SEMICLASSICAL ANALYSIS OF MANY-FERMION SYSTEM IN THE GENERALIZED COHERENT-STATE REPRESENTATION

Toru Suzuki and Hiroshi Kuratsuji[+)]

Niels Bohr Institute, 2100 Copenhagen Ø, Denmark
[+)]Kyoto University, 606 Kyoto, Japan

1. Introduction

Time-dependent Hartree-Fock method is obtained as a restriction of the full, time-dependent Schrödinger dynamics of many-fermion system to a family of determinantal wave functions. Confronted with various facets of this restriction, one may try to extend the method from each point of view. Thus, for instance, shortcomings associated with its single-particle aspect might be remedied by introducing two-body correlation effect explicitly. One of such prominent aspects of the TDHF method is the so-called 'classical' aspect which makes it difficult to extract an unambiguous quantum mechanical information from the outcome of the method: In TDHF, one-body motion in a mean field is treated quantum mechanically, while the time evolution of the field itself is governed by a nonlinear equation which conflicts with the superposition principle. This implies that the concept of wave function is not inherent in the method. Although the TDHF equation contains certain quantum mechanical informations, it is this aspect of a wave function where lies the mission of a standard semiclassical method to 'sew a quantum flesh to classical bones'.[1)] In order to surmount the limitation of TDHF in this respect, therefore, one must develop a relevant semiclassical method of many-fermion system based on classical TDHF bones. A natural framework to accomplish this purpose is provided by a path-integral method applied to many-fermion system, which is known to yield TDHF equation in a classical limit.[2-5)] We adopt here the approach developed in Refs.4,6-8) which utilizes a quantum mechanical representation based on generalized coherent states.

2. Representation Based on Slater Determinants

TDHF equation describes a continuous time-development of a determinant and thus defines a path in a space of a set of parameters which labels determinants. On the other hand, actual time evolution of a full wave function takes place in a quantum mechanical Hilbert space.

Path integral method enables one to find a relationship between the two, by describing the latter in terms of a path concept in a parameter space which fixes a quantum mechanical representation basis. Hence, one is led to adopt a representation in which a whole family of determinants is taken as a basis set.

To safely define the concept of path one must impose the following conditions:

(I) The set of basis states $\{|Z>\}$ which defines a labelling of paths should be complete. This implies that there is a 'resolution of unity' relation in the state space considered:

$$\int d\mu(Z)|Z><Z| = 1 \qquad (1)$$

under a proper integration measure $d\mu(Z)$.

(II) For a description of closed systems, the Hamiltonian should not have a matrix element which makes a wave function escape from the space.

The family of determinants satisfies the condition (I): Actually, it is highly nonorthogonal and overcomplete. The relation (1) reflects the closed property of a Lie algebra of particle-hole operators, and the determinants $\{|Z>\}$ are regarded as generalized coherent states[9] associated with the corresponding Lie group. More specifically, if one takes the Thouless' form of parametrization,

$$|Z> = \mathcal{N}_Z^{-\frac{1}{2}}\exp\{\Sigma_\mu Z_\mu (a^\dagger b^\dagger)_\mu\}|0>, \quad a_p|0> = b_h|0> = 0 \qquad (2)$$

where $\mathcal{N}_Z$ is a normalization factor and $\{Z_\mu;\ \mu=(ph)=1,2,\dots,M\}$ are complex numbers, then the integration measure is given by [8]

$$d\mu(Z) = \text{const.}\times \det\{\ g\ \}\cdot\Pi_\mu dReZ_\mu dImZ_\mu$$
$$g_{\mu\nu}(Z^*,Z) = \partial^2\log\mathcal{N}_Z/\partial Z_\mu^*\partial Z_\nu \qquad (3)$$

In this case, the r.h.s. of (1) denotes unity in the state space having fixed number of particles determined by $|0>$. The expression (3) is, in fact, quite general. To describe a system having pairing correlations, for instance, one has simply to replace p-h creation operators by pair creation operators and $|0>$ by the fermion vacuum.

The condition (II) is satisfied if the Hamiltonian can be expressed as a polynomial of the elements of the Lie algebra as is usually the case.

