MEAN-FIELD THEORY AND RANDOM TWO-BODY COLLISIONS

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A microscopic theory to derive the collision term extending time-dependent mean-field theories for finite systems is discussed. It is based on a random-matrix model for the residual interaction in a diabatic basis of single-particle states. The structure of the energy-conserving function in a finite system with discrete s.p. levels is investigated. The effect of statistical two-body collisions on the equilibration of the occupation numbers is shown analytically in a schematic model. A numerical calculation demonstrates the influence of a collision term on TDHF results.

1. Introduction

The extension of time-dependent mean-field theory to include incoherent effects of the residual interaction leads to the derivation of collision terms. The consideration of incoherent processes that cause a destruction of phase relations in the interacting many-body system is believed to be important in particular in view of the dissipative phenomena that have emerged in the study of heavy-ion collisions[1-3]. Whereas the time-dependent Hartree-Fock approximation incorporates one-body damping, truly irreversible processes provided by random two-body collisions are not accounted for.

Different approaches to derive the collision term in the context of nuclear physics have been proposed. They exploit either Green's function methods[4] of quantum statistical mechanics[5], a perturbation expansion for the density operator[6,7], a time-averaging procedure of the TDHF-equations[8], or stochastic properties of the residual interaction[9,10] as in statistical spectroscopy. These methods lead to a collision term in the equation of motion for the one-body density matrix that is similar to the one in the Boltzmann equation for infinite, homogeneous systems, properly modified[11] to include Fermi statistics. A major difference occurs in finite systems because the single-particle energy levels are discrete, and change in time as the mean field gets distorted. As a consequence the energy-conserving δ-function of the Boltzmann collision term opens up in a finite system, and collisions between nucleons in single-particle levels

that are several MeV apart become possible. Overall energy is still conserved through a balance of the energy content of both mean-field and collision term. This has been discussed in detail in the theory of Grangé et al.[10] and will be sketched in the review of that theory in section 2 .

In any of the theories[5-10] that lead to a collision term coherent effects of the residual interaction are not considered. Since they are likely to be unimportant at sufficiently high excitation energy this approximation appears reasonable if the emphasis is on the understanding of dissipative effects based on a time-dependent mean-field theory such as TDHF.

The theory[10] is irreversible already on the A-body level, as a consequence of the ensemble-averaging. The reduction of the equation of motion for the average A-body density matrix to the two- und further to the one-body level gives then rise to the appearance of the collision term. The hierarchy for the ensemble-averaged density matrices that emerges in the reduction process differs from the usual hierarchy obtained when taking subsequent traces of the Liouville equation.

In section 2, the theory is first outlined on the A-body level, the conservation laws are discussed, the reduction to a set of coupled equations for the ensemble-averaged one- and two-body density matrices is performed, and the collision term in the one-body equation of motion is investigated. This part is to a large extent based on Ref. 10 where details can be found.

The role of the collision term in dissipative processes in finite nuclei is discussed in section 3 . The structure of the energy-conserving function is investigated and the time scales that are relevant for the equilibration are presented. The analytical model[12] describes schematically the equilibration in the single-particle occupation numbers on the basis of a nonlinear partial differential equation. Whereas it has not yet been coupled to the time-evolution of the mean field, a numerical investigation[13] with a relaxation ansatz for the collision term exhibits how the Wigner transform of the one-body density approaches statistical equilibrium as a consequence of random two-body collisions which are absent in a pure mean-field approach. Several conclusions are drawn in section 4 .

2. A Random-Matrix Model to derive the Collision Term

In the mean-field approximation, the equations of motion for the A-body density matrix $\rho^{(A)}(\vec{x}_1,..,\vec{x}_A;\vec{x}_1',..,\vec{x}_A';t)$

can be written as

$$i\,\dot{\varrho}^{(A)}_{HF} = \left[H_{HF}, \varrho^{(A)}_{HF} \right] \tag{2.1}$$

with the self-consistent mean-field Hamiltonian

$$H_{HF} = \sum_{i=1}^{A} h^{(i)}_{HF} = \sum_{i=1}^{A} t_i + \sum_{i=1}^{A} u_i(\varrho^{(1)}) \quad , \tag{2.2}$$

the one-body mean-field

$$u_1 \varrho^{(1)} = \operatorname{Tr}_2 \Big\{ v(1,2) \left(\varrho^{(1)}(2,2;t)\varrho^{(1)}(1,1';t) - \varrho^{(1)}(1,2;t)\varrho^{(1)}(2,1';t) \right) \Big\} \tag{2.3}$$

and $v(1,2)$ the two-body interaction. Here, $\varrho^{(A)}_{HF}$ is a product of the $\varrho^{(1)}$, and by taking the trace over (A-1) particles in Eq. (2.1) the time-dependent Hartree-Fock equation

$$i\,\dot{\varrho}^{(1)} = \left[h_{HF}(\varrho^{(1)}), \varrho^{(1)} \right] \tag{2.4}$$

is recovered.

To go beyond this mean-field approximation in order to include the incoherent effects of the residual interaction, we have formulated in [10] a random-matrix model. As discussed in the introduction, we thus restrict ourselves to effects of the residual force which lead to irreversible behaviour. The matrix elements of the full two-body interaction

$$v = \sum_{i<j}^{A} v(i,j) \tag{2.5}$$

(which is usually replaced by an effective or density-dependent interaction in the construction of the mean field) are assumed to exhibit stochastic behaviour in a suitably chosen basis. Technically, the matrix $\hat{v}$ is replaced by an ensemble of statistically distributed matrices. To determine the ensemble, we require in [10] that the mean field (2.3) is the same for all its members and does not fluctuate. This leads us to construct an ensemble of Hamiltonians

$$H = T + v + V \tag{2.6}$$

with the actual effective operator v that serves only to determine the mean field, and V an ensemble of operators with mean value zero. The random force V is taken to be a sum of two-body operators $V(i,j)$.

