FROM TDHF TO NUCLEAR HYDRODYNAMICS[+]

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Important developments in the nuclear many-body problem took place in the 1950's. Brueckner formulated a many-body theory based on the assumption that nuclear interactions are basically due to two-nucleon forces[1]. The theory can treat these very strong forces including the strong repulsion at short distances by partial summation of series expansions. The ensuing effective interaction, the Brueckner Reaction-matrix includes in particular many-body effects due to the Pauli exclusion principle and due to the nucleon propagation in a mean field of the other nucleons. These are effects that we now know are essential for the understanding of nuclear saturation and binding.

It is generally accepted that the basic assumption of two-body interactions in nuclei is essentially correct. A critical study does however indicate that three-body forces cannot be neglected[2]. This can also be formulated by saying that mesonic degrees of freedom are not negligible in ground-state nuclei. So, although the picture of a nucleus as consisting of individual nucleons (or quasi-nucleons) moving about independently and interacting through effective interactions is essentially correct it may not be completely so. However it is the picture we shall assume for the time being. An alternative picture would for example be a relativistic mean field theory that explicitly involves a coupling to the mesonic fields[3].

In the 60's and 70's Brueckner theory was applied extensively in Nuclear (or Brueckner) Hartree-Fock calculations[4]. One can state as a summary that these calculations were successful in that nuclear sizes, binding-energies and level-spectra are in fair agreement with experimental data. A partially parallell development took place at this time. The Brueckner reaction-matrix or effective force can be parametrized by a velocity and density-dependent force; usually referred to as a Skyrme-force. Such a parametrization has been rather successful[5]. This approach has the advantage that, because it is fitted to particular data one may increase the predictive power as regards other data or nuclei. Because of the rather lengthy calculations involving several approximations and truncations as well as starting from a semi-phenomenological rather than the "true" force the Brueckner Hartree-Fock calculations falls rather short of a precision fitting. An accurate fit is on the other hand possible by using a simple parametrized force

such as the Skyrme-force.

That it is indeed possible to describe nuclei by a simple two-body force is perhaps the most interesting and important finding resulting from this work in the 60's and 70's. The relative simplicity of the force has made it possible to extend the Hartree-Fock ground-state calculations to TDHF-calculations[6]. Although the extension of nuclear ground-state calculations to nuclear collisions may seem straight-forward and require no further justification some caution is required and important corrections may be necessary. It is this latter point that this talk will be concerned with.

In the ground-state the quasi-nucleons interacting via a Brueckner reaction-matrix (or Skyrme-force) have an infinite mean free path. This corresponds to an independent particle picture. This also follows quite generally from the properties of a fermion-system for which the mean-free path $\lambda \sim 1/T^2$ where T is the temperature. This result implies that the transport-coefficients are strongly temperature-dependent near absolute zero temperature. This result was obtained for nuclear matter by Tomonaga in 1938[7].

It is also well known that a nucleon above the fermisurface has a finite mean free path. This fact is contained in the absorptive part of the optical potential for nucleon-nucleus interactions.

The mean free path depends therefore both on the temperature of the fermion-system and on the momentum of the particle. This dependence is found to be[8]

$$\frac{1}{\tau_p(\varepsilon_p,T)} = \frac{(m^*)^3}{16\pi^4\hbar^6} W \frac{(\pi T)^2 + (\varepsilon_p - \mu)^2}{1+\exp((\mu-\varepsilon_p)/T)} \qquad (1)$$

τ_p being the lifetime of a quasiparticle of energy ε_p, μ the chemical potential and W a transition rate. It is indicated explicitly that τ_p is a function of ε_p and T. It is also, of course, a function of the density.

Our discussion above suggests a possible correction to TDHF. A finite quasi-particle life-time or "relaxation"-time (finite mean free path) will modify the transport-properties. The TDHF- (or Vlasov) equation that includes one-body collisions only (with the mean field) should be modified or extended to include also two-body collisions. For this modification to play a role the typical relaxation-time should of course be short and at least comparable with typical reaction times.