3.Path Integral Expression for the Propagator

Once the conditions I,II are established, it is straightforward to express the propagator of the system, $U(t_f,t_i)\equiv\exp\{iH(t_f-t_i)/\hbar\}$ for a time-independent Hamiltonian H, in a path integral form:[4)]

$$U(t_f,t_i) = \lim_{n\to\infty}\int\cdots\int\prod_{k=0}^{n}d\mu(Z_k)\cdot|Z_f\rangle\langle Z_i|\cdot\exp(iS_n\{Z\}/\hbar)$$
$$S_n\{Z\} = -\sum_{k=1}^{n}\varepsilon(i\hbar\log\langle Z_k|Z_{k-1}\rangle+\langle Z_k|H|Z_{k-1}\rangle/\langle Z_k|Z_{k-1}\rangle) \quad (4)$$

where $Z_k=\{(Z_k)_\mu\}$ $(Z_0=Z_i, Z_n=Z_f)$ denotes the k-th meshpoint on the path at time $k\varepsilon+t_i$ and $\varepsilon=(t_f-t_i)/n$. (For a well-behaved Hamiltonian, one can alternatively use a diagonal expression, $H=\int d\mu(Z)|Z\rangle H'(Z^*,Z)\langle Z|$, where H' is a contravariant symbol of H[9)], to obtain a slightly different form of a path integral.)

The integration over Z_k (k=1,...,n-1) may be evaluated in the stationary phase phase approximation (SPA), from which one obtains a variational condition (for $\varepsilon\to 0$),

$$\delta S = 0, \quad S\{Z(\cdot)\} = \int_{t_i}^{t_f}\langle Z(t)|(i\hbar\partial_t-H)|Z(t)\rangle dt \quad (5)$$

leading to the TDHF equation of motion.

4. Formal Properties of the TDHF Equation and Its Solutions

Under the parametrization (2) of determinants, the TDHF equation of motion takes a form

$$dZ_\mu/dt = \{Z_\mu,\mathcal{H}\} = -(i/\hbar)\Sigma_\nu(g^{-1})_{\mu\nu}\partial\mathcal{H}/\partial Z_\nu^* \quad (6)$$

where the Poisson bracket is defined by

$$\{A,B\} \equiv -(i/\hbar)\Sigma_{\mu\nu}(g^{-1})_{\mu\nu}(\partial A/\partial Z_\mu\cdot\partial B/\partial Z_\nu^*-\partial B/\partial Z_\mu\cdot\partial A/\partial Z_\nu^*) \quad (7)$$

and the 'classical' Hamiltonian is given by $\mathcal{H}\equiv\langle Z|H|Z\rangle$. For any quantum mechanical operator F satisfying a similar condition (II) as for H, one can associate a 'classical' counterpart given by the covariant symbol $\mathcal{F}(Z^*,Z)\equiv\langle Z|F|Z\rangle$.

One-body nature of TDHF can be seen in the following identity[8)] for the commutator of any one-body operator F and an arbitrary operator G:

$$\langle Z|[F,G]|Z\rangle = \{\mathcal{F},\mathcal{G}\} \quad (8)$$

Accordingly, if there is a one-body operator F which is commutable with the Hamiltonian, its classical image is also conserved:

$$[F,H] = 0 \quad\to\quad d\mathcal{F}/dt = \{\mathcal{F},\mathcal{H}\} = 0 \quad (9)$$

The form of (6) suggests that the structure of the TDHF equation is quite analogous to the classical Hamilton's equation of motion[10)]. Indeed, as summarized below, the TDHF solutions are regarded as classical trajectories in a 2M-dimensional phase space characterized by the metric tensor $g_{\mu\nu}$. (It is not fully classical, however, in the sense of the 'large-N' limit.)

i) Since (6) is a coupled first order differential equation in the time variable, a single point on a path at any fixed time is sufficient to uniquely determine the solution.

ii) The system is conservative so that the classical energy $E_c = \mathscr{H}$ is a constant of motion. Hence the trajectories are confined to a 2M-1 dimensional hypersurface of the phase space. If one can find M-1 additional constants of motion, the system is integrable. This is not generally the case for $M \geq 2$, and the phase space may be separated into regular and irregular regions[11,12].

iii) For a given solution having energy E_c, the classical (TDHF) action is given by

$$S_c = \int_i^f \omega - E_c(t_f - t_i),$$
$$\omega = (i\hbar/2)\cdot(dZ_\mu \partial/\partial Z_\mu - dZ^*_\mu \partial/\partial Z^*_\mu)\log \mathcal{N}_Z \tag{10}$$