To formulate the statistical properties of V in analogy with methods known from statistical spectroscopy[14] and nuclear reactions[15] a

suitable basis of single-particle states has to be chosen. At any time t we consider the antisymmetric matrix elements of V in a basis of single-particle states $|\alpha(t)\rangle$, $|\beta(t)\rangle$ defined by

$$H_{HF}(t)\,|\alpha(t)\rangle = \varepsilon_\alpha(t)\,|\alpha(t)\rangle \tag{2.7}$$

and written as $\langle\alpha\beta|V|\gamma\delta\rangle_t$. These matrix elements have a Gaussian distribution with mean value zero (V describes the statistical fluctuations around the mean field), and second moment

$$\overline{\langle\alpha\beta|V|\gamma\delta\rangle_t \langle\gamma'\delta'|V|\alpha'\beta'\rangle_{t'}} = \overline{V^2_{\alpha\beta\gamma\delta}}\left(\delta_{\alpha\alpha'}\delta_{\beta\beta'} - \delta_{\alpha\beta'}\delta_{\beta\alpha'}\right)\cdot$$
$$\cdot\left(\delta_{\gamma\gamma'}\delta_{\delta\delta'} - \delta_{\gamma\delta'}\delta_{\gamma'\delta}\right)\exp\left[-\frac{(t-t')^2}{2\tau^2_{corr}}\right]. \tag{2.8}$$

The bar denotes the ensemble average. The strength $\overline{V^2}$ of the residual force is independent of time. To connect the states $|\alpha(t)\rangle$ at different times a diabatic basis of single-particle states is chosen[10], thus ensuring maximum overlap $\langle\alpha(t)|\alpha(t')\rangle$ for $t \neq t'$ at Landau-Zener crossings[16]. In this basis, the nondiagonal elements $\langle\alpha(t)|\beta(t')\rangle$ $(\alpha \neq \beta)$ never attain large values, in contrast to an adiabatic basis. The statistical assumptions (2.8) are suitable for the diabatic states. The price to be paid is a rather large value $\tau_{corr} = 2\cdot10^{-21}$ s of the correlation time that determines the decrease in the overlap of the wavefunctions. This, however, will not cause serious problems in the further development of the theory because the rapid change of the s.p. energies with time in the diabatic basis results in an additional, shorter time scale that provides the justification of a weak-coupling approximation. The technical problem of the construction of diabatic states from Eq. (2.7) will not be considered here.

Because of the stochastic nature of V, the A-body density matrix becomes a random variable. Extending the mean-field equation (2.1), its equation of motion becomes

$$i\dot{\rho}^{(A)} = \left[H_{HF}, \rho^{(A)}\right] + \left[V, \rho^{(A)}\right]. \tag{2.9}$$

The ensemble-average of this equation can now be taken. The result will be seen to obey an irreversible equation which, upon taking the trace, essentially reduces to a mean-field equation with a collision term. This program is described in the following parts of this section.

2.1. The equation of motion for the ensemble-averaged A-body density matrix

To obtain the formal solution of Eq. (2.9) it is convenient to transform to the interaction representation

$$\rho^I = O_t \rho^{(A)} O_t^{-1} \quad , \quad V^I = O_t V O_t^{-1} \tag{2.10}$$

with the time-ordered mean-field propagators

$$O_t = \underrightarrow{\exp}\left[i \int_{t_o}^{t} ds\, H_{HF}(s) \right] \tag{2.11}$$

$$O_t^{-1} = \underleftarrow{\exp}\left[-i \int_{t_o}^{t} ds\, H_{HF}(s) \right]$$

so that

$$i\dot{\rho}^I = [V^I, \rho^I] \tag{2.12}$$

with the formal solution

$$\rho^I = \underleftarrow{\exp}\left\{ -i \int_{t_o}^{t} [V^I(s), \cdot]\, ds \right\} \rho^I(t_o). \tag{2.13}$$

To perform the ensemble-average of this equation, it is convenient to use a cumulant expansion[10,17] for the ensemble-average of the time-ordered exponential, to obtain (after transforming back to Schrödinger representation)

$$i\dot{\overline{\rho^{(A)}}} = \left[H_{HF}, \overline{\rho^{(A)}}\right] - i O_t^{-1} \left[\overline{V^I(t)}, \int_{t_o}^{t} \left[\overline{V_{SC}^I(s)}, \overline{\rho^I(s)} \right] ds \right] O_t \ . \tag{2.14}$$

Given the statistical assumptions, this result is correct for any strength of V. However, V_{SC}^I involves time-ordered cumulants of arbitrary order and therefore the result is not yet very useful. The derivation of an integral equation for V_{SC}^I is extremely tedious. The lowest-order nonvanishing contributions are

$$\begin{aligned} V_{SC}^I(s) \simeq V^I(s) + \int_{t_o}^{s} dt_3 \int_{t_o}^{t_3} dt_4 \Big\{ & \left[\overline{V^I(s)}, \left[\overline{V^I(t_3)}, \left[\overline{V^I(t_4)}, \cdot \right] \right] \right] \\ & + \left[\overline{V^I(s)}, \left[\overline{V^I(t_3)}, \left[V^I(t_4), \cdot \right] \right] \right] - \left[V^I(t_3), \left[\overline{V^I(s)}, \left[\overline{V^I(t_4)}, \cdot \right] \right] \right] \\ & - \left[V^I(t_4), \left[\overline{V^I(s)}, \left[\overline{V^I(t_3)}, \cdot \right] \right] \right] \Big\} . \end{aligned} \tag{2.15}$$