One should also be aware of another correction. It was pointed out above that the effective interaction in itself depends on the "medium" both through the exclusion-principle and through the mean

field. These effects are contained in Brueckner's reaction matrix. When working with a Skyrme-force these effects are approximated by a density-dependence. This may be sufficient for fitting ground-state properties but it may not be adequate for the more complicated excitations reached in heavy-ion collisions. Some information on these modifications of the force is obtained through the work of Faessler et al[9] and by Buchler et al[10].

There are consequently two modifications that we should be concerned about when extending the low-energy TDHF-formalism to higher energies.

1. The modification of the two-body force.
2. A modification of the transport-equation to include two-body collisions.

I shall during the rest of this talk only concern myself with the second modification. (These effects are actually not that separate from each other but it depends on the approach one is taking to these problems).

To implicate the effect of two-body collisions we shall rely on traditional non-equilibrium statistical mechanics methods. Two-body collisions will be assumed to affect transport-properties in the medium; i.e. heat-conduction and viscosity. The effect of these collisions will be a randomnization, i.e. an increase of entropy. Collective, coherent energy will be transferred to heat because of these collisions. In TDHF entropy is conserved; in the fine grained scale.

In previous work we have included this effect of two-body collisions by the time-relaxation method[11,12,13,14]. This implies replacing the collision-term in the transport-equation (K is the two-body interaction)

$$\left(\frac{\partial f}{\partial t}1\right)_{coll} = \sum f_1 f_2 (1-f_3)(1-f_4)\, K^2\, \delta(\varepsilon_1+\varepsilon_2-\varepsilon_3-\varepsilon_4) \qquad (2)$$

by

$$\left(\frac{\partial f}{\partial t}1\right)_{coll} = \frac{f_1 - f_0}{\tau} \qquad (3)$$

This approximation is usually referred to as the time-relaxation-method. (In some cases it is exact). Here f_0 is a distribution obtained by <u>locally</u> equilibrating the distribution f. The density, current-, and energy-density are conserved quantities in the two-body collision and this suffices to define f_0.

The relaxation-time would in the case of one particle outside a fermi-sphere be obtained from eq. (1). In a heavy ion collision the distribution of momenta is more complicated. One is dealing with

strong deformations of the ground state fermispheres. In general will the relaxation-time be a functional of the distribution-function f. Eq. (1) could however be used as a guide-line as to functional dependence. Information (at T = 0) is also obtainable from the imaginary part of the optical model potential. I shall here also, briefly, review some calculations of the collision term in nuclei. The collision-term is shown graphically in fig. 1. It shows explicitly that one is concerned with a 1h-2h1p coupling. In Brueckner-theory this diagram is referred to as a second order rearrangement term. More specifically we are dealing with the imaginary part of the energy-

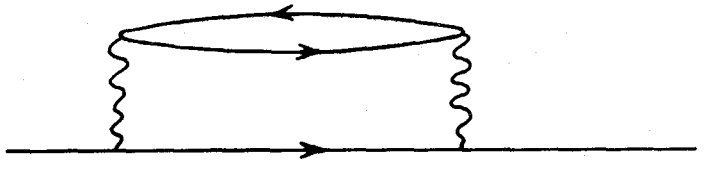

Fig. 1 The two-body collision diagram.

conserving part of this diagram.

This diagram has been calculated with a K-matrix calculated from realistic forces, with the aim of obtaining the optical model. Good agreement with optical model parameters is obtained[15]. Of particular interest is for our discussion of course the agreement with the absorptive part. This same diagram has also been calculated for hole-states in nuclei; the result agrees with experimental widths obtained in knock-out reactions. The interaction used in this calculation[16] is a simple approximation to the K-matrix based on the Moszkowski-Scott separation method. This same interaction was used in a calculation appropriate for heavy-ion collisions: the lifetime of a configuration consisting of two fermi-spheres separated by a momentum P; the relative momentum between two ions. The result is shown in fig. 8 of ref.[11]. The two fermi-spheres had sharp surfaces corresponding to zero temperature. A characteristic of this calculation is the strong P-dependence. As P increases the available phase-space increases fast.

