

MICROSCOPIC CALCULATIONS ON PAIR-CONDENSED  
STATES OF INFINITE FERMI SYSTEMS

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Abstract. An analysis of the spatial correlations in the BCS pair-condensed state of strongly interacting Fermi systems is presented. The short-range correlations among the particles are described by state-independent correlations of the Jastrow type. Preliminary results of the Fermi hypernetted chain (FHNC) evaluation of the radial distribution function, the one-particle momentum distribution and the pair distribution of neutron and nuclear matter are also given.

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## 1. Introduction.

The pairing phenomena in Fermi systems have been the object of many theoretical and experimental studies, and remarkable theories have been developed which explain several physical effects of the superfluid and superconducting systems. Most of these theories rely on the assumption that the particles interact through an effective weak interaction. It is an interesting problem to study the relationship of such an effective interaction with the realistic interaction, acting among the particles, by means of a microscopic calculation.

The successes of the variational theory, based on correlated trial wave functions, in describing the ground state of strongly interacting systems is encouraging for applying this approach to the study of pair-condensed states. These states must be described by correlated state vectors of the form  $|\Psi_S\rangle = \hat{F}|\Phi_S\rangle$ , where  $|\Phi_S\rangle$  is a model state vector which adequately describes the weak-interacting system and the operator  $\hat{F}$  contains the short-range correlations. Some recent interesting calculations<sup>(1-3)</sup> have been performed, in the framework of the above theory, for both  $^3\text{He}$  and neutron matter. These calculations are based on some approximations made either in the way of taking into account of the short-range correlations of the operator  $\hat{F}$ , or in the evaluations of the mean values of the various cluster terms on the model state vector  $|\Phi_S\rangle$ .

In this contribution, some preliminary results of a microscopic calculation on the pair-condensed states of neutron and nuclear matter, described by trial state vector  $|\Psi_S\rangle$ , are presented. The model state vector considered is the  $|\text{BCS}\rangle$  state vector, which is adequate for describing the S-wave pairing. The correlations induced by the operator  $\hat{F}$  are taken to be state-independent and of the Jastrow type. Such an ansatz, even if not sufficiently adequate when strong state-dependent interactions are present among the particles, should still provide for accurate realistic calculations on the pairing phenomena in these strongly-interacting systems.

The technique<sup>(4)</sup> adopted here does not involve any of the approximations which are present in the earlier calculations of the same kind<sup>(1-3)</sup>. Such a method allows for the evaluation of the two-body distribution functions and the one- and two-body density matrices, through the solution of a set of integral equations of the FHNC type. The numerical effort needed to solve these equations is almost the same as in the case of normal Fermi systems. This feature allows for further improve-

ments of the trial state vector. On one side other types of pairing can be included in the model state vector  $|\phi_g\rangle$ , on the other side, state-dependent correlations can be introduced by using similar treatments as in the case of normal Fermi systems.

In section 2 some details on the trial state vector considered and the physical quantities calculated are given. The FHNC method is briefly discussed in section 3. In section 4 the results for the structure function, the one-particle momentum distribution and the pair distribution are presented.

## 2. Trial state vector and density matrices.

In a variational theory of strongly-interacting systems the trial wave function must describe adequately the short-range correlations induced by the potential. In the case of a BCS pair-condensed state of a Fermi fluid this requirement can be achieved by means of the following state vector

$$|\psi_S\rangle = \hat{F}|\text{BCS}\rangle. \quad (1)$$

The model state vector is given by

$$|\text{BCS}\rangle = \prod_{\vec{k}} (u_{\vec{k}} + v_{\vec{k}} a_{\vec{k}\uparrow}^+ a_{-\vec{k}\downarrow}^+) |0\rangle, \quad (2)$$

where  $|0\rangle$  is the vacuum state,  $a_{\vec{k}\sigma}^+$  is the creation operator for a fermion in the single particle orbital  $(\vec{k}, \sigma)$  and the amplitudes  $u_{\vec{k}}$  and  $v_{\vec{k}}$  are real variational functions, subjected to the condition  $u_{\vec{k}}^2 + v_{\vec{k}}^2 = 1$  which ensures the normalization property  $\langle \text{BCS} | \text{BCS} \rangle = \langle 0 | 0 \rangle = 1$ . The correlation operator  $\hat{F}$  is a second quantized operator which becomes the N-particle correlation operator  $\hat{F}_N$  in the space of N-particle states. As a consequence, a more precise definition of  $|\psi_S\rangle$  is given by<sup>(3)</sup>

