1/Degeneracy Expansion of Collective Quantum Fields and Higher Effective Actions

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I Introduction

The low lying collective excitations of complicated many-body systems display, to a certain approximation, quasiclassical behavior. This means that the action of "characteristic" collective fluctuations is large with respect to $\hbar$ such that their presence is suppressed by a "quantum Boltzmann factor" $e^{iA/\hbar}$. I would like to point out two alternative systematic ways of expanding physical amplitudes according to fluctuation sizes. One has a somewhat restricted range of applications and is useful only for model systems with large degeneracy. The other one is quite general and appropriate to the treatment of realistic physical nuclei which are characterized by strong exchange forces as well as pairing and higher cluster formation, for example alpha particle clusters.

If a system is described by a single Bose quantum field $\varphi(x) = \varphi(\mathbf{x},t)$ all properties of the system are contained in the set of all Green's functions[1]

$$G^{(n)}(x_1,\dots,x_n) \equiv \langle 0|T(\varphi(x_1)\cdot\ldots\cdot\varphi(x_n))|0\rangle \tag{1}$$

where $\varphi(x)$ is the fully interacting Heisenberg field and $|0\rangle$ the ground state. These Green's functions can be studied simultaneously by considering the single generating functional

$$Z[j] \equiv \langle 0|T e^{\frac{i}{\hbar}\int dx\, j(x)\varphi(x)}|0\rangle \tag{2}$$

whose functional derivatives are equal to $G^{(n)}$:

$$G^{(n)}(x_1,\dots,x_n) = Z[j]^{-1} \frac{\hbar}{i} \frac{\delta}{\delta j(x_1)} \cdots \frac{\hbar}{i} \frac{\delta}{\delta j(x_n)} Z[j] \Big|_{j=0} \tag{3}$$

Such an object has a simple, quasiclassical limit which can efficiently be evaluated by using the path integral representation

$$Z[j] = \int \mathcal{D}\varphi(x)\, e^{\frac{i}{\hbar}\left(\mathcal{A}[\varphi] + \int dx\, j(x)\varphi(x)\right)} \tag{4}$$

where $\mathcal{A}[\varphi]$ is the action and $e^{\frac{i}{\hbar}(\mathcal{A}[\varphi]+\int dx\, j\varphi)}$ the "quantum Boltzmann factor" for each field configuration in the presence of the external current. If one formally takes the limit $\hbar \to 0$, this integral is dominated by the saddle point which leads to the classical field equation

$$\left[\frac{\delta}{\delta\varphi(x)} \mathcal{A}[\varphi] + j(x)\right]_{\varphi=\varphi_{cl}(x)} = 0 \tag{5}$$

and the partition function

$$Z[j] \sim Z_{cl}[j] = e^{\frac{i}{\hbar}\left(\mathcal{A}[\varphi_{cl}] + \int dx\, j(x)\varphi_{cl}(x)\right)} \tag{6}$$

Quantum corrections can be calculated by setting[2]

$$\varphi(x) = \varphi_{cl}(x) + \sqrt{\hbar}\,\varphi'(x) \tag{7}$$

and expanding the action $\mathcal{A}[\varphi]$ in powers of $\varphi'(x)$:

$$Z[j] = Z_{cl}[j] \cdot \int D\varphi' \exp i \left\{ \int dx_1 dx_2\, \varphi'(x) \frac{\delta^2 A[\varphi_{cl}]}{\delta\varphi_{cl}(x_1)\delta\varphi_{cl}(x_2)} \varphi'(x_2) \right.$$

$$\left. + \hbar^{1/2} \int dx_1 dx_2 dx_3 \frac{\delta^3 A[\varphi_{cl}]}{\delta\varphi_{cl}(x_1)\delta\varphi_{cl}(x_2)\delta\varphi_{cl}(x_3)} \varphi'(x_1)\varphi'(x_2)\varphi'(x_3) + \ldots \right\} \quad (8)$$

$$= Z_{cl}[j] + \hbar\, Z^{(1)}[j] + \hbar^2\, Z^{(2)}[j] + \ldots$$

The quadratic piece in $\varphi'(x)$ can be considered as a new propagator and the higher powers $\hbar^{\frac{n}{2}-2}\varphi'^{\,n}$ as interaction terms. Calculating the associated Feynman graphs leads to the expansion of $Z[j]$ in powers of $\hbar$.

Moreover, if time dependent oscillating solutions are known, it is straightforward to find the quantized energy level and their quasiclassical corrections. To lowest order in $\hbar$, this goes as follows: Consider a family of such orbits $\varphi^T_{cl}(x)$ differing only by the period T. Denote $\frac{2\pi}{T}$ by ω and define a Lagrangian-like quantity

$$L(\omega) \equiv \mathcal{A}[\varphi^T_{cl}]/T \quad (9)$$

as a function of ω . The variable ω may be considered as the velocity-like Lagrange coordinate $\dot{q}$. Consequently we define

$$p(\omega) \equiv \frac{\partial L(\omega)}{\partial \omega} \mathrel{\hat{=}} \frac{\partial L}{\partial \dot{q}} \quad (10)$$

as the associated canonical momentum and the Legendre transform

$$E(p) = p\omega - L(\omega) \mathrel{\hat{=}} p\dot{q} - L \quad (11)$$

as the energy variable. The quasiclassical quantization rule is then

$$p(\omega_n) = 2\pi n \hbar \tag{12}$$

(or $2\pi(n+\frac{1}{2})\hbar$ if the orbits have turning points) and the energy levels are found as

$$E_n = p(\omega_n)\cdot\omega_n - L(\omega_n) \tag{13}$$

For multiply periodic orbits, the other frequencies are treated in complete analogy. These statements can be proved directly from the path integral[3].