A physically reasonable approximation for low bombarding energies is the weak-coupling limit formulated in [10]: Only those pairs of V^I are averaged that are time-ordered nearest neighbours. Then, $V_{SC}^I(s) = V^I(s)$ in Eq. (2.14), and using the weak-coupling limit once again to pull out ρ^I from under the time integral the result[10] is

recovered

$$i\dot{\overline{\rho^{(A)}}} = \left[H_{HF}, \overline{\rho^{(A)}}\right] - i\left[\overline{V(t)}, \left[\overline{W}(t), \overline{\rho^{(A)}}(t)\right]\right] \qquad (2.16)$$

where $W(t) \equiv \int_{t_0}^{t} ds\; O_t^{-1} O_s V(s) O_s^{-1} O_t$. This equation represents the desired transport equation that describes the irreversible approach of the ensemble-averaged A-body density matrix towards statistical equilibrium. It is consistent with the conservation laws of particle number and energy. Taking the trace of Eq. (2.16) over all variables and using the cyclic invariance of the trace, we find immediately that the norm of $\overline{\rho^{(A)}}$ and hence the particle number is conserved.

The proof that the first moment of energy is also conserved has been given in [10]. It requires to show that

$$\frac{d}{dt}\,\overline{Tr_{1\ldots A}\left\{H\rho^{(A)}\right\}} = Tr_{1\ldots A}\left\{H_{HF}\,\dot{\overline{\rho^{(A)}}} + \overline{V\dot{\rho}^{(A)}}\right\} \qquad (2.17)$$

vanishes. The first term on the r.h.s. is derived as in the usual mean-field theory[18]. Its trace would vanish for $\overline{\rho^{(A)}}$ replaced by $\rho_{HF}^{(A)}$ Here it is rewritten using Eq. (2.16) to become

$$-Tr_{1\ldots A}\left\{H_{HF}\left[\overline{V(t)}, \left[\overline{W}(t), \overline{\rho^{(A)}}(t)\right]\right]\right\}$$

whereas the second term becomes using Eq. (2.9)

$$i^{-1}\,Tr_{1\ldots A}\left\{\overline{V\left[H_{HF}, \rho^{(A)}\right]}\right\}.$$

The two terms cancel because a comparison between Eqs. (2.9) and (2.16) shows that

$$\overline{V\rho^{(A)}} = -i\,\overline{V}\left[\overline{W}, \overline{\rho^{(A)}}\right].$$

Overall mean-energy conservation is thus due to a balance of the energy content of the time-dependent mean field, and of the collision term. This is in contrast to both conventional mean-field theory (where the energy contained in the mean field only is conserved), and collision terms in Boltzmann-type equations. A reduction of the equation for the average A-particle density matrix to the one-particle level as described in the next section has to preserve overall energy conservation[10].

2.2 Reduction to the one- and two-body level

A hierarchy of coupled equations for the average density matrices $\overline{\rho^{(k)}}$ (k = 1...A) can now be obtained by taking subsequent traces over the corresponding variables in the A-body equation (2.16). Due to the ensemble-averaging which accounts for the statistical effect of the residual force, this hierarchy differs from the usual one obtained by taking traces over the Liouville equation. There, the one-body density matrix is coupled to the two-body level only (and factorization of $\rho^{(2)}$ yields the Hartree-Fock equations). Now the ensemble-averaged $\overline{\rho^{(1)}}$ will be coupled to the two- and three-body level.

To perform the trace over the collisional part of Eq. (2.16), a diabatic approximation to the matrix elements of the single-particle propagator is used,

$$\langle\alpha(t)|\exp\left\{-i\int_s^t d\tau\, h_{HF}(\tau)\right\}|\beta(s)\rangle \simeq \delta_{\alpha\beta}\cdot\exp\left\{-i\int_s^t d\tau\, \varepsilon_\alpha(\tau)\right\}\,. \qquad (2.18)$$

Hence, the collision term is evaluated under the assumption that in the mean-field propagation of any single-particle state $|\beta(s)\rangle$ no occupation probability is redistributed into other states. The neglect of this redistribution due to terms of the type $\langle\alpha(t)|\partial/\partial t|\beta(t)\rangle$, $\alpha\neq\beta$ puts an upper limit on the collective velocity[10] which may be smaller than the one imposed by the weak-coupling limit used to derive Eq. (2.16). Both approximations are believed to be valid for typical collective velocities of shape deformations in deeply inelastic heavy-ion reactions at several MeV per nucleon above the Coulomb barrier.

With the approximation (2.18) the reduction to equations for the average one- and two-body density matrices can be performed. The result is written as

$$i\dot{\overline{\rho}}^{(1)}_{\mu_1\nu_1} = \left[h^{(1)}_{HF}\,,\,\overline{\rho^{(1)}}\right]_{\mu_1\nu_1} + i\,C^{(1)}_{\mu_1\nu_1}\left(\overline{\rho^{(2)}},\overline{\rho^{(3)}}\right) \qquad (2.19)$$

$$i\dot{\overline{\rho}}^{(2)}_{\mu_1\mu_2\nu_1\nu_2} = \left[h^{(1)}_{HF}+h^{(2)}_{HF}\,,\,\overline{\rho^{(2)}}\right]_{\mu_1\mu_2\nu_1\nu_2} + i\,C^{(2)}_{\mu_1\mu_2\nu_1\nu_2}\left(\overline{\rho^{(2)}},\overline{\rho^{(3)}}\right). \qquad (2.20)$$

Both the collision term $C^{(1)}$ on the one-body level, and $C^{(2)}$ on the two-body level are functions of $\overline{\rho^{(2)}}$ and $\overline{\rho^{(3)}}$ which survive the reduction of the second term in Eq. (2.16). They contain the effect of the residual interaction V to any order in the weak-coupling limit and lead to a violation of time-reversal invariance in both equations.