$$|\psi_S\rangle = \sum_N \sum_{(m_N)} F_N |\phi_N^{(m)}\rangle \langle \phi_N^{(m)} | \text{BCS} \rangle, \quad (3)$$

where  $|\phi_N^{(m)}\rangle$  is a N-particle state specified by the single particle states  $m_N = (m_1, m_2, \dots, m_N)$ . The correlation factor is taken to be of the Jastrow type, which means that the coordinate representation of the state vector  $F_N |\phi_N^{(m)}\rangle$  is given by

$$\langle \vec{x}_1 \dots \vec{x}_N | F_N |\phi_N^{(m)}\rangle = \prod_{i>j=1}^N f(r_{ij}) \prod_{m_1}^{m_N} \{\phi_{m_1}(\vec{x}_1) \dots \phi_{m_N}(\vec{x}_N)\}, \quad (4)$$

where  $\vec{x}_i$  represents both the spatial and the spin variables. It must be noticed that  $|\psi_S\rangle$  does not correspond to a state with a definite number of particles. This implies that, in variational calculations, the correlation factor  $f(r_{ij})$  and the BCS amplitudes are constrained to the condition  $\langle \hat{N}_{op} \rangle / \Omega = \rho$  where  $\hat{N}_{op}$  is the particle-number operator and  $\rho$  is the density of the system. If this condition is satisfied the particle-number dispersion can be neglected in the case of a system with a very large number of particles<sup>(4)</sup>.

The successive density matrices furnish a complete description of a quantum fluid. The one-body density matrix can be defined as<sup>(5)</sup>

$$n_{\sigma_1\sigma_2}^{(1)}(\vec{x}_1, \vec{x}_1) = \frac{v}{\mathcal{N}^0} \langle \Psi_S | \psi_{\sigma_1}^\dagger(\vec{x}_1) \psi_{\sigma_1}(\vec{x}_1) | \Psi_S \rangle = \delta_{\sigma_1\sigma_1} n(r_{11}) \quad , \quad (5)$$

where  $\psi_\sigma^\dagger(\vec{r})$  ( $\psi_\sigma(\vec{r})$ ) is the creation (annihilation) operator for a fermion with spin projection  $\sigma$  at the field point  $\vec{r}$ , and  $\mathcal{N}^0$  is the normalization constant. The last equality in eq.(5) holds since the system considered is homogeneous and unpolarized. Analogously, the two-body density matrix is defined as

$$n_{\substack{\sigma_1\sigma_1 \\ \sigma_2\sigma_2}}^{(2)}(\vec{x}_1, \vec{x}_1, \vec{x}_2, \vec{x}_2) = \frac{v^2}{\mathcal{N}^0} \langle \Psi_S | \psi_{\sigma_1}^\dagger(\vec{x}_1) \psi_{\sigma_1}(\vec{x}_1) \psi_{\sigma_2}^\dagger(\vec{x}_2) \psi_{\sigma_2}(\vec{x}_2) | \Psi_S \rangle \quad . \quad (6)$$

In the following, only the terms which are diagonal in the spin variables will be considered and denoted as  $n_{\sigma_1\sigma_2}^{(2)}(\vec{x}_1, \vec{x}_1, \vec{x}_2, \vec{x}_2)$ . It is of interest to know whether this quantity shows the off-diagonal long range order proper of the BCS pairing. The diagonal elements of the two-body density matrix represent the spin-dependent distribution functions

$$g^{\sigma_1\sigma_2}(r) = \frac{1}{2} n_{\sigma_1\sigma_2}^{(2)}(\vec{x}_1, \vec{x}_1, \vec{x}_2, \vec{x}_2) \quad , \quad (7)$$

which are the fundamental quantities for evaluating the expectation values of the energy. In particular, the mean value of a two-body spin independent operator can be expressed in terms of the radial distribution function

$$g(r) = \frac{1}{v} \sum_{\sigma_1\sigma_2} g^{\sigma_1\sigma_2}(r) \quad . \quad (8)$$