In recent years, semiclassical expansion methods around the solution of field equations have been applied also to collective quantum fields[4]. Such fields appear in fermion systems if the fermion degrees of freedom are eliminated in favor of particle-hole or particle-particle composite fields[5] by a simple trick of quadratic completion[6]. For a typical fermion Lagrangian

$$Z = \int D\psi D\psi^+ \exp\left[i \left\{ \int dx\, \psi^+(x)(i\partial_t-\xi)\psi(x) - \frac{1}{2}\int dxdx'\, V(x,x')\psi^+(x)\psi(x)\psi^+(x')\psi(x')\right\}\right] \tag{14}$$

one may, for example, introduce a trivial multiplicative complete square involving the fermionic density

$$(\det V)^{1/2} \int D\rho\, e^{\frac{i}{\hbar}\frac{1}{2}\int dxdx'\, V(x,x')(\rho(x)-\psi^+(x)\psi(x))(\rho(x')-\psi^+(x')\psi(x'))} \tag{15}$$

after which the fermion integral becomes quadratic, resulting in the partition function of a fermion system without two-particle interactions but moving in an average external field determined by $\rho(x)$:

$$Z_1[\rho] \equiv e^{\frac{i}{\hbar}\mathcal{A}'[\rho]} = \int \mathcal{D}\psi \mathcal{D}\psi^{+} \exp\Big\{ i \int dx\, \psi^{+}(x) \Big(i\partial_t - \xi - \int dx'\, V(x,x')\rho(x') \Big) \psi(x) \Big\} \tag{16}$$

Here, the integral over the Fermi fields can easily be performed, resulting in the fermionic action

$$\mathcal{A}'[\rho] = -i\hbar \,\mathrm{tr}\, \log \Big(i\partial_t - \xi - \int dx'\, V(x,x')\rho(x') \Big)$$

with the symbol tr denoting the functional trace in all field labels x. What remains is a purely collective formulation of the original fermionic theory in terms of the density field $\rho(x)$:

$$Z_1 = (\det V)^{1/2} \int \mathcal{D}\rho(x)\, e^{\frac{i}{\hbar}\left\{ \mathcal{A}'[\rho] + \frac{1}{2}\int dx\, dx'\, V(x,x')\rho(x)\rho(x') \right\}} \tag{17}$$

This path integral contains the same physical information as (14), albeit in another field language. Alternatively one can complete the square using a bilocal density

$$\exp\Big\{ -\frac{i}{\hbar}\,\frac{1}{2} \int dx\, dx'\, V(x,x') \big| \rho(x,x') - \psi^{+}(x)\psi(x') \big|^2 \Big\} \tag{18}$$

or a pair field

$$\exp\Big\{ \frac{i}{\hbar}\,\frac{1}{2} \int dx\, dx'\, V(x,x') \big| \Delta(x,x') - \psi(x)\psi(x') \big|^2 \Big\} \tag{19}$$

In this case, the collective action redescribes the many fermion system in terms of either of these two collective fields.

Finally, it is possible to choose an arbitrary complete square[5]

$$(\det V_1)^{1/2}(\det V_2)^{1/2} \int D\rho(x_1x_2)D\Delta(x_1x_2)D\Delta^+(x_1x_2)$$

$$\exp \frac{i}{2\hbar} \{ \int dx_1dx_2dx_1'dx_2' \, V_1(x_1x_2x_1'x_2')(\rho(x_1x_2)-\psi^+(x_1)\psi(x_2))(\rho^+(x_1'x_2')-\psi^+(x_1')\psi(x_2'))$$

$$+ \int dx_1dx_2dx_1'dx_2' \, V_2(x_1x_2x_1'x_2')(\Delta(x_2x_2')-\psi(x_2)\psi(x_2'))(\Delta^+(x_1'x_1)-\psi^+(x_1')\psi^+(x_1))\} \qquad (20)$$

as long as we require $V_1+V_2 = [\alpha\delta(x_1-x_2)\delta(x_1'-x_2')V(x_1,x_2)]-\beta[x_1\leftrightarrow x_1']-(1-\alpha-\beta)[x_2\leftrightarrow x_2']$. such that the quartic four fermion interaction is eliminated in favor of quadratic terms.

Historically, this type of approach has been extremely useful for the study of second order phase transitions in several physical systems, such as superconductors and superfluid ^{3}He.[5] In these systems, an attractive interaction leads to the formation of Cooper pairs and therefore the pair field (19) appears in the collective version

Hand in hand went the application of this approach to many soluble field theoretic models in two space-time dimensions (Schwinger model, Thirring model, degenerate shell model[7]) and collective quantum fields have opened up new ways of describing and understanding their structural properties.

As a matter of fact, there is a whole new class of soluble models which can be constructed by means of collective quantum fields: It consists of all field theories in which the same field occurs many times, say N, with N tending to infinity[8]. In this case the action has the generic form

$$\mathcal{A} = \int dx \sum_{n=1}^{N} \psi_n^+(x)(i\partial_t - \xi)\psi_n(x) \qquad (21)$$

$$- \frac{1}{N} \frac{1}{2} \int dx\,dx' \, V(x,x') \sum_{n=1}^{N} \psi_n^+(x)\psi_n(x) \sum_{n=1}^{N} \psi_n^+(x')\psi_n(x')$$

where the $\frac{1}{N}$ factor ensures the existence of the limit $N \to \infty$. This interaction can be eliminated by the complete square

$$(\det V)^{N/2} \int D\rho(x)\, e^{\frac{i}{\hbar}\frac{N}{2}\int dx dx' V(x,x')(\rho(x)-\frac{1}{N}\sum_{n=1}^{N}\psi_n^+(x)\psi_n(x))(\rho(x')-\frac{1}{N}\sum_{n=1}^{N}\psi_n^+(x')\psi_n(x'))} \tag{22}$$

Due to the complete generacy of the N levels, the fermion integral is the same for everyone of the $n=1,2,3,\ldots$ fields ψ_n and the collective path integral becomes

$$Z_1 = (\det V)^{N/2} \int \mathcal{D}\rho\, e^{N\frac{i}{\hbar}\mathcal{A}_{coll}[\rho]} \tag{23}$$

$$\equiv (\det V)^{N/2} \int \mathcal{D}\rho\, e^{N\frac{i}{\hbar}\left(\mathcal{A}'[\rho] + \frac{1}{2}\int dx dx' V(x,x')\rho(x)\rho(x')\right)}$$

Now we see that the number N appears as an overall factor such that in the limit of large N the path integral is dominated by the saddle point in just the same way as the previous fundamental field theory (4) in the quasiclassical limit $\hbar \to 0$. Thus, in the collective quantum field, the quantity $\frac{1}{N}$ plays the same role as the expansion parameter $\hbar$ in (6). In the limit $N \to \infty$, the partition function is given by the solution of the field equation

$$\frac{\delta}{\delta\rho}\mathcal{A}_{coll}[\rho]\Big|_{\rho=\rho_\infty} = \frac{\delta}{\delta\rho}\Big[-i\hbar\, \mathrm{tr}\log\left(i\partial_t - \xi - \int dx' V(x,x')\rho(x')\right) \tag{24}$$

$$+\frac{1}{2}\int dx dx' V(x,x')\rho(x)\rho(x')\Big]_{\rho=\rho_\infty} = 0$$

resulting in

$$Z_1 \underset{N\to\infty}{\longrightarrow} Z_\infty \equiv \det V^{N/2}\, e^{N\frac{i}{\hbar}\mathcal{A}_{coll}[\rho_\infty]} \tag{25}$$

The corrections in $\frac{1}{N}$ can be obtained by expanding the exponent in powers of $\rho' \equiv (\rho - \rho_\infty)/\sqrt{N}$ and calculating Feynman graphs using the quadratic part of ρ' as an inverse propagator and the higher powers $\left(\frac{1}{N}\right)^{\frac{n}{2}-2} \rho'^{\,n}$ as vertices.