The major difficulty in a detailed calculation of this type is the computation of the effective interaction K in eq. (2) or the diagram in Fig. 1. (or transition rate in eq. (1)). The strong interactions in the nucleus makes a detailed calculation important. The interac-

tion K is a functional of the distribution in momentum-space. We are here actually concerned with the previously modification "1". This may not be so important when calculating the mean field but is probably important for estimating the relaxation-time.

The most detailed calculation from which we can deduce relaxation-times seems to date be that of Izumoto et al.[17]. The purpose of their calculation was actually to calculate the heavy-ion optical potential. As one step in their calculation the authors calculate the imaginary part of single hole states, from which we find the relaxation times shown in Fig. 2. The authors only calculate lifetimes of holes. Their results can probably rather safely be extrapolated into

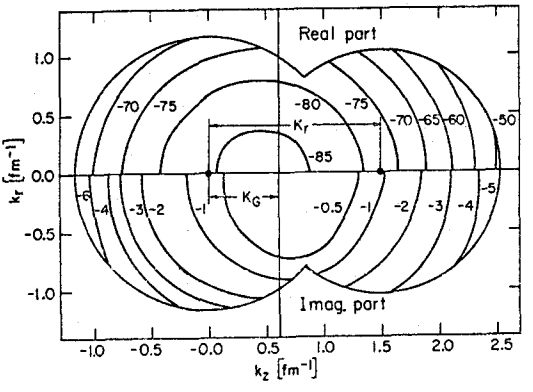

Fig. 2 Single-hole potentials as a function of momentum of a hole-state. The configuration is two zero-temperature fermi-spheres, with radii 1.163 and 1.040 fm^{-1} respectively, separated by a relative momentum K_r = 1.5 fm^{-1}. Relaxation time is $\tau = .32 \cdot V_{im}^{-1} \cdot 10^{-21}$s from Ref.[17].

particle-states. One should note the strong state-dependence. The lifetime varies from $\sim 10^{-21}$ to $\sim 10^{-22}$s from the small to the high momenta. The lifetimes also vary with the separation of the fermi-spheres i.e. with excitation. From eq. (1) we find that a temperature-increase, smearing of the fermi-surfaces would also decrease the relaxation-time. Results obtained by Bertsch[18] suggest however that for small excitations, whether the energy goes into heating or deformation of the fermi-surface is irrelevant; the lifetime depends only on the total excitation energy. This is an approximation that we shall adopt instead of improved estimates.

Although we find that the relaxation of a nucleon-state depends on both the temperature of the medium and the momentum of the nucleon we

have only included the temperature-dependence in our calculations. For technical reasons it has so far been prohibitive to include the momentum-dependence. Following Bertsch's result we have also equated temperature with excitation energy. The definition of a temperature (even local temperature) is in a non-equilibrium system strictly not allowed. Our "definition" will simply be in terms of the excitation energy[12].

At low temperature we have used the functional form obtained by Tomonaga[7]. (His is a low temperature expansion). At high temperatures we have used data described in detail in ref.[11]. Two functions have been used with units in MeV and 10^{-21} seconds.

$$\tau_R = \tau(T) = T^{-2} + T^{-1} \tag{4a}$$

$$\tau_M = \tau(T) = T^{-2} + .0058 \tag{4b}$$

The transport-equation is written in terms of the density-matrix and is now:

$$i\hbar \frac{\partial \rho}{\partial t} + \frac{\hbar^2}{m} \nabla_r \cdot \nabla_x \rho - D\rho = -i\hbar(\rho-\rho_0)/\tau(T) \tag{5}$$

with $\rho \equiv \rho(\underline{r},\underline{x},t)$ and $D = U(\underline{r}+\frac{1}{2}\underline{x})-U(\underline{r}-\frac{1}{2}\underline{x})$ where $\underline{x}$ is a non-locality coordinate.