One can utilize an invariance property associated with the 1-form ω-Edt to express (10) in the following form:

$$S_c = S_f - S_i, \quad S_{f,i} \equiv \int_0^{f,i} \omega - E_c dt \tag{11}$$

The integration in (11) is taken along a path from some fixed point O on the energy surface to $Z_f(Z_i)$. The integration paths O→i, O→f may be taken arbitrarily as fas as they lie on the same surface of the phase space flow.

iv) The exterior derivative of the 1-form ω,

$$d\omega = -i\hbar g_{\mu\nu} dZ^*_\mu \wedge dZ_\nu \tag{12}$$

is the absolute integral invariant of Poincaré and defines a symplectic structure of the phase space. Starting with (12) one can construct a series of integral invariants, the highest of which is the phase space volume element

$$\Omega = \text{const.}\times \det\{g\} \Lambda_\mu (dZ^*_\mu \wedge dZ_\mu) \tag{13}$$

This expression agrees with the invariant measure (3). The invariance of (13) along TDHF paths is the Liouville's theorem.

5. Semiclassical Propagator

In the lowest order SPA the expression (4) reduces to

$$\hat{U}(t_f, t_i) = \int d\mu\{Z\} \, |Z(t_f)\rangle e^{iS_c\{Z\}/\hbar} \langle Z(t_i)| \tag{14}$$

where the integration is now taken over the classical(TDHF) solutions. From the property i) above, the integration is replaced by the one on phase space points at some fixed time t_0; $d\mu\{Z\} \rightarrow d\mu(Z(t_0))$. Due to the property iv), the time t_0 can be arbitrary chosen.

The semiclassical propagator (14) is also obtained naturally if one tries to recover the superposition principle starting with a (single path) TDHF propagator,

$$U_{TDHF}(t_f,t_i) = |Z(t_f)>e^{iS_c\{Z\}/\hbar}<Z_i| \qquad (15)$$

which is defined for a given initial determinant $|Z(t_i)>=|Z_i>$. The propagator (15) satisfies the exact Schrödinger equation, $(i\hbar\partial_t - H)U=0$, if the Hamiltonian happens to be a one-body operator. Even in this case, however, U_{TDHF} does not coincide with the exact propagator: The initial ($t_f=t_i$) value of U_{TDHF} does not give the correct expression, unity, but the projector to $|Z_i>$. To obtain an expression which coincides with the exact propagator for one-body Hamiltonians in terms of TDHF solutions, one has simply to take a linear superposition of U_{TDHF} for different initial determinants so as to satisfy the correct initial condition. This linear combination is built in as the resolution-of-unity relation (1). Consequently, one recovers the expression (14).

The propagator U obeys a number of relations which may or may not be related to a symmetry of the Hamiltonian. Similarly, the semiclassical propagator $\tilde{U}$ satisfies such relations as time-translation/reversal symmetry, Hermitian conjugate property, etc.[8] In SPA, $\tilde{U}$ satisfies also a product relation, $\tilde{U}(t_f,t_i) \simeq \tilde{U}(t_f,t_m)\cdot\tilde{U}(t_m,t_i)$, from which follows the unitarity of $\tilde{U}$ within the same approximation.

6. Quantization of Bound States

Stationary states are characterized by periodic time dependence. Since the classical energy is conserved along TDHF trajectories, one can extract time dependence in a similar manner. This enables one to deduce a semiclassical analog of a stationary wave function and to obtain quantization conditions. In the following we discuss integrable classical systems where the trajectories lie on (M-dimensional) invariant tori in 2M-dimensional phase space. For non-integrable systems, there appear chaotic trajectories, the quantization condition of which is yet to be established[12,13].

The invariant tori are determined by M constants of motion, including energy. Therefore, a phase space point can be specified as $Z=\{Z_\mu\} \leftrightarrow \{E,\upsilon,\theta\}$ where the energy E and $\upsilon=\{\upsilon_\alpha;\alpha=1,\ldots,M-1\}$ are the 'classical numbers' which define an invariant torus, while $\theta=\{\theta_\beta;\beta=$

$1,\ldots,M\}$ is the coordinate on the torus. In terms of these variables the invariant integration measure (3) may be written as $d\mu(Z)=\tilde{\rho}(E,\upsilon,\theta)\times dEd\upsilon d\theta$ under a suitable transformation of the weight function $\tilde{\rho}$.