The density matrices are often studied in momentum space since the corresponding quantities are more closely related to experimental measurements. The one-particle momentum distribution, which is expressed by the relation

$$n(k) = \frac{1}{\mathcal{N}^0} \langle \Psi_S | a_{k\sigma}^\dagger a_{k\sigma} | \Psi_S \rangle = \frac{1}{v} \int d\vec{r} n(r) \exp\{i\vec{k}\cdot\vec{r}\} \quad , \quad (9)$$

gives the occupation probability of the single particle orbital ( $\vec{k}\sigma$ ) in the state vector  $|\Psi_S\rangle$ . It is interesting to evaluate for a realistic choice of  $|\Psi_S\rangle$  how much such a probability differs from the corresponding one of the weak-interacting system. The pair distribution, defined as

$$n^{\sigma_1\sigma_2}(\vec{k}_1, \vec{k}_2) = \frac{1}{\mathcal{N}^0} \langle \Psi_S | a_{k_1\sigma_1}^\dagger a_{k_1\sigma_1} a_{k_2\sigma_2}^\dagger a_{k_2\sigma_2} | \Psi_S \rangle \quad , \quad (10)$$

may be evaluated by taking the Fourier inverse of the matrix  $n_{\sigma_1\sigma_2}^{(2)}$ . It is known that for the superfluid phase of fermions such a distribution is a smoothly varying function of the momenta  $\vec{k}_1$  and  $\vec{k}_2$ , except when the single particle states are related by the pairing condition (in the case of BCS pairing the relation is  $\vec{k}_2\sigma_2 = -\vec{k}_1-\sigma_1$ ). In this case, the probability  $n^{\sigma_1\sigma_2}(\vec{k}_1, \vec{k}_2)$  increases by a finite amount  $\chi_F^2(k_1)$  with respect to the value of the uncorrelated pairs. For the pure BCS state vector one gets  $\chi_F(k_1) = u(k_1)v(k_1)$ .

Other interesting quantities, which are related to the two-body distribution functions, are the structure functions expressed by the relation

$$S^{\sigma_1\sigma_2}(k) = \delta_{\sigma_1\sigma_2} + \frac{\rho}{v} \int d\vec{r} (g^{\sigma_1\sigma_2}(\vec{r}) - 1) \exp\{i\vec{k}\cdot\vec{r}\} \quad . \quad (11)$$

The liquid structure function  $S(k) = \frac{1}{v} \sum_{\sigma_1\sigma_2} S^{\sigma_1\sigma_2}(k)$  describes the correlations between the density fluctuations which are present in the trial state  $|\psi_S\rangle$ .

### 3. FHNC approach.

The FHNC procedure to calculate the quantities defined in the previous section is discussed in detail in ref.(4). Here we summarize the results of that derivation and briefly discuss the structure of the radial distribution function and of the density matrices. The FHNC scheme, to be used in connection with a trial state vector of the form as given in eq.(3), strictly follows the standard FHNC procedure derived<sup>(6)</sup> for the Jastrow-Slater ansatz. An excellent review of this procedure has been made by ROSATI<sup>(7)</sup> (see also ref.(8) for the case of polarized Fermi systems). The role played there by the squared Slater determinant is assumed here by the following quantities

$$\Delta_p(\vec{r}_1, \dots, \vec{r}_p) = \sum_{s_1 \dots s_p} \sum_{\substack{m_1 \dots m_p \\ n_1 \dots n_p}} \phi_{n_1}^*(\vec{x}_1) \dots \phi_{n_p}^*(\vec{x}_p) \phi_{m_1}(\vec{x}_1) \dots \phi_{m_p}(\vec{x}_p) \cdot \langle \text{BCS} | a_{n_p}^\dagger \dots a_{n_1}^\dagger a_{m_1} \dots a_{m_p} | \text{BCS} \rangle, \quad (12)$$

where  $\phi_m(x)$  is the normalized single particle wave function associated with the state  $m=(\vec{k}\sigma)$  and where for each index  $m_i$  ( $n_i$ ) the sum is extended to all the single particle states. A detailed analysis<sup>(4)</sup> of the above quantities shows that they can be written in the cluster form

$$\Delta_p(\vec{r}_1, \dots, \vec{r}_p) = \rho_0^p \{ 1 + \sum_{i>j=1}^p Y_2(\vec{r}_i, \vec{r}_j) + \sum_{i>j>k=1}^p Y_3(\vec{r}_i, \vec{r}_j, \vec{r}_k) + \dots \}, \quad (13)$$