There can also be the generic interaction forms

$$e^{-\frac{i}{\hbar}\frac{1}{2N}\int dx\,dx'\, V(x,x')\left|\sum_{n=1}^{N}\psi_n^{+}(x)\psi_n(x')\right|^2} \tag{26}$$

$$e^{-\frac{i}{\hbar}\frac{1}{2N}\int dx\,dx'\, V(x,x')\left|\sum_{n=1}^{N}\psi_n(x)\psi_n(x')\right|^2} \tag{27}$$

in which case one obtains collective bilocal density fields

$$\rho(x,x') \sim \frac{1}{N}\sum_{n=1}^{N}\psi_n^{+}(x)\psi_n(x') \tag{28}$$

or collective pair fields

$$\Delta(x,x') \sim \frac{1}{N}\sum_{n=1}^{N}\psi_n(x)\psi_n(x') \tag{29}$$

In each case, the limit $N\to\infty$ leads to an exact solution of this theory via the extremum. Moreover, periodic solutions can be quantized by rules like (12).

Fascinated by the successes[5,7], several groups have applied collective quantum fields to realistic nuclear problems[9-11]. They immediately ran, however, into difficulties. These can be traced back to the amgiguity in the choice of the collective field (15), (18), or (19). In nuclear problems, there are usually important correlations in both $\rho(x)$ and $\rho(x,x')$, due to the presence of exchange forces, and often also in $\Delta(x,x')$ if there are strong pairing forces. In fact, calculations based only on a density field lead, in lowest order, to Hartree equations, which often fail to reproduce experimental results. This is why inclusion of $\rho(x,x')$ becomes necessary, which is achieved via the Hartree-Fock equation. The pairing field, finally, is taken into account by means of the Hartree-Fock-Bogoljubov (HFB) equations. By exploiting the freedom in introducing the collective fields it is in principle possible[5] to choose V_1 and V_2 such that to lowest order in V all three forces are properly taken into account. The special natural role of the HFB equations as a lowest approximation, however, reveals itself only after a study of fluctuations of ρ and Δ fields. These are quite different for different choices of V_1 and V_2. The HFB fields mini-mize the fluctuation corrections to lowest order in V.[10] Within the col-lective field approach this requirement is quite unnatural. In order to see this consider the collective field theory based on the pure density field $\rho(x)$. Within this theory the natural lowest correction is of the one-ρ loop type which from (17) can be seen to be

$$\left[\det V(1-VG_\rho G_\rho)\right]^{-1/2}$$

where G_ρ are the propagators of the Fermi field moving in the presence of the average background field ρ, as determined by (16). The first factor V cancels the $(\det V)^{1/2}$ in front of (17). Expanding the remainder in powers of V gives

$$\exp\left\{-\frac{1}{2}\sum_{n=1}^{\infty}\frac{1}{n}\left(VG_\rho G_\rho\right)^n\right\} \tag{30}$$

The n=1 term consists of the exchange part of the potential energy V but calculated within the Hartree basis. If V has the property that its exchange part is zero there is no such correction.

Now, the Hartree-Fock-Bogoljubov field is distinct by the fact that n=1 fluctuation correction for the general collective fields (20) van-ishes. Those for $n \geq 2$, however, will in general be non-zero. Since the potential V is in no way small, there is no special reason to select the n=1 term over the others.

It must be stressed that restricting the freedom in the choices of V_1 and V_2 is of crucial importance if one wants to extract physical in-formation from approximations which always begin with mean fields. While mathematically any choice V_1 and V_2 leads to the same theory, this can in general, be guaranteed only due to violent fluctuations of ϱ and Δ fields. Approximating ϱ and Δ by their mean values and small fluctuations around these may lead to results which have nothing to do with physical properties of the system. In general one can say that integrating out, in a partition function, a subset of degrees of freedom (and this is what we have done when going to ϱ and Δ fields) can create completely fictitious spectra in the remaining fluctuating fields. A simple mathematical model can illustrate this point. Consider the partition function of a system with two degrees of freedom, both of energy ε :

$$Z_1 = \int \frac{dx}{\sqrt{2\pi\varepsilon}} \frac{dy}{\sqrt{2\pi\varepsilon}}\, e^{-\varepsilon \frac{x^2+y^2}{2}} = 1 \qquad (31)$$

Integrating out the y variable leaves

$$Z_1 = \int \frac{dx}{\sqrt{2\pi\varepsilon}}\, e^{-\varepsilon \frac{x^2}{2}} \qquad (32)$$

which is a system with only one degree of freedom of energy ε . Thus only one of the physical frequencies can be recovered from the second form of the partition function.

Let us now look at the same partition function but change variables from x, y to ξ, η :

$$x = \mathrm{ch}\,\Theta\, \xi + \mathrm{sh}\,\Theta\, \eta$$
$$y = \mathrm{sh}\,\Theta\, \xi + \mathrm{ch}\,\Theta\, \eta$$

Then it reads

$$Z_1 = \int \frac{d\xi}{\sqrt{2\pi\varepsilon}} \frac{d\eta}{\sqrt{2\pi\varepsilon}}\, e^{-\frac{\varepsilon}{2} \mathrm{ch}\,\Theta \left(\eta + \mathrm{th}\,2\Theta\, \xi\right)^2 - \frac{\varepsilon}{2} \frac{1}{\mathrm{ch}\,\Theta} \xi^2} \qquad (33)$$

If we now integrate out the η variable we obtain

$$Z_1 = \int \frac{d\xi}{\sqrt{2\pi\varepsilon}} \frac{1}{\sqrt{ch\,2\theta}} e^{-\frac{\varepsilon}{2}\frac{1}{ch\,2\theta}\xi^2} \tag{34}$$

and see that the remaining fluctuation in ξ has frequency $\varepsilon/ch\,2\theta$ which has nothing to do with either of the physical frequencies. Certainly, the final integral over ξ leads to the same total partition function (31)

$$Z_1 = 1$$

but the intermediate "action"

$$\mathcal{A}[\xi] = \frac{\varepsilon}{2}\frac{1}{ch\,2\theta} - \frac{1}{2}\log ch\,2\theta \tag{35}$$

gives no insight into the physical fluctuations.