The exact time evolution of a wave function is given by

$$|\Psi(t_f)\rangle = U(t_f,t_i)|\Psi(t_i)\rangle = \Sigma_m |m\rangle e^{iE_m(t_f-t_i)/\hbar}\langle m|\Psi(t_i)\rangle \quad (16)$$

where $|m\rangle$ denotes a stationary eigenstate of the Hamiltonian specified by a set of quantum numbers m. One can analogously write down the equation for a semiclassical wave function $\tilde{\Psi}$ as

$$\begin{aligned} |\tilde{\Psi}(t_f)\rangle &= \tilde{U}(t_f,t_i)|\tilde{\Psi}(t_i)\rangle \\ &= \int\tilde{\rho}dEd\upsilon d\theta|E\upsilon\theta_f\rangle e^{iS_f/\hbar}e^{-iS_i/\hbar}\langle E\upsilon\theta_i|\tilde{\Psi}(t_i)\rangle \end{aligned} \quad (17)$$

where $\theta_i(\theta_f)$ denotes the initial(final) point of a path on a torus and $S_i(S_f)$ has been introduced in (11). The equation (17) can be regarded as an integral equation for $\tilde{\Psi}$, which has a solution

$$\langle E\upsilon\theta|\tilde{\Psi}(t)\rangle = \tilde{A}(E,\upsilon)\exp\{i(\int^{\theta}\omega-Et)/\hbar\} \quad (18)$$

By comparing (16) with (17) and (18) one thus obtains $E=E_m$ for stationary states. Quantization conditions for the 'classical numbers' $\{E,\upsilon\}$ are obtained from a requirement of one-valuedness of the wave function (18) on the invariant torus[7]:

$$\oint\omega = 2\pi\hbar\cdot n \quad (n: \text{integer}) \quad (19)$$

where the integrations are over topologically independent closed loops on the torus. This is the quantization condition of Einstein[14].

After the quantization, the m-th semiclassical wave function (18) is defined on the m-th invariant torus having $E=E_m$ and $\upsilon=\upsilon_m$, i.e., $\tilde{\Psi}_m(\theta)=\text{const.}\times\exp(i\int^{\theta}\omega/\hbar)$. Because of the nonorthogonal character of the basis states $|Z\rangle$, one may extend the definition of the semiclassical wave function over the whole phase space as

$$\langle Z|\tilde{m}\rangle = \int\tilde{\rho}(Z_m)d\theta_m\langle Z|Z_m\rangle\tilde{A}(Z_m)\exp\{i\int^{Z_m}\omega/\hbar\} \quad (20)$$

where $Z_m=\{E_m,\upsilon_m,\theta_m\}$. In SPA, the expression (20) reduces to (18) on the torus.

7. Illustrative Examples

<Harmonic Oscillator>

The Hamiltonian of one-dimensional harmonic oscillator is given by

$$H = \hat{p}^2/2m + m\omega^2\hat{x}^2/2 = (\varepsilon/2)(b^{\dagger}b+bb^{\dagger}) + (\kappa/2)(b^{\dagger}b^{\dagger}+bb) \quad (21)$$

where the boson creation/annihilation oberators are defined through

$$\hat{x} = \xi(b^\dagger+b), \quad \hat{p} = i\eta(b^\dagger-b), \quad 2\xi\eta = \hbar \tag{22}$$

The basis vector is the boson coherent state

$$|Z> = \exp(-Z^*Z/2+Zb^\dagger)|0>, \quad b|0> = 0 \tag{23}$$

which describes a wave packet centered at

$$x = \xi(Z^*+Z), \quad p = i\eta(Z^*-Z) \tag{24}$$

with dispersion $\Delta x=\xi$ and $\Delta p=\eta$. The classical equation of motion is given by the ordinary Hamilton's equation. For a given initial condition, $Z(t=0)=Z_0$, the classical solution is

$$Z(t) = Z_0\cos\omega t - i(\hbar\omega)^{-1}(\varepsilon Z_0+\kappa Z_0^*)\sin\omega t \tag{25}$$

having a classical action $S_c=-\varepsilon t/2$. One can show that the semiclassical propagator $\tilde{U}$ coincides with the exact one in this case by substituting the solution (25) into the expression (14) and then performing an gaussian integration over the classical solutions.