where  $\rho_0 = \frac{v}{(2\pi)^3} \int d\vec{k} v^2(k)$  is the density of the uncorrelated state vector  $|\text{BCS}\rangle$ . The quantities  $Y_q(\vec{r}_{i_1}, \dots, \vec{r}_{i_q})$  are symmetric functions of the coordinates  $\vec{r}_{i_1}, \dots, \vec{r}_{i_q}$  and are characterized by allowed products of the following two types of statistical correlations

$$l_V(r) = \frac{v}{(2\pi)^3 \rho_0} \int d\vec{k} v^2(k) \exp\{i\vec{k} \cdot \vec{r}\}, \quad (14)$$

$$l_U(r) = \frac{v}{(2\pi)^3 \rho_0} \int d\vec{k} u(k)v(k) \exp\{i\vec{k} \cdot \vec{r}\}.$$

By introducing the complex function  $L(r) = -\frac{1}{v} l_V(r) + \frac{i}{v} l_U(r)$ , as statistical correlation factor, the various terms contributing to  $Y_q$  may be constructed in the following way: firstly, all the possible

terms in which the factors  $L(r_{ij})$  appear in the form of closed separate loops are considered. Then each term is multiplied by  $(-2v)^{n_L/2}$  where  $n_L$  is the number of the closed loops and  $n_2$  the number of loops involving two indices only. Finally, only the real part of each product corresponding to a closed loop is taken.

It can be easily verified that the quantity  $\Delta_p$  satisfies the cluster property, namely  $\Delta_p(\vec{r}_1, \dots, \vec{r}_p) + \Delta_p(\vec{r}_1, \dots, \vec{r}_q) \Delta_{p-q}(\vec{r}_{q+1}, \dots, \vec{r}_p)$  if the subset  $\vec{r}_1, \dots, \vec{r}_q$  of particles is removed far away from the rest. This property allows for a complete cancellation among the unlinked terms of the expansion of the radial distribution function and the density matrices, which, ultimately, result to be expressed by series of linked but reducible terms.

The procedure to sum the above series has been derived by FANTONI and ROSATI<sup>(9)</sup> in their calculation of the one- and two-body distribution functions in finite Fermi systems and is discussed in detail in ref.(4). As a result, the radial distribution function has the following structure

$$g(r_{12}) = 1 + N_{dd}(r_{12}) + X_{dd}(r_{12}) + 2 \frac{c_d}{c} (N_{de}(r_{12}) + X_{de}(r_{12})) + \left(\frac{c_d}{c}\right)^2 (N_{ee}(r_{12}) + X_{ee}(r_{12})) \quad (15)$$

The functions  $N_{mn}(r_{12})$  ( $X_{mn}(r_{12})$ ) are given by the sum of nodal (non-nodal) irreducible renormalized cluster terms<sup>(+)</sup> of the type specified by the subscripts (see ref.(8) for the notations). The vertex correction  $c$  corresponds to the sum of all cluster terms with only one external point and is related to the density of the system through the equation  $\rho = c \rho_0 = \langle \hat{N}_{op} \rangle / \Omega$ . The vertex correction  $c_d$  is the sum of the cluster terms in which the external point is involved by dynamical correlations  $h(r) = f^2(r) - 1$  only.

Five coupled integral equations must be solved for evaluating the nodal functions  $N_{mn}(r_{12})$  and other two algebraic equations enter in the evaluation of the vertex corrections.

The one-body density matrix is also expressed in terms of irreducible

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(+) A renormalized cluster term is a cluster term multiplied by a proper vertex correction for each of its indices. A vertex correction associated with an index  $i$  of a cluster term  $\Gamma$  corresponds to the sum of all the allowed products of dynamical and statistical correlations having only the index  $i$  in common with  $\Gamma$ .



quantities, as follows

$$n(r_{11}) = v\rho_0 c_\xi^2 \text{Re}\{L(r_{11}) + N_{\xi\xi\text{CC}}(r_{11}) + \mathcal{E}_{\xi\xi\text{CC}}(r_{11})\} \exp\{N_{\xi\xi}(r_{11}) + \mathcal{E}_{\xi\xi}(r_{11})\}. \quad (16)$$

The vertex correction  $c_\xi$  is the sum of the cluster terms in which the external point is involved by dynamical correlations  $\xi(r) = f(r) - 1$  only. Four coupled integral equations must be solved for evaluating the functions  $N_{\xi\xi}(r)$  and  $N_{\xi\xi\text{CC}}(r)$ . Owing to the fact that the diagonal part of the density matrix gives the density  $\rho$  of the system, the following sum rules hold

$$\begin{aligned} n(0) &= c\rho_0, \\ n^* &= c_\xi^2 \exp\{N_{\xi\xi}(0) + \mathcal{E}_{\xi\xi}(0)\} = 1. \end{aligned} \quad (17)$$