The first equation follows from the second one by tracing over $\mu_2 = \nu_2$ with the collision term on the one-body level given by

$$C^{(1)}_{\mu_1\nu_1} = \sum_{\gamma} C^{(2)}_{\mu_1\gamma\nu_1\gamma} \quad . \tag{2.21}$$

For simplicity I restrict now the discussion of the collision term to its diagonal elements and obtain

$$\begin{aligned} C^{(2)}_{\mu\gamma\mu\gamma} = & \sum_{\alpha\beta} \overline{V^2_{\mu\gamma\alpha\beta}} \cdot G(\varepsilon_\mu + \varepsilon_\gamma - \varepsilon_\alpha - \varepsilon_\beta) \cdot \\ & \cdot \Big[\Big\{ \overline{\rho^{(2)}_{\alpha\beta\alpha\beta}} - \overline{\rho^{(3)}_{\mu\alpha\beta\mu\alpha\beta}} - \overline{\rho^{(3)}_{\gamma\alpha\beta\gamma\alpha\beta}} \Big\} \\ & - \Big\{ \overline{\rho^{(2)}_{\mu\gamma\mu\gamma}} - \overline{\rho^{(3)}_{\mu\gamma\alpha\mu\gamma\alpha}} - \overline{\rho^{(3)}_{\mu\gamma\beta\mu\gamma\beta}} \Big\} \Big] \end{aligned} \tag{2.22}$$

where the normalization

$$\sum_{\alpha_2 \cdots \alpha_A, \beta_2 \cdots \beta_A} \overline{\rho^{(A)}_{\alpha_1 \cdots \alpha_A ; \beta_1 \cdots \beta_A}} = (A-1)! \, \overline{\rho^{(1)}_{\alpha_1\beta_1}} \tag{2.23}$$

has been used. On the one-body level, the off-diagonal elements $C^{(1)}_{\mu\nu}$, $\mu \neq \nu$ can be found in Eq. (8) of Ref. 10. The off-diagonal elements on the two-body level have not yet been derived.

In Eq. (2.22), the strength of the residual interaction is $\bar{V}^2$, and the energy-conserving function G is obtained as

$$\begin{aligned} G(\varepsilon_\mu + \varepsilon_\gamma - \varepsilon_\alpha - \varepsilon_\beta) = \mathrm{Re} \Big[\int_{t_0}^{t} ds \, \exp \Big\{ -i \int_{s}^{t} d\tau \big(& \varepsilon_\alpha(\tau) + \varepsilon_\beta(\tau) \\ & - \varepsilon_\mu(\tau) - \varepsilon_\gamma(\tau) \big) - \frac{(t-s)^2}{2\tau_{corr}^2} \Big\} \Big] . \end{aligned} \tag{2.24}$$

The discussion of G is deferred to the next section. Here let us proceed with the equations of motion for $\overline{\rho^{(1)}}$ and $\overline{\rho^{(2)}}$. The coupled set of equations (2.19), (2.20) is not closed. It is evident that the natural way to obtain a closed set consists in a factorization of $\overline{\rho^{(3)}}$ in both equations with direct and exchange contributions:

$$\begin{aligned} \overline{\rho^{(3)}_{\alpha_1\alpha_2\alpha_3\beta_1\beta_2\beta_3}} = & \; \overline{\rho^{(1)}_{\alpha_1\beta_1}} \; \overline{\rho^{(2)}_{\alpha_2\alpha_3\beta_2\beta_3}} - \overline{\rho^{(1)}_{\alpha_2\beta_1}} \; \overline{\rho^{(2)}_{\alpha_1\alpha_3\beta_2\beta_3}} + \\ & + \overline{\rho^{(1)}_{\alpha_3\beta_1}} \; \overline{\rho^{(2)}_{\alpha_1\alpha_2\beta_2\beta_3}} \, . \end{aligned} \tag{2.25}$$

The resulting equations for $\overline{\rho^{(1)}}$ and $\overline{\rho^{(2)}}$ provide a basis for the calculation of expectation values of one- and two-body operators which is thought to be rather complete. The only feature that is missing, but may be important in low-energy heavy-ion collisions are fluctuations of the mean field.

In the next section, we consider the equation of motion for $\overline{\rho^{(1)}}$ only. Hence, also $\overline{\rho^{(2)}}$ is written as an antisymmetric product of $\overline{\rho^{(1)}}$. Various simplifications will be used in order to discuss the effect of the collision term on dissipative processes in finite fermion systems.

3. Role of the Collision Term in a Finite System

The equation of motion for the average one-body density matrix is written as

$$i \langle \mu | \dot{\overline{\rho^{(1)}}} | \nu \rangle = \langle \mu | [h_{HF}, \overline{\rho^{(1)}}] | \nu \rangle + i C_{\mu\nu}(\overline{\rho^{(1)}}) \tag{3.1}$$

The matrix elements of $\dot{\overline{\rho^{(1)}}}$ are given by

$$\langle \mu | \dot{\overline{\rho^{(1)}}} | \nu \rangle = \frac{d}{dt} \overline{\rho^{(1)}_{\mu\nu}} - \sum_{\gamma} \langle \dot{\mu} | \gamma \rangle \overline{\rho^{(1)}_{\gamma\nu}} - \sum_{\gamma} \overline{\rho^{(1)}_{\mu\gamma}} \langle \gamma | \dot{\nu} \rangle \tag{3.2}$$

In occupation-number representation the diagonal elements

$$\overline{\rho^{(1)}_{\mu\mu}} = n_\mu(t)\, \varphi_\mu(\vec{x},t)\, \varphi_\mu^*(\vec{x}',t) \tag{3.3}$$

with single-particle occupation numbers $n_\mu(t) \equiv n(\varepsilon_\mu, t)$, $0 \le n_\mu \le 1$ are governed by

$$\frac{\partial n_\mu}{\partial t} = \sum_{\alpha\beta\gamma} \overline{V^2}_{\mu\gamma\alpha\beta}\, G(\varepsilon_\mu + \varepsilon_\gamma - \varepsilon_\alpha - \varepsilon_\beta) \left[(1-n_\mu)(1-n_\gamma) n_\alpha n_\beta - (1-n_\alpha)(1-n_\beta) n_\mu n_\gamma \right] . \tag{3.4}$$