From (33) we see that integrating out part of the field variables has the effect of producing constraints on the remaining fields which, in general, create a new spectrum, completely different from the physical one[13].

Thus in realistic nuclear applications, collective quantum fields are generally of little use+. Notice that these difficulties do not occur in the above described successful applications of the method which were selected such that at least one of the following conditions was satisfied:

+Unless one decides to sum over many collective field fluctuations, brute force on the computer, via Monte Carlo calculations, as has been done recently for the ground state energy of exactly soluble models (see S. Koonin's and J. Negele's lectures at this meeting).

1) Close to a second order phase transition, only the collective field which destabilizes at the critical point is a relevant variable. All others are irrelevant.[5]
2) In soluble models, the choice of the collective field is irrelevant since any of the three forms gives an exact reformulation of the same partition function.[7]
3) In the limit $N \to \infty$, one channel is always uniquely selected.[8]

Thus we arrive at the following alternative:

Either we must restrict our attention to model systems which consist of a highly degenerate or at least almost degenerate shell. Then the degeneracy can play the role of N and $1/N=1/\text{degeneracy}$ becomes a good expansion parameter for the collective quantum fields, which leads to a systematic sequence of approximations.

Or we must develop another systematic way of dealing with collective phenomena which is capable of treating the three two-particle correlation on the same footing, while retaining all the other attractive features of the collective quantum field theory. These can be characterized as follows:

1) The quantum problem is reduced to an extremal principle for classical collective variables.
2) The semiclassical quantization rules can directly be imposed.

We shall see that such a method indeed exists.

While this alone would justify its detailed study it has another important feature which, in general, makes it far superior to the collective field approach.

3) There is a straightforward extension which permits the description of collective effects in any higher multiparticle clusters.

The method is based on higher effective actions which is an adaption of thermodynamics to field theory.

Let us first recall the standard effective action as it appears in all textbooks on quantum theory[1] (For brevity, we shall from now on set $\hbar=1$).

II Simple Effective Action

Consider the logarithm of the generating functional

$$W[j] = \frac{1}{i} \log Z[j] \tag{36}$$

It is well-known, and easy to verify,[14] that its functional derivatives are the connected Green's functions, i.e.

$$G_c^{(n)}(x_1,\ldots,x_n) = \frac{\delta}{i\delta j(x_1)} \cdots \frac{\delta}{i\delta j(x_n)} W[j] \tag{37}$$

In particular

$$\Phi(x) \equiv G_c^{(1)}(x) = \langle \varphi(x) \rangle = \frac{\delta W[j]}{\delta j(x)} \tag{38}$$

is the vacuum expectation of the field in the presence of the external current j. The effective action is defined as the Legendre transform

$$\begin{aligned}\Gamma[\Phi] &= W[j] - \int dx \, \frac{\delta W[j]}{\delta j(x)} \, j(x) \\ &= W[j] - \int dx \, \Phi(x) \, j(x)\end{aligned} \tag{39}$$

in which j has to be eliminated in favor of Φ by inverting (38).

By definition, this effective action is a functional of a classical object which has no more fluctuations since $\Phi(x)$ is the expectation of the fluctuating field $\varphi(x)$. Still, $\Gamma[\Phi]$ contains all quantum mechanical information on the system: First of all, ground state and large amplitude collective oscillations can be found from extremizing $\Gamma[\Phi]$ with time independent or time dependent fields $\Phi(\underset{\sim}{x},t)$. This directly follows follows (39): By construction, the effective action has the derivative from

$$\frac{\delta \Gamma}{\delta \Phi(x)} = \int dx' \frac{\delta W[j]}{\delta j(x')} \frac{\delta j(x')}{\delta \Phi(x)} - \int dx' \, \Phi(x') \frac{\delta j(x')}{\delta \Phi(x)} - j(x) = -j(x) \tag{40}$$

which vanishes in the absence of external currents. Second, with little effort it can be shown that higher Green's functions can be reconstructed by putting all possible tree diagrams built from vertices and branches where the vertices are the functional derivatives of $\Gamma[\Phi]$ of third and higher order

$$\Gamma^{(n)}(x_1,\ldots,x_n) \equiv \frac{\delta}{\delta\Phi(x_1)} \cdots \frac{\delta}{\delta\Phi(x_n)} \Gamma[\Phi] \tag{41}$$

which are the one-particle irreducible (OPI) amputated Green's functions, shortly called vertex functions, and where the branches are the full connected propagators[+]

$$G_c(x_1,x_2) = i\,\Gamma^{(2)^{-1}}(x_1,x_2) \equiv G(x_1,x_2) \tag{42}$$

The construction of $\Gamma[\Phi]$ can in fact proceed by calculating G and the vertex functions and composing from these the effective functional[1]

$$\Gamma[\Phi] = \sum_{n=0}^{\infty} \frac{1}{n!} \int dx_1 \ldots dx_n\, \Gamma^{(n)}(x_1,\ldots,x_n)\, \Phi(x_1)\cdot\ldots\cdot\Phi(x_n) \tag{43}$$

This functional can then be extremized to find ground state and collective excitations.