The two-body density matrix can be expressed as a series of irreducible renormalized cluster terms with four external indices. A subset of these cluster terms is characterized by terms having two unlinked products of correlation functions, each involving a couple of external indices. As a result, the matrix has the following structure

$$\begin{aligned} n_{\uparrow\uparrow}^{(2)}(\vec{r}_1, \vec{r}_1', \vec{r}_2, \vec{r}_2') &= n_{\downarrow\downarrow}^{(2)}(\vec{r}_1, \vec{r}_1', \vec{r}_2, \vec{r}_2') = \\ &= f(r_{12})f(r_{1,2'})\{n(r_{11})n(r_{22}) - n(r_{12})n(r_{1,2'}) + \text{Linked portions}\}, \\ n_{\uparrow\downarrow}^{(2)}(\vec{r}_1, \vec{r}_1', \vec{r}_2, \vec{r}_2') &= n_{\downarrow\uparrow}^{(2)}(\vec{r}_1, \vec{r}_1', \vec{r}_2, \vec{r}_2') = \\ &= f(r_{12})f(r_{1,2'})n(r_{11})n(r_{22}) + \chi_F(r_{12})\chi_F(r_{1,2'}) + \text{Linked portions}, \end{aligned} \quad (18)$$

where the pairing function  $\chi_F(r)$  is given by

$$\chi_F(r) = v\rho_0 c_\xi^2 \text{Im}\{L(r) + N_{\xi\xi\text{CC}}(r) + \mathcal{E}_{\xi\xi\text{CC}}(r)\} \exp\{N_{\xi\xi}(r) + \mathcal{E}_{\xi\xi}(r)\}. \quad (19)$$

It can be easily verified that the matrix has an independent particle behaviour in the limit in which the particles 1 and 2 are far away from each other. On the other side, the function  $\chi_F(r)$  of eq.(19) shows the BCS pairing behaviour of the matrix, namely  $n_{\sigma_1\sigma_2}^{(2)}(\vec{r}_1, \vec{r}_1', \vec{r}_2, \vec{r}_2') \rightarrow \delta_{\sigma_1, -\sigma_2} \chi_F(r_{12}) \chi_F(r_{1,2'})$  when  $|\vec{r} - \vec{r}'|$  approaches infinity. The pair distribution is given by

$$n_{\sigma_1\sigma_2}(\vec{k}_1, \vec{k}_2) = n(k_1)n(k_2) + \chi_F^2(k_1) \delta_{\sigma_1, -\sigma_2} \delta_{\vec{k}_1, -\vec{k}_2} + O(1/A), \quad (20)$$

where the function  $\chi_F(k) = \frac{1}{v} \int d\vec{r} \chi_F(r) \exp(i\vec{k} \cdot \vec{r})$  measures the shift

from the value for uncorrelated pairs. It is to be noted that the structures of the two-body density matrix and of the pair distribution show that a state-independent Jastrow-Slater wave function does not exhibit long-range ordering. In fact, in that case, the statistical correlation  $l_U(r)$  vanishes, and, consequently,  $\chi_F(r)=0$  .

#### 4. Numerical results and discussion.

In this section some numerical results for the quantities previously discussed are presented in correspondence to simplified models of neutron ( $\nu=2$ ) and symmetrical nuclear matter ( $\nu=4$ ). In the latter case, the model state vector  $|\phi_g\rangle$  considered is of the type  $|\text{BCS}\rangle_n |\text{BCS}\rangle_p$ , where  $|\text{BCS}\rangle_n$  and  $|\text{BCS}\rangle_p$ , both given by eq.(2), refer to the neutrons and protons, respectively. The two-body correlation factor of the Jastrow ansatz has been taken of the form

$$\begin{aligned} f(r) &= 1 - \exp\{-\alpha(r-d)\} , & r > d \\ &= 0 & , \quad r < d \end{aligned} \quad (21)$$

where  $d$  is the hard-core radius of the potential. This simple one-parameter form of the correlation factor should give a reasonable good description of the short-range correlations, as it results from earlier calculations on the ground state of nuclear matter<sup>(8)</sup>. The potentials considered are the OMY potentials<sup>(10)</sup> with hard-core radii  $d=0.4$  fm and  $d=0.6$  fm, which are denoted respectively by OMY4 and OMY6.