This is similar to the collision term in the Boltzmann equation modified for Fermi statics. It is a balance equation with a gain- and a loss term for the occupation numbers of the time-dependent single-particle levels. The energy-conserving function G, however, has a more complicated structure to be discussed below. The nondiagonal elements of the density matrix obey

$$i \frac{d \overline{\rho^{(1)}_{\mu\nu}}}{dt} = (\varepsilon_\mu - \varepsilon_\nu) \overline{\rho^{(1)}_{\mu\nu}} + i \langle \dot{\mu} | \nu \rangle \overline{\rho^{(1)}_{\nu\nu}} + i \langle \mu | \dot{\nu} \rangle \overline{\rho^{(1)}_{\mu\mu}} + i \Gamma_{\mu\nu} \overline{\rho^{(1)}_{\mu\nu}} \tag{3.5}$$

The $\Gamma_{\mu\nu}$-term is the collisional contribution to the time evolution

of $\overline{\rho^{(1)}_{\mu\nu}}$. From the structure of the collision term, it is expected that these nondiagonal contributions $\Gamma_{\mu\nu}$ equilibrate faster than the diagonal ones which refer to the s.p. occupation numbers. If the latter reach their equilibrium (the Fermi distribution) within τ_{equ}, the inequality

$$\frac{1}{\Gamma_{\mu\nu}} < \tau_{equ} \tag{3.6}$$

holds. We now proceed to investigate the relaxation in the occupation numbers, and defer the coupling to the time-evolution of the mean field to the last section.

3.1. Structure of the energy-conserving function

The energy-conserving function in the equation (3.4) for the single-particle occupation numbers has been given in Eq. (2.24). If the mean field and the single-particle energies were independent of time as in an infinite homogeneous system, the limit correlation time $\tau_{corr} \to \infty$ can be taken. Then, with $t_o \to -\infty$, $t \to \infty$, one obtains

$$G(\varepsilon_\mu + \varepsilon_\gamma - \varepsilon_\alpha - \varepsilon_\beta) \longrightarrow \pi\, \delta(\varepsilon_\mu + \varepsilon_\gamma - \varepsilon_\alpha - \varepsilon_\beta) \tag{3.7}$$

and the collision term coincides with the one obtained usually[4]. It leads to equilibration in an infinitely extended system, with a Fermi-Dirac distribution for the occupation probabilities in the equilibrium limit.

In a finite system, the single-particle energies depend upon the mean field, and thus upon time. The associated time scale τ_{sp} is shorter than the correlation time τ_{corr} for the single-particle wavefunctions:

$$\tau_{sp} < \tau_{corr} \tag{3.8}$$

whereas the latter is of the order of typical interaction times ($2 \cdot 10^{-21}$s), τ_{sp} is of the order of 10^{-22}s [19]. As a consequence, the width of the energy-conserving function is not determined by τ_{corr} but instead by the rapid change of the single-particle energies as the system deforms. Hence, we evaluate G neglecting the effect of τ_{corr}:

$$G(\varepsilon_\mu + \varepsilon_\gamma - \varepsilon_\alpha - \varepsilon_\beta) \simeq Re\left[\int_{t_o}^{t} ds \exp\left\{-i\int_{s}^{t} d\tau\, \Delta\varepsilon(\tau)\right\}\right] \tag{3.9}$$

where

$$\Delta\varepsilon(\tau) \equiv \varepsilon_\alpha(\tau) + \varepsilon_\beta(\tau) - \varepsilon_\gamma(\tau) - \varepsilon_\mu(\tau).$$

In a first-order expansion of $\Delta\varepsilon(\tau)$,

$$\Delta\varepsilon(\tau) \simeq \Delta\varepsilon(t) + (\tau - t)\,\Delta\dot{\varepsilon}(t) \tag{3.10}$$

the G-function becomes

$$G_{\mu\gamma\alpha\beta} = \int_0^{t-t_0} du \cos\left[\Delta\varepsilon \cdot u - \frac{1}{2}\Delta\dot{\varepsilon} \cdot u^2\right] \tag{3.11}$$

which is integrated to obtain

$$G_{\mu\gamma\alpha\beta} = \left[\frac{\pi}{|\Delta\dot{\varepsilon}|}\right]^{1/2} \left\{\left[\frac{1}{2} - C(z)\right] \cos\left(\frac{\pi}{2} z^2\right) + \left[\frac{1}{2} - S(z)\right] \sin\left(\frac{\pi}{2} z^2\right)\right\} \tag{3.12}$$

with the Fresnel integrals

$$S(z) = \int_0^z \sin\left(\frac{\pi}{2} u^2\right) du\,, \quad C(z) = \int_0^z \cos\left(\frac{\pi}{2} u^2\right) du$$

and $z = |\Delta\varepsilon| / \sqrt{\pi |\Delta\dot{\varepsilon}|}$.

A rather accurate approximation to this result is written as [20]

$$G_{\mu\gamma\alpha\beta} \simeq \left[\frac{\pi}{|\Delta\dot{\varepsilon}|}\right]^{1/2} \cdot \left[2 + 4.142\,z + 3.492\,z^2 + 6.670\,z^3\right]^{-1} . \tag{3.13}$$

The result is shown in Figure 1 .