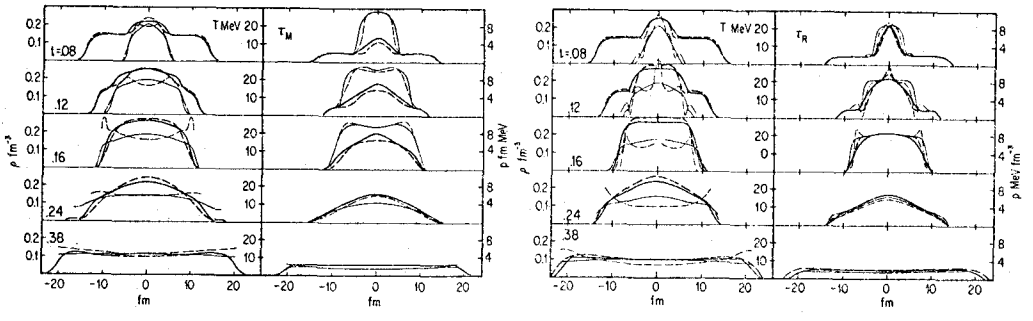

Fig. 3 Comparison between fluid results (full curves) and TDHF (broken curves) for $\tau = \tau_M$, left figure, and for $\tau = \tau_R$, right figure. The set of curves to the left in each figure show densities (heavy lines with scale to the left) and temperatures (thin lines with scale to the right). The set of curves to the right show paral lel (thin lines) and perpendicular (full lines) internal pressures. Reference axis is the beam direction.

Fig. 3 show results for collisions between two nuclear slabs at 20 MeV/A CM energy. The broken lines are the results obtained calculat-

ing density and temperature as well as pressure, both transverse and longitudinal. Details of this calculation is given in ref.[13]. It should be observed that for the shorter relaxation time, τ_R, the pressure is isotropic while for the longer, τ_M, it is highly unisotropic.

Directly related to this is the more or less obvious fact that for small relaxation-times the properties of the system approaches that of an ideal fluid, i.e. viscosity and heat-conduction goes to zero. It is of interest to find out how the nuclear matter would approach a fluid from a gas state as the temperature (excitation) is increased.

A microscopic derivation of fluid-equations starts from a transport-equation. Fluid-equations have been derived[14] from eq. (5) by a moment-expansion of the Wigner-function, which is the Fourier-transform of $\rho(\underline{r},\underline{x},t)$ with respect to $\underline{x}$. A truncation is made including only the third moment and expressing heat-conduction in terms of $\kappa\cdot\nabla T$, where κ is the heat conductivity. The ensuing equations have been solved (see ref.[14] for details) and the result is shown in Fig. 3 (full lines) for comparison with the results from the transport-equation). It is seen that the fluid approximation to the transport-equation is indeed quite good for these relaxation-times at 20 MeV/A CM. It should be pointed out however that these fluid-equations have a highly unisotropic pressure-tensor. It is probably not approximable by a viscosity term.

A summary of our findings can be given as follows. Nuclear matter behaves at low temperatures $T < 5$ MeV like a gas with long mean free paths. TDHF is applicable for collisions up to $\sim$ 5 MeV/nucleon. As the temperature is increased the importance of two-body collisions increases; the relaxation time decreases. For $T \gtrsim 10$ MeV (for collisions above 20 MeV/nucleon in CM nuclear matter behaves as an ideal fluid. In the intermediate region nuclear matter has to be treated by a transport-equation like eq. (5) with a finite relaxation time. The behaviour is therefore more complicated in this intermediate region. The energy and temperature limits given above are highly approximate. It is rather the goal of heavy-ion experimental and many-body theoretical work to describe this transition from gas to fluid. The property of nuclear matter described say in terms of heat-conductivity and viscosity is largely unknown at these energies and temperatures.

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