In all the calculations the parameter  $\alpha$  has been kept fixed to the value which gives the minimum of the corresponding normal state energy expectation value.

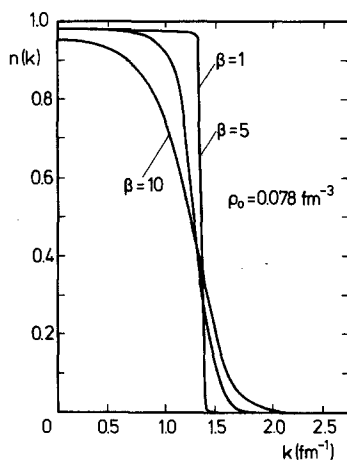


Fig. 1- One particle momentum distribution of the OMY4 model of neutron matter at  $\rho_0=0.078 \text{ fm}^{-3}$  for different values of the parameter  $\beta$  (in MeV units). The value  $\alpha=5.4 \text{ fm}^{-1}$  has been taken for the trial parameter of the two-body correlation function.

The BCS amplitude has been taken of the form

$$v^2(k) = \{1 + \exp((\hbar^2 k^2 / 2m - \lambda) / \beta)\}^{-1}, \quad (22)$$

where  $\lambda$  and  $\beta$  are two variational parameters. Such a variational choice for  $v^2(k)$  has been adopted by PAULICK and CAMPBELL<sup>(2)</sup> in their calculations on superfluid  $^3\text{He}$ . The results of these calculations surprisingly show that the normal phase of  $^3\text{He}$  is always preferred to the superfluid one. Whether this feature is due to a bad variational choice of  $v^2(k)$  or to the approximations made in the calculations is not clear.

In a realistic calculation one should consider the values for the parameters  $\alpha, \lambda$  and  $\beta$  which give the minimum of the energy expectation value, provided that  $c\rho_0 = \rho$ . In these preliminary calculations our attention is limited to an analysis of the behaviour of the radial distribution function and the density matrices as a function of the BCS amplitudes and the density of the system. For this reason different values of the parameters  $\beta$  and  $\rho_0$  (which is simply related to  $\lambda$ ) have been considered. The FHNC/O approximation has been used, namely the contributions from elementary cluster terms have been neglected.

In Fig.1 the results for the one-particle momentum distribution of the OMY4 model of neutron matter are presented in correspondence to three different values of the parameter  $\beta$  at  $\rho_0 = 0.078 \text{ fm}^{-3}$ . For increasing values of  $\beta$  the shape of  $n(k)$  becomes more smooth and more similar to the corresponding shapes of Bose systems. The convergence of the iterative procedure for solving the FHNC equations is quite fast, 5-6 iterations being sufficient for all the cases considered.

$\beta$ (MeV)	1	5	10
$\rho$ ( $\text{fm}^{-3}$ )	0.0776	0.0747	0.0714
$n(0)/\rho$	1.	1.	1.001
$n^*$	1.001	1.001	1.

Table I - Density  $\rho = c\rho_0$  and results on the sum rule check for the OMY4 model of neutron matter at  $\rho_0 = 0.078 \text{ fm}^{-3}$ .

The FHNC/O approximation seems to be sufficiently good for these calculations, since the sum rules are always very well fulfilled, as

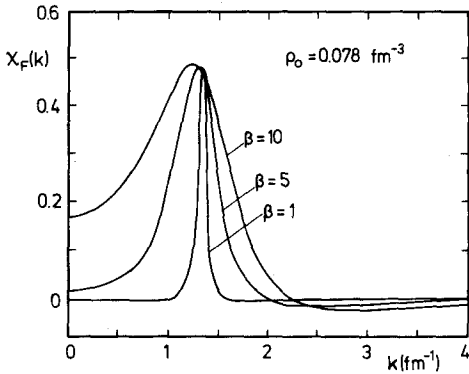


Fig. 2- Same as Fig.1, but for the Fourier transform of the pairing function.