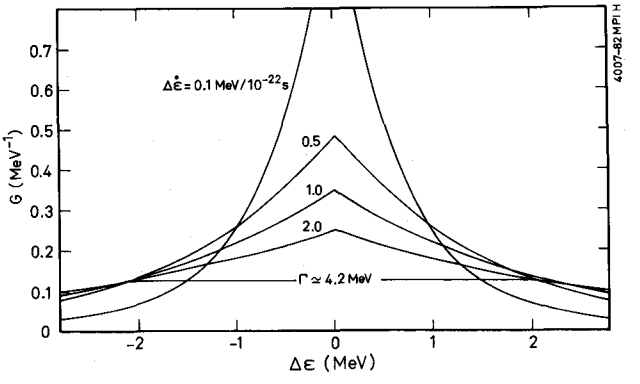

Fig. 1 : The energy-conserving function $G(\Delta\varepsilon)$ in a finite fermion system for different velocities $\Delta\dot{\varepsilon}$ (Ref.21).

The width of the energy-conserving function is seen to increase in proportion with $(\Delta\dot{\varepsilon})^{1/2}$. For heavy-ion collisions, the difference $\Delta\dot{\varepsilon} = (\dot{\varepsilon}_\alpha + \dot{\varepsilon}_\beta - \dot{\varepsilon}_\gamma - \dot{\varepsilon}_\mu)$ has typically the value

$$\Delta\dot{\varepsilon} \simeq 1\ \mathrm{MeV} / 10^{-22}\,\mathrm{s}. \tag{3.14}$$

This value is obtained using

$$\frac{d(\Delta\varepsilon)}{dt} = \frac{d(\Delta\varepsilon)}{dQ}\frac{dQ}{dt}$$

and a collective speed of $\dot{Q} = Q_{max}/(4\cdot 10^{-21}\mathrm{s})$ where Q_{max} is the deformation at the scission point. The associated characteristic time scale τ_{sp} becomes

$$\tau_{sp} = (\Delta\dot{\varepsilon})^{-1/2} \simeq 2.5\cdot 10^{-22}\,\mathrm{s} \tag{3.15}$$

which is indeed an order of magnitude shorter than the correlation time. This short value also provides the posteriori justification of the weak-coupling limit that has been used in the derivation of the previous section. Due to the finite correlation time $\tau_{corr} > \tau_{sp}$, the energy-conserving function will have exponential tails which are not included in Fig. 1 .

The collision term in a finite system is thus linked to the time-dependence of the mean field: Fast changes of the mean field result in a large width of the energy window provided by G, and a rapid action of the collision term which converts ordered nucleonic motion coherent with the mean field into incoherent motion, until the single-particle occupation numbers have acquired a Fermi distribution and the mean field does not change any further.

3.2. A schematic model for the action of the collision term:

Whereas TDHF corresponds to constant single-particle occupation numbers $n_\mu = 1$ or 0, the collision term describes the equilibration in the fermion system due to the action of the residual force. To emphasize its effect on the occupation numbers according to the reduced equation (3.4), let us outline a schematic model that leads to analytical solutions[12] and still preserves the essential physical ingredients such as Fermi statistics, particle-number and energy conservation.

The collision term is written in the form of a master equation with gain and loss term, respectively

$$\frac{\partial n_\mu}{\partial t} = (1-n_\mu)\sum_\beta W_{\beta\to\mu}\, n_\beta - n_\mu \sum_\beta W_{\mu\to\beta}\,(1-n_\beta) \tag{3.16}$$

where

$$W_{\mu\to\beta} = \sum_{\alpha\gamma} \overline{V^2}_{\mu\gamma\alpha\beta}\; G(\varepsilon_\mu+\varepsilon_\gamma-\varepsilon_\alpha-\varepsilon_\beta)\,(1-n_\alpha)\,n_\gamma \qquad (3.17)$$

and $W_{\beta\to\mu}$ accordingly. The aim is to transform the collision term into a tractable differential equation which contains the essential physical ingredients. The nonlinearity in the transition function W is disregarded; improvement with an iteration scheme appears possible. With the replacement (g_β the s.p. level density)

$$\sum_\beta W_{\beta\to\mu}\, n_\beta \longrightarrow \int d\beta\, W_{\beta\mu}\, g_\mu \cdot n_\beta \qquad (3.18)$$

$$\sum_\beta W_{\mu\to\beta}\,(1-n_\beta) \longrightarrow \int d\beta\, W_{\mu\beta}\, g_\beta\,(1-n_\beta),$$

the additional assumption $W_{\beta\mu} = W_{\mu\beta}$, and a Taylor expansion of n_β and $g_\beta \cdot n_\beta$ around $\varepsilon_\beta = \varepsilon_\mu$ the nonlinear partial differential equ.[12] for $n \equiv n(\varepsilon_\mu, t)$ is obtained

$$\frac{\partial n}{\partial t} = -\frac{\partial}{\partial \varepsilon}\left[v\cdot n(1-n) + n^2\,\frac{\partial D}{\partial \varepsilon}\right] + \frac{\partial^2}{\partial \varepsilon^2}\left[D\cdot n\right]. \qquad (3.19)$$

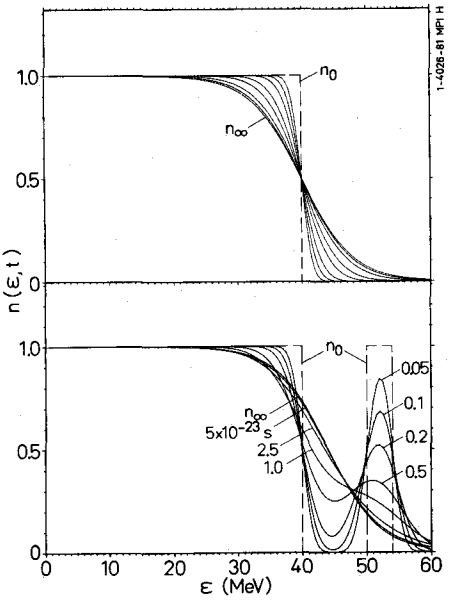

Fig. 2: Equilibration in a finite fermion system due to the action of the residual force, modelled by the analytical solution of Eq. (3.19) (Ref.12).