[+]The inverse is understood in the functional matrix sense, i.e.

$$\int dx_2\, \Gamma^{(2)}(x_1,x_2)\, \Gamma^{(2)^{-1}}(x_2,x_3) = \delta(x_1-x_3)$$

Such an expansion would, however, not be useful in studying quasiclassical limits. For this, $\Gamma[\Phi]$ has to be calculated in the loop expansion which collects, at each loop level, infinitely many powers of Φ such that $\Gamma[\Phi]$ becomes a non-perturbative object. This goes as follows[14]. One inserts into the action

$$\varphi(x) = \Phi(x) + \varphi'(x) \tag{44}$$

and expands

$$\begin{aligned} A[\varphi] = A[\phi+\varphi'] &= A[\phi] + \int dx \, \frac{\delta A[\phi]}{\delta\phi(x)} \, \varphi'(x) \\ &+ \frac{1}{2!} \int dx_1 dx_2 \, \frac{\delta^2 A[\phi]}{\delta\phi(x_1)\delta\phi(x_2)} \, \varphi'(x_1)\varphi'(x_2) \\ &+ \frac{1}{3!} \int dx_1 dx_2 dx_3 \, \frac{\delta^3 A[\phi]}{\delta\phi(x_1)\delta\phi(x_2)\delta\phi(x_3)} \, \varphi'(x_1)\varphi'(x_2)\varphi'(x_3) \\ &+ \ldots \end{aligned} \tag{45}$$

and considers the Feynman graphs composed of the Φ-dependent propagator

$$G_\Phi = i\left[\frac{\delta^2 \mathcal{A}[\Phi]}{\delta\Phi\,\delta\Phi}\right]^{-1} \tag{46}$$

and the Φ-dependent interactions

$$\frac{\delta^{(n)} \mathcal{A}[\Phi]}{\delta\Phi(x_1)\cdot\ldots\cdot\delta\Phi(x_n)} \, , \quad n \geq 3 \tag{47}$$

If one adds up all vacuum graphs which are one-particle irreducible, then the result is $\Gamma[\Phi]$. The expansion can be ordered by the number of loops which increases with the number of explicit $\hbar$'s in each term. Thus effective action is indeed a non-perturbative object. It has been used efficiently for the study of phase transitions in Bose systems.

Notice that this method is far superior to the original quasiclassical expansion since each loop term with an explicit power in $\hbar$ is equivalent to a whole infinite set of diagrams in the expansion (8).

The effective action can be used to quantize periodic orbits. Let $\Phi^T(x)$ be an oscillating extremum of period T, then we can define the effective Lagrangian

$$L(\omega) \equiv \Gamma[\Phi^T]/T$$

and proceed just as in equs. (9) to (13).

III 1/Degeneracy Collective Effective Actions

This leads us to the first proposal for going beyond time-dependent Hartree-Fock equations. If a many-body system consists of a large number of degenerate or of almost degenerate single particle levels, say N, we can introduce this number explicitly into the interaction (15) and rewrite it in the form (20). The result is a collective partition function of the form (22). There could also exist strong pairing forces, in which case there would be an additional term like (29). This would bring in a collective $\Delta(x,x')$ field which we shall omit, for brevity. It is then possible to perform the limit $N\to\infty$ and obtain a $1/N$ expansion of the partition function analogous to the semiclassical expansion (8).

According to Ch. II, however, a much better non-perturbative approach can be based on the loop expansion of the collective effective action. Just as before in the effective action of the fundamental field φ, each loop sums up a whole infinite set of powers in 1/N. In addition, higher loops are suppressed by higher powers in 1/N. In order to calculate this effective action we introduce a fictitious external collective source $N\, i \int dx\, j(x)\, \rho(x)$ and calculate

$$Z[j] \equiv e^{\,i W[j]} = \int \mathcal{D}\rho \; e^{N i \left[\mathcal{A}_{coll}[\rho] + \int dx\, j(x)\rho(x)\right]} \tag{48}$$

The expectation of the density field is then given by

$$\bar{\rho}(x) = \langle \rho(x) \rangle = \frac{\delta W[j]}{\delta j(x)} \tag{49}$$

and we can define a collective effective action

$$\begin{aligned} \Gamma_{coll}[\bar{\rho}] &= W[j] - \int dx \frac{\delta W}{\delta j(x)}\, j(x) \\ &= W[j] - \int dx\, \bar{\rho}(x)\, j(x) \end{aligned} \tag{50}$$

This has the same pleasant properties as the previous one defined for the fundamental field. It can be expanded in a series of loops, which are the one-particle irreducible vacuum graphs, with $\bar{\varsigma}$- dependent propagators $G_{\bar{\varsigma}}$ and vertices $\delta^n \Gamma / \delta\bar{\varsigma} \ldots \delta\bar{\varsigma}$. Moreover, higher numbers of loops are depressed by factors 1/N such that, for high degeneracy, these have a decreasing importance.

The extremum of $\Gamma_{coll}[\bar{\varsigma}]$ determines non-perturbative collective ground state and large amplitude collective excitations.

For periodic orbits, writing the effective collective action as $L_{coll}(\omega) = \Gamma_{coll}[\bar{\varsigma}^T]/T$ leads to a quantization just as described before.

IV Two-Particle Effective Action

As discussed in the introduction, for general nuclei the 1/degeneracy limit cannot be used and density, exchange, and pairing correlations have to be considered on the same footing. This can be done by another type of effective action which is constructed directly involving the composite fields $\psi^+(x)\psi(x')$, $\psi(x)\psi(x')$, $\psi^+(x)\psi^+(x')$. For this, one introduces into the generating fermion functional with external bilocal source terms[16,17] and forms

$$Z[\mu,\lambda] \equiv e^{iW[\mu,\lambda]} = \int D\psi D\psi^+ \exp\{i\, A[\psi,\psi^+]$$

$$+ i \int dx dx' \left[\psi^+(x)\mu(x,x')\psi(x) + \frac{1}{2}\left(\psi^+(x)\psi^+(x')\lambda(x,x') + \lambda^+(x,x')\psi(x)\psi(x')\right)\right]\} \quad (51)$$

The derivatives with respect to μ and λ lead to the bilocal density and pair expectations

$$\frac{\delta W[\mu,\lambda]}{\delta \mu(x,x')} = \langle T\, \psi^+(x)\psi(x')\rangle \equiv \rho(x,x') \quad (52)$$

$$\frac{\delta W[\mu,\lambda]}{\delta \lambda^+(x,x')} = \frac{1}{2}\langle T\, \psi(x)\psi(x')\rangle \equiv \frac{1}{2}\Delta(x,x')$$

$$\frac{\delta W[\mu,\lambda]}{\delta \lambda(x,x')} = \frac{1}{2}\langle T\, \psi^+(x)\psi^+(x')\rangle \equiv \frac{1}{2}\Delta(x',x)^+ \equiv \frac{1}{2}\Delta^+(x,x') \quad (53)$$

which are just the normal and anomalous Green's functions of the system. The two can be treated simultaneously by employing a bispinor notation for the fields ψ, ψ^+:

$$\varphi \equiv \begin{pmatrix}\psi \\ \psi^+\end{pmatrix} \quad (54)$$

and rewriting Z_1 in the form

$$Z_1[K] = e^{iW[K]} = \int \mathcal{D}\varphi \; e^{iA[\varphi] + \frac{i}{2}\varphi^T K \varphi} \tag{55}$$

where K contains the sources μ and λ in a two-by-two matrix

$$K \equiv \begin{pmatrix} \lambda^+ & -\mu^T \\ \mu & \lambda \end{pmatrix} \tag{56}$$