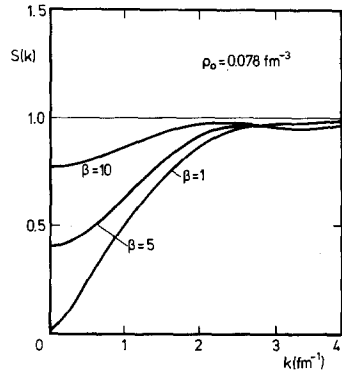


Fig. 3- Same as Fig. 1, but for the Structure function.

can be seen from Table I, where the values of  $n(0)/\rho$ ,  $n^*$  and the density  $\rho$  obtained at  $\rho_0 = 0.078 \text{ fm}^{-3}$  are given. Figs. 2 and 3 display the behaviour of the function  $\chi_F(k)$  and of the structure function  $S(k)$ , respectively. The Fourier transform of the pairing function results to be peaked around the value  $k = (6\pi^2 \rho / v)^{1/3}$ . The effects due to the short-range correlations are not very large at small values of  $k$ , namely, in that region  $\chi_F(k)$  does not differ very much from the pure BCS estimate  $u(k)v(k)$ . The differences are more pronounced at higher values of  $k$ ,

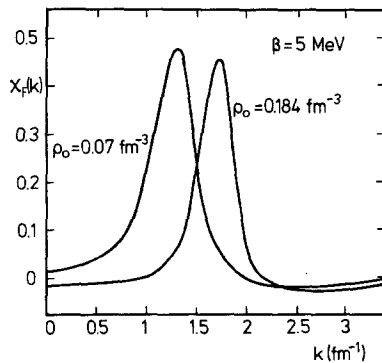


Fig. 4- Fourier transform of the pairing function of the OMY4 model of neutron matter at two different values of  $\rho$ . The parameters  $\alpha$  and  $\beta$  have been kept fixed at the values  $\alpha = 5.4 \text{ fm}^{-1}$ ,  $\beta = 5 \text{ MeV}$ .

where  $\chi_F(k)$  has much more structure than the corresponding quantity of the uncorrelated system. In Fig.4 the behaviour of  $\chi_F(k)$  as a function of the density parameter  $\rho_0$  is displayed. Analogous results have been obtained for the OMY6 model of neutron matter.

The momentum distribution and the function  $\chi_F(k)$  of the OMY6 model of nuclear matter, calculated at  $\rho_0 = 0.185 \text{ fm}^{-3}$  and with  $\beta = 5 \text{ MeV}$  are given in Fig.5. The dashed and solid curves refer to the pure BCS and the correlated BCS evaluations, respectively.

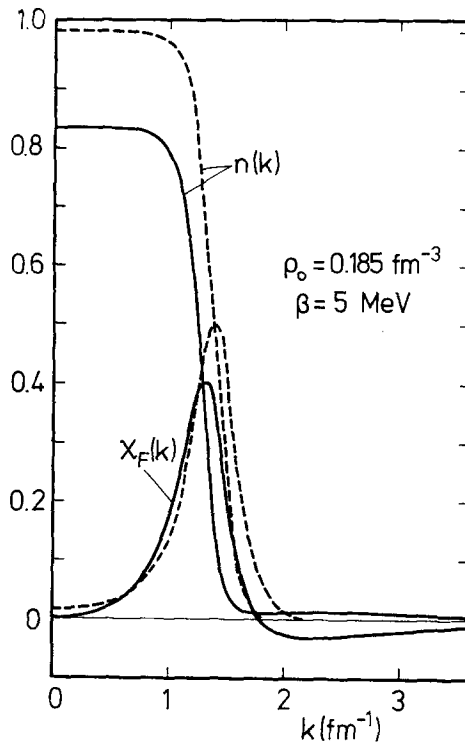


Fig. 5- One particle momentum distribution and Fourier transform of the pairing function for the OMY6 model of nuclear matter. The following values for the trial parameters have been considered:  $\alpha = 4.5 \text{ fm}^{-1}$ ,  $\beta = 5 \text{ MeV}$  and  $\rho_0 = 0.185 \text{ fm}^{-3}$ . The dashed and the solid curve represent respectively the pure BCS and the correlated BCS estimates.

The comparison between these curves gives an estimate of the effects due to the short-range correlations.

A variational calculation without any constraints on the trial parameters seems to be possible. The study of the Euler-Lagrange equation which minimizes the quantity  $\langle \hat{H} - \mu \hat{N}_{\text{op}} \rangle$  appears to be necessary in order to determine the proper behaviour of the two-body correlation function and the BCS amplitude.

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