Transport coefficients have been introduced via moments of the transition function ($x = \varepsilon_\beta - \varepsilon_\mu$)

$$D = \tfrac{1}{2} g_\mu \int dx\, W(\mu,x) x^2$$
$$v = g_\mu^{-1} \frac{d}{d\varepsilon_\mu} (g_\mu \cdot D). \qquad (3.20)$$

In Eq. (3.19), I have expressed the dissipative effects in the s.p. occupation through the drift term v, the diffusive effects through the diffusion term D. Both transport coefficients combine to determine the speed of the relaxation process as well as the equilibrium distribution. Unlike in case of linear diffusion equations of the Fokker-Planck type[1-3], solutions of Eq. (3.19) are rather difficult to obtain. This is a consequence of the nonlinear terms which are due to the exclusion principle. They are essential for the proper description of the time evolution.

The Eq. (3.19) has been solved analytically in Ref. 12 through a nonlinear transformation in the limit of constant transport coefficients. As is to be expected, it has a Fermi-type equilibrium limit

$$n_\infty(\varepsilon) = \left[1 + \exp\left(-\frac{v}{D}(\varepsilon - \varepsilon_F)\right)\right]^{-1}. \qquad (3.21)$$

Analytical solutions for any initial distribution $n_0(\varepsilon)$ (dashed curve in Fig. 2) are obtained as[12]

$$n(\varepsilon,t) = \int_{-\infty}^{\infty} n_0(x) f(x,\varepsilon;t)\, dx \Big/ \left[\int_{-\infty}^{\infty} f(x,\varepsilon;t)\, dx\right] \qquad (3.22)$$

where

$$f(x,\varepsilon;t) = \exp\left\{-\tfrac{1}{2} D^{-1}\left[vx - 2v\int_0^x n_0(y)\, dy\right]\right\} \exp\left[-(\varepsilon - x)^2/(4Dt)\right].$$

These solutions are shown in Fig. 2 for schematic initial conditions. In the lower part, projectile particles can be viewed as occupying states in the continuum region of the potential, and the residual force acts to equilibrate the system towards the equilibrium limit (3.21). Here we have not coupled the time evolution of the occupation numbers to the mean-field evolution.

In the model, the equilibration time is obtained as a ratio of the transport coefficients

$$\tau_{equ} = 4D/v^2 \qquad (3.23)$$

and thus, since $\tau_{equ} \propto D/v^2 \propto 1/\overline{V^2}$, essentially by the strength of the residual interaction that enforces the equilibration process. A microscopic calculation of the transport coefficients and the investigation of their dependence on the collective velocity (through the function G) is desirable. In Fig. 2, I have chosen $D = 20 \cdot 10^{23}\,\mathrm{MeV}^2\,\mathrm{s}^{-1}$

and $v=-5\cdot10^{23}$MeV s^{-1} such that $-D/v$ = 4 MeV (to be compared with the temperature) and $\tau_{equ} = 3.2\cdot10^{-23}$s .

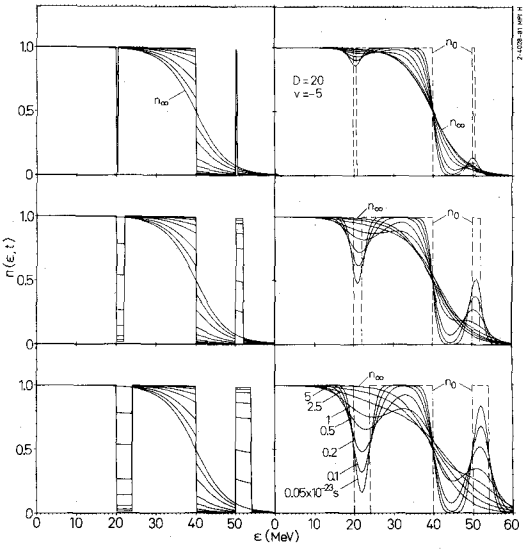

Fig. 3 : Comparison of the results for the collision term (r.h.s.) with the relaxation ansatz (l.h.s.) for various initial distributions (Ref. 12).

In Fig. 3 the results of the collision term are compared to the corresponding relaxation ansatz (l.h.s.)[22]

$$\frac{\partial n}{\partial t} = (n_\infty - n)/\tau_{equ} \tag{3.24}$$

which enforces equilibration towards n_∞ . The sequence of time is the same as on the r.h.s. For small times, and especially for small initial disturbances, the relaxation ansatz leads to a slower equilibration. Numerical solutions of the full occupation-number equation obtained by Toepffer and Wong for an infinite system[23] exhibit a similar behaviour. It may be worthwhile to replace the relaxation ansatz by the solution of Eq. (3.19) in actual extended mean-field calculations. This has not yet been done. In the next section, the modification of numerical TDHF-results through the action of a phenomenological collision term[13] is reported.

3.3. Effect of a Collision Term on TDHF-Results

Following the work of Richert et al.[13] we have studied the effect of a collision term on one-dimensional TDHF-results in a reaction between two slabs. The TDHF equations have been rewritten in terms of the Wigner transform

$$f(x,k;t) = (2\pi)^{-1} \int_{-\infty}^{+\infty} ds \exp(-iks)\, \varrho(x+\tfrac{1}{2}s, x-\tfrac{1}{2}s) \qquad (3.25)$$

of the one-body density matrix as

$$\left(\frac{\partial}{\partial t} + \frac{k}{m}\frac{\partial}{\partial x}\right) f(x,k;t) + F_{pot}(f) = 0 \qquad (3.26)$$

with the Wigner transform $F_{pot}(f)$ of the self-consistent single-particle potential. The effective interaction of Ref. 24 with parameters adapted[13] to a genuine one-dimensional system is used. A phenomenological collision term which has not yet been related to the theory presented above is added to the r.h.s. of Eq. (3.26)

$$I_{coll} = (2\tau)^{-1} \left[f(x, 2\bar{k}-k; t) - f(x,k;t) \right]. \qquad (3.27)$$

