise information on the isoscalar and isovector monopole parts of the effective interactions.

Note: Free use has been made of results obtained by/or with A. Cortes and A. Zuker.

References

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