For brevity, vector notation has been used to denote functional contraction of indices

$$\varphi^T K \varphi \equiv \int dx\,dx' \, \varphi(x) \, K(x,x') \, \varphi(x') \tag{57}$$

Now the derivative of W with respect to K gives

$$\frac{\delta W[K]}{\delta K(x,x')} = \frac{1}{2} G(x,x') \tag{58}$$

where the matrix

$$G(x,x') = \begin{pmatrix} \Delta(x,x') & -g(x',x) \\ g(x,x') & \Delta^+(x,x') \end{pmatrix} \tag{59}$$

collects the two types of Green's functions.

Using this functional $W[K]$ one introduces the effective action of bilocal fields as

$$\Gamma[G] \equiv W[K] - W_K \; K \tag{60}$$

and sees again that it is extremal in G :

$$\frac{\delta \Gamma[G]}{\delta G} = -\frac{1}{2} K \; = 0 \tag{61}$$

It is easy to construct $\Gamma[G]$ which consists of a free piece[16,17]

$$\Gamma^{(0)} = \frac{i}{2} \operatorname{tr}\left(G_0^{-1} G T\right) - \frac{i}{2} \operatorname{tr} \log iG^{-1} , \tag{62}$$

where G_0 is the free propagator of the doubled $\varphi = \begin{pmatrix} \psi \\ \psi^+ \end{pmatrix}$ field

$$G_0(x,x') \equiv \left< T\, \varphi(x)\, \varphi(x') \right>\Big|_{V=0} = \begin{pmatrix} 0 & G_{0\,12}(x-x') \\ -G_{0\,12}(x'-x) & 0 \end{pmatrix} \tag{63}$$

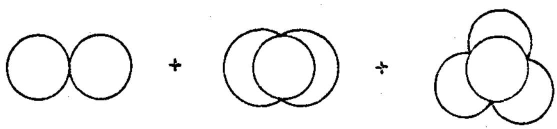
with

$$G_{0\,12}(x-x') = \int \frac{dE}{2\pi} \frac{d^3p}{(2\pi)^3} \, e^{-iE(t-t') + i\mathbf{p}(\mathbf{x}-\mathbf{x}')} \frac{i}{E - \left(\frac{\mathbf{p}^2}{2m} - \mu\right)} ,$$

plus interacting pieces $\Gamma^{int}[G]$. The rule of composing these is the following: Consider all vacuum diagrams which do not fall into pieces by cutting two internal lines. These are called two-particle irreducible (TPI). Associate with each leg the full Green's function G and perform the integration just as in an ordinary Feynman diagram. The lowest three terms are[16,17]

$$\Gamma^{int}[G] = -\frac{1}{8} VGG + \frac{i}{48} (VGG)^2 + \frac{1}{48} (VGG)^3 + \cdots \tag{64}$$

corresponding to the TPI vacuum graphs



(65)

Keeping only the first graph, extremization $\delta\Gamma/\delta G = 0$ leads to the equation of motion

$$iG^{-1} = iG_0^{-1} - \frac{1}{2} VG \tag{66}$$

which is recognized as the time-dependent Hartree-Fock-Bogoljubov[15] equation. The higher diagrams give specific prescriptions on how to go beyond this if the most important correlations are of the two-particle kind.

V Effective Action with Two and Four Particle Correlations

In nuclear physics, four-particle clusters are known to play an important role. This suggests the introduction of an effective action in which four-particle correlations are taken into account explicitly[17,18]. The consequence is one more variable in the extremal principle and this always improves the quality of the extremum. Thus we introduce, in addition to the bilocal source, another one which couples to four particles and consider

$$Z[K,\gamma] = e^{iW[K,\gamma]} = \int \mathcal{D}\varphi \exp\{ i\mathcal{A}[\varphi] + \frac{i}{2}\varphi^T K \varphi - \frac{i}{4!}\gamma\varphi\varphi\varphi\varphi \} \tag{67}$$

The derivative of W with respect to γ gives the four-particle Green's function

$$-\frac{\delta}{\delta\gamma} W[K,\gamma] = \frac{1}{4!} \langle 0 | T\varphi(x_1)\cdots\varphi(x_4)|0\rangle = \frac{1}{4!} G^{(4)}(x_1 \ldots x_4) \tag{68}$$

This can be separated into connected and disconnected parts as

$$G^{(4)}(x_1 \ldots x_4) = G_c^{(4)} + (G \times G + 2\,\text{perm.}) \tag{69}$$

Moreover, the four-particle connected part has singular factors, namely a single particle propagator at each leg. For calculations it is useful to work with the smoothest possible object, which sometimes can be approximated by a constant. Therefore, we remove those factors and introduce the four particle vertex function α via

$$G_c^{(4)}(x_1 \ldots x_4) \equiv -i\,\alpha\, GGGG \tag{70}$$

The higher effective action is now introduced as a functional of G and α :

$$\Gamma[G,\alpha] = W[K,\gamma] - W_K K - W_\gamma \gamma \tag{71}$$

where K and γ have to be expressed in terms of G and α by inverting the defining equations

$$W_K = \frac{1}{2} G \tag{72}$$

$$W_\gamma = \frac{i}{4!} \alpha GGGG - \frac{3}{4!} GG \tag{73}$$

By construction, the effective action $\Gamma[G,\alpha]$ is extremal in G and α :

$$\frac{\delta \Gamma[G,\alpha]}{\delta G} = -\frac{1}{2} K + \frac{1}{4} \gamma G - \frac{i}{6} \alpha G^3 \gamma \tag{74}$$

$$\frac{\delta \Gamma[G,\alpha]}{\delta \alpha} = -\frac{i}{4!} \gamma G^4 \tag{75}$$

and can now be used to obtain non-trivial ground state configurations of two- and four-particle distributions as well as quantized large-amplitude periodic orbits.

Notice that equation (75) is essential in calculating the non-perturbative formation of four-particle clusters. As a matter of fact, it can be considered as a "gap equation" for α-particle condensation[19].