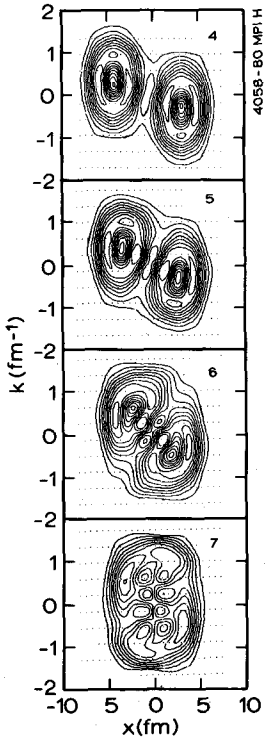

Fig. 4 : Time evolution of the Wigner transform f(x,k;t) of the single-particle density matrix in the entrance phase of the collision. Times in 10 fm/c (Ref.13).

It imposes an equilibration in momentum space towards the local mean momentum

$$\bar{k}(x,t) = \int k\, f(x,k;t)\, dk \,/\, \int f(x,k;t)\, dk \qquad (3.28)$$

with a relaxation time τ . Through the value of τ a connection with microscopic derivation of the collision term is feasible. In [13] we have treated it as a parameter.

To solve the equation for the Wigner function supplemented with the collision term, f(x,k;t) has been expanded on a basis of harmonic functions, and the truncated set of coupled differential equations for the expansion coefficients has been solved numerically. Results for a collision of two one-dimensional slabs containing 8 nucleons each are shown in Figs. 4 and 5 for a bombarding energy of 7.5 MeV/nucleon. At this energy three-dimensional TDHF-calculations for $^{16}O + {}^{16}O$ and similar systems are known to exhibit a low-ℓ cutoff for fusions ("fusion window") [25] .

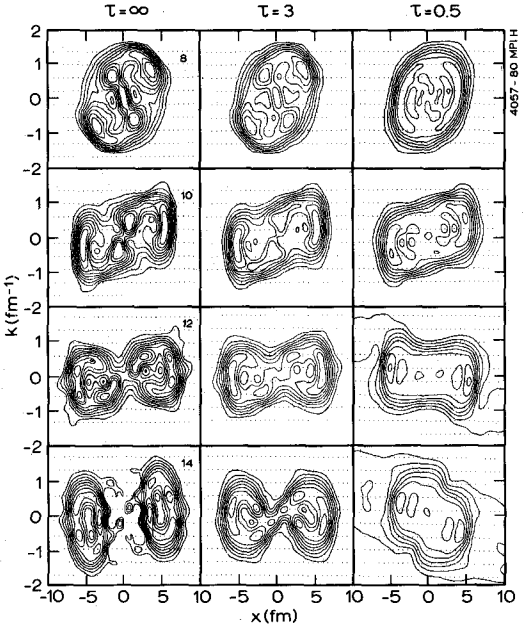

Fig. 5 : Time evolution of the Wigner function in the exit phase of the collision for three different strengths τ (in 10 fm/c) in the relaxation ansatz for the collision term (Ref. 13).

For reasonable strengths of the collision term as expressed by the equilibration time (τ = 5 ... 30 fm/c) very little modification of the TDHF results is found in the initial phase of the collision (Fig. 4). The effect of the residual two-body forces becomes apparent during the final stages where they lead to a slowing-down of the translational motion beyond what is found from the effects of the mean field, and eventually induce fusion.

This is illustrated in the contour plots of Fig. 5 for the Wigner function in the exit phase of the collision where the TDHF result is compared with results for two different strengths of the collision term. For small τ the collisions drive the system towards the equilibrated compound nucleus. The equilibration cannot be described in a pure mean-field calculation without the effect of two-body collisions. Similar results have been obtained by Köhler[26]. Our results are also likely to affect predictions of two- and three-dimensional TDHF-calculations for the occurrence of the fusion window. Experimental evidence for the fusion window has not been found[27], and it may well turn out to be an artifact of the TDHF-approximation.

4. Conclusions

We have been concerned with the extension of time-dependent mean-field theory in order to include the incoherent effect of the residual interaction. The consideration of random two-body collisions appears as a mandate if the goal is a microscopic understanding of dissipative processes starting from a mean-field approach. In the initial stages of a collision problem, TDHF may be a good approximation to the time-evolution of the many-body system, but at later stages the collision term becomes important, leads to a steady increase of entropy, and may eventually induce fusion (that is statistical equilibrium), unless the collision partners separate before due to the dynamical evolution of the process.

The random-matrix model of Grangé et al.[10] has been formulated in section 2 in such a way as to include the residual interaction for arbitrary strength. The corresponding equation for the average A-body density matrix has a memory kernel which is extremely difficult to handle. The subsequent restriction of the theory to the tractable weak-coupling limit is sufficiently accurate for nuclear collisions at energies of several MeV/nucleon above the Coulomb barrier.

The results of the theory describe dissipative behaviour. In actual calculations, they should yield expectation values of one-body operators somewhat different from the ones obtained in TDHF: The mean energy loss in a collision will be larger, the mass drift at high excitation energies is expected to increase, predictions for a fusion window will be modified. A better microscopic understanding of the measured mean values in nuclear collisions appears to be possible, beyond the theoretical interest in the origin of dissipation.

The collision term will certainly also contribute to the expectation values of two-body operators. Coupled equations for the ensemble-averaged one- and two-body density matrix have been derived to treat this problem. However, statistical fluctuations of the mean field itself have not been considered in the theory for reasons of technical feasibility, and this may, indeed, limit the microscopic understanding of the large fluctuations observed in dissipative nuclear collisions on the basis of the present theory.

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