VI Ways Beyond Landau's Theory of Fermi Liquids

In nuclear physics, Landau's theory of Fermi liquids has greatly helped studying low lying excited states. Within the present framework, this theory constitutes a certain lowest approximation which arises as follows[17,20]:

Let G_0 be the ground state extremum of $\Gamma[G]$. Then we can write

$$G = G_0 + \delta G \tag{76}$$

and expand, up to quadratic order in δG,

$$\begin{aligned}\Gamma[G] &= \Gamma[G_0] + \delta^2 \Gamma[G_0, \delta G] + \ldots \\ &= \Gamma[G_0] + \frac{1}{2!} \left.\frac{\delta^2 \Gamma[G]}{\delta G\, \delta G}\right|_{G_0} \delta G\, \delta G + \ldots\end{aligned} \tag{77}$$

where the linear term is absent due to the extremality of $\Gamma[G_0]$. Inserting for $\Gamma[G]$ the lowest non-trivial approximation to (62) and (64) we find[16]

$$\delta^2 \Gamma[G_0, \delta G] = \frac{i}{4} \delta G\, G_0^{-1} \times G_0^{-1} \delta G - \frac{1}{8} V\, \delta G\, \delta G \tag{78}$$

For simplicity, let us assume a translationally invariant Fermi liquid where all particles can be labeled by a momentum $\underset{\sim}{p}$ with an energy $\varkappa(\underset{\sim}{p})$. At low temperature, the particles occupy all levels up to an energy $\varkappa(\underset{\sim}{p}) = \mu \sim E_F$ which is called Fermi energy. In rotational invariant systems, this corresponds to a sphere in momentum space with all $|\underset{\sim}{p}|$ smaller than the Fermi momentum p_F. The free particle propagator is $(p \equiv (E, \underset{\sim}{p}) \equiv (p_0, \underset{\sim}{p}))$

$$G_{0_{12}}(p) = \frac{i}{E - \xi(\underset{\sim}{p})} \qquad \xi(\underset{\sim}{p}) \equiv \varkappa(\underset{\sim}{p}) - \mu \tag{79}$$

Thus we can form a single loop integral

$$-i \int \frac{dp_0}{2\pi} G_{0_{12}}(p + \frac{q}{2})\, G_{0_{12}}(p - \frac{q}{2})$$
$$= i \int \frac{dp_0}{2\pi} \frac{1}{p_0+\frac{q_0}{2} - \xi(\underset{\sim}{p}+\frac{\underset{\sim}{q}}{2})} \frac{1}{p_0-\frac{q_0}{2} - \xi(\underset{\sim}{p}-\frac{\underset{\sim}{q}}{2})} \tag{80}$$

where $\underset{\sim}{q}$ is the total and $\underset{\sim}{p}$ the relative momentum of the two particle lines in the loop. This can be rewritten as

$$\frac{1}{q_0-\xi(\underset{\sim}{p}+\frac{\underset{\sim}{q}}{2})+\xi(\underset{\sim}{p}-\frac{\underset{\sim}{q}}{2})} \int \frac{dp_0}{2\pi} \left[- \frac{i}{p_0+\frac{q_0}{2} - \xi(\underset{\sim}{p}+\frac{\underset{\sim}{q}}{2})} + \frac{i}{p_0-\frac{q_0}{2} - \xi(\underset{\sim}{p}-\frac{\underset{\sim}{q}}{2})}\right] \tag{81}$$

Now, the dp_0 integral can be performed most conveniently by rotating the contour such that it coincides with the imaginary $p_0 = ip_4$ axis. In order to allow for non-zero temperature we may split the integral into a sum over Matsubara frequencies

$$\int \frac{dp_0}{2\pi} \to iT \sum_{p_4 = 2\pi T\cdot(n+\frac{1}{2})} \tag{82}$$

Then (81) becomes

$$\frac{1}{q_0 - \xi(\underset{\sim}{p}+\frac{\underset{\sim}{q}}{2})+\xi(\underset{\sim}{p}-\frac{\underset{\sim}{q}}{2})} \left(n(\underset{\sim}{p}+\frac{\underset{\sim}{q}}{2})-n(\underset{\sim}{p}-\frac{\underset{\sim}{q}}{2})\right) \tag{83}$$

where $n(\underset{\sim}{p}) = (e^{\xi(\underset{\sim}{p})/T} + 1)^{-1}$ is the Fermi distribution.

In the long-wavelength the limit $q \to 0$ this becomes

$$(q_0 - \frac{\partial \xi}{\partial p} \cdot q)^{-1} \frac{\partial n}{\partial \xi} \frac{\partial \xi}{\partial p} \cdot q \tag{84}$$

But $\partial \xi / \partial p$ is the group velocity of the particles $v(p)$. Moreover, for low temperature, the derivative $\partial n / \partial \xi$ is strongly peaked at the surface of the Fermi sea $\varkappa \sim E_F$ such that we can write approximately

$$-i G_{0_{12}} G_{0_{12}} \approx -(q_0 - v \cdot q)^{-1} v \cdot q \; \delta(\xi - E_F) \tag{85}$$

If we neglect pairing effects which would be carried by the diagonal parts of G, equation (85) can be inserted into (78) to write the effective action $\delta^2 \Gamma[G_0, \delta G]$ in the approximate form

$$\delta^2 \Gamma[G_0, \delta G] \approx \frac{1}{4} \delta G \left(\frac{q_0}{v \cdot q} - 1 \right) \left(\frac{\partial n}{\partial \xi} \right)^{-1} \delta G - \frac{1}{8} V \delta G \, \delta G \tag{86}$$

Extremizing this in δG we obtain the differential equation in momentum space[19]

$$(q_0 - v \cdot q) \delta G = v \cdot q \; \frac{\partial n}{\partial \xi} \; \frac{1}{2} V \delta G \tag{87}$$

This is recognized as Landau's equation for quasiparticle densities with a collision term.

The development of the effective action $\Gamma[G]$ to higher orders in δG allows for a straightforward extension of this equation, adding on the right hand side of (87)

$$\underset{\sim}{v} \cdot q \; \frac{\partial n}{\partial \xi} \sum_{n=2}^{\infty} \delta^2 \Gamma^{(n)} / \delta G \, \delta G \Big|_{G=G_0} \delta G \qquad (88)$$

where $\Gamma^{(n)}$ denotes the graphs (65) with powers $V^2, V^3, \ldots$etc. But there is an even more dramatic way of going beyond Landau's theory. Being in the possession of the higher effective action $\Gamma[G, \alpha]$ involving also the four particle vertices we can take the extremum G_0 and α_0 and expand $\Gamma[G, \alpha]$ quadratically around this. The equations for $\delta G = G - G_0$ and $\delta\alpha = \alpha - \alpha_0$ account for the possibility of small oscillations in densities <u>and vertices</u>, and solving these equations will lead to far better approximations than any finite number of higher corrections (88).

VII Higher Effective Action in Collective Fields

If a system has high degeneracy and strong four-particle clusters, there is the possibility of using higher effective actions in conjunction with the fluctuating collective field theory (48). For this we can add, in the exponent, the bilocal source term

$$N \frac{i}{2} \int dx\, dx' \, \rho(x)\, K(x,x')\, \rho(x') \tag{89}$$

and obtain a generating functional

$$Z[j,K] = e^{Ni\, W[j,K]} \tag{90}$$

which can then be used to calculate an effective action

$$\Gamma[\bar{\rho}, \bar{\alpha}] = W[j,K] - W_j\, j - W_K\, K \tag{91}$$

where

$$W_j \equiv \bar{\rho} \tag{92}$$

and $\bar{\alpha}$ is defined by

$$W_K = \frac{1}{2} \bar{\alpha} + \frac{3}{2} \bar{\rho} \bar{\rho} \tag{93}$$

This effective action can then be treated in close analogy with the functional $\Gamma[G,\alpha]$ discussed in Chapter IV.

VIII Outlook

We have pointed out a variety of possible ways of going beyond time-dependent Hartree-Fock equations. Each of them has a parameter which in some sense is small and systematically organizes an approximation scheme.

In simple models of many-body systems, one possible parameter is the inverse degeneracy of the single particle levels. Then the lowest approximation consists in extremizing the collective action which leads to the time-dependent Hartree equations. The solutions can be corrected by powers 1/degeneracy=1/N. There exist also non-perturbative ways of going beyond the leading order in 1/N. They are based on forming the collective effective action and expanding it in vacuum diagrams organized by the number of loops, which for high N are suppressed by increasing powers in 1/N.

If there are strong multiparticle clusters, higher effective collective actions must be used which give even more dramatic non-perturbative corrections in 1/N.

For small degeneracies, the most efficient way to go beyond time-dependent Hartree is provided by the higher effective actions in the composite fields $\psi^+\psi$, $\psi\psi$, $\psi^+\psi^+$. Here, the lowest approximation always leads to the time-dependent Hartree-Fock-Bogoljubov equation. The expansion in loop diagrams is characterized by increasing leading orders in $\hbar$. If four-particle variables are included, a non-perturbative gap equation is obtained also for α -particle condensation.

In all these approaches, a quantum mechanical many-body system is described in terms of classical c-number variables whose field equations follow from an extremal principle.

The quantization of multiple periodic solutions presents no problem since we are always in possession of an action and there are simple rules of determining the discrete set of allowed orbits.

Much detailed work will be necessary in order to see whether the proposed methods lead to a significant improvement of our understanding of complicated many-nucleon systems.

References

1) See, for example, C. Itzykson and J.B. Zuber, Quantum Field Theory, McGraw-Hill, New York, 1980.

2) R.P. Feynman and A.R. Hibbs, Quantum Mechanics and Path Integrals, New York, 1965. The extension to fluctuating field theory was first used extensively by
S.A. Langer, Ann. Phys. (N.Y.) 41, 108(1967),
later by
A.A. Belavin, A.M. Polyakov, A.S. Schwartz, and Yu.S. Tyupkin, Phys. Lett. 59B, 85(1975).
G. 't Hooft, Phys. Rev. Lett. 37, 8(1976).

3) R.F. Dashen, B. Hasslacher, and A. Neveu, Phys. Rev. D10, 4114(1974).

4) H. Kleinert and H. Reinhardt, Nucl. Phys. A332, 331(1979).

5) H. Kleinert, Fortschr. Phys. 26, 565(1978).

6) R.L. Stratonovich, Dok. Akad. Nauk. SSSR 115, 1097(1957), (Sov. Phys. Dokl. 2, 416(1958).

7) H. Kleinert, Phys. Lett. 69B, 9(1977) and Lectures presented at NATO Advanced Study Institute on Non-Linear Equations in Physics and Mathematics, Istanbul, August 1977, ed. A.O. Barut (Reidel, Dardrecht, Holland, 1978).

8) T.H. Berlin and M. Kac, Phys. Rev. 86, 821(1952).
H.E. Stanley, Phys. Rev. Lett. 20, 589(1968), Phys. Rev. 176, 780 (1968).
E. Helfand, Phys. Rev. 183, 562(1969).
M. Kac and C.J. Thompson, Proc. Norwegian Acad. Science (1971).
D. Gross and A. Neveu, Phys. Rev. D10, 3225(1974).

9) D. Ebert and H. Reinhardt, Nucl. Phys. A298, 60(1978), for recent work see H. Reinhardt's lecture at this meeting.

10) S. Levit, Phys. Rev. C21, 1594(1980), S. Levit et al., Phys. Rev. C21, 1603(1980), C22, 1979(1980), for recent work see J. Negele's lecture at this meeting.

11) Y. Alhassid and S.E. Koonin, Phys. Rev. C23, 1590(1981).

12) The coherent state path integrals, advanced by J.P. Blaizot and H. Orland, J. Physique Lett. 41, 531(1980) and Phys. Rev., Urbana preprint, 1981, are in principle capable of including exchange and pairing forces. Their approach is, however, very complicated to handle in practical calculations. See also
H. Kuratsuji and T. Suzuki, Phys. Lett. 92B, 19(1980), J. Math. Phys. (N.Y.) 21, 472(1980) and T. Suzuki's lecture at this meeting.

13) This has been of importance in calculating the lifetimes of metastable states
I.H. Duru, H. Kleinert, and Y. Ünal, Journ. Low Temp. Phys. 42, 137(1981).

14) For a detailed discussion as well as references to the earlier work see
H. Kleinert, Fortschritt. Phys. 30, 187(1982), Phys. Lett. A84, 259 (1981).

15) H. Kleinert, Lett. Nuovo Cimento 31, 521(1981).

16) H. Kleinert, Fortschr. Phys. 30, (in press).

17) H. Kleinert, Phys. Lett. A84, 199(1981).

18) H. Kleinert, Journ. of Phys. 68, 239(1982).

19) H. Kleinert, Phys. Lett. A84, 202(1981).