In the following we adopt an optimal choice for the parameters in (2), $\xi=\sqrt{\hbar/2m\omega}$, $\eta=\sqrt{m\hbar\omega/2}$, which gives $H=\hbar\omega(b^\dagger b+\frac{1}{2})$. The invariant tori of this system are ellipses in the p-x plane. It is easily seen that the quantization condition (19) reproduces the exact spectrum, $E_n=\hbar\omega(n+\frac{1}{2})$, under the above choice of parameters ξ,η. In order to compare the wave functions, we shall first express the exact n-th eigenstate in the coherent-state representation:

$$<Z|\Psi_n> = <Z|(b^\dagger)^n/\sqrt{n!}|Z> = (n!)^{-\frac{1}{2}}(Z^*)^n e^{-Z^*Z/2} \tag{26}$$

The peak of the wave function (26) occurs at $|Z|=\sqrt{n}$, i.e., $p^2/2m + m\omega^2x^2/2 = \hbar\omega n$, which coincides with the n-th quantized torus. One can calculate a full semiclassical wave function over the p-x plane from (20). The integration measure on the torus is obtained from $d\mu(Z) =d\mathrm{Re}Z\, d\mathrm{Im}Z/\pi=dE d\theta/2\pi\hbar\omega$ where $\tan\theta=m\omega x/p$. By taking a simple choice, $\tilde{A}=1$, one can perform the line integration over the ellipse and obtain

$$<Z|\tilde{\Psi}_n> = \mathrm{const.}\times(n!)^{-1}n^{n/2}e^{-n/2}(Z^*)^n e^{-Z^*Z/2} \tag{27}$$

which has the same functional dependence on Z as the exact wave function (26). The factor $(n!)^{-\frac{1}{2}}$ in (26) is also reproduced for large n.

<R(4) model>

The model consists of two orbits specified by a signature $\sigma=\pm1$, each having Ω(=even)-fold degeneracy distinguished by a quantum number $k=\pm1,\pm2,\ldots,\pm\Omega/2$. The Hamiltonian contains a pairing and a particle-hole interaction, the latter of which may be regarded as a prototype of the quadrupole force:

$$H = -(G/8)(P^\dagger P + PP^\dagger) - (X/4)QQ$$
$$P^\dagger = \Sigma_\sigma S_+(\sigma), \quad Q = \Sigma_\sigma \sigma \cdot S_0(\sigma) \tag{28}$$
$$S_+(\sigma) = \tfrac{1}{2}\Sigma_k c^\dagger_{k\sigma} c^\dagger_{\bar{k}\sigma}, \quad S_0(\sigma) = \tfrac{1}{2}\Sigma_k (c^\dagger_{k\sigma} c_{k\sigma} - c_{k\sigma} c^\dagger_{k\sigma})$$

The operators S_+, $S_- \equiv (S_+)^\dagger$ and S_0 constitute a quasi-spin for each $\sigma=\pm 1$ orbit. Hence the Hamiltonian possesses $R(4) \sim SU(2)\times SU(2)$ symmetry and is easily diagonalized in the basis of either group. The model has been analyzed in connection with the attenuation factors in the interacting boson approximation[15]. A semiclassical quantization problem has been studied in ref.16) for the case of particle number $N=\Omega$. We discuss below a general case based on the framework given above.

Since the model is described by the orientation of two quasi-spins, the number of degrees of freedom M is two and the phase space is four-dimensional. The Hamiltonian is commutable with the number operator, $\hat{N} = \Sigma_{\sigma k} c^\dagger_{k\sigma} c_{k\sigma}$, which is a one-body operator. Therefore, even when the basis vectors (determinants) break a number-conservation, the classical (TDHFB) equation of motion allows two constants of motion, energy and the average particle number. This means that the system is integrable and the classical trajectories lie on two-dimensional invariant tori. The basis vectors can be parametrized by the classical orientation ψ_σ, ϕ_σ $(\sigma=\pm 1)$ of quasi-spins as

$$|\{\psi,\phi\}\rangle = \mathcal{N} \exp\{\Sigma_\sigma \tan(\psi_\sigma/2) e^{-i\phi_\sigma} S_+(\sigma)\}|0\rangle \tag{29}$$

where $|0\rangle$ denotes a south pole ($S_0=-S=-\Omega/4$) state. The TDHF equation describes a motion of the quasi-spin vectors on a sphere with radius S. If one introduces four physical variables instead of $\{\psi,\phi\}$ as

$$n = \langle\hat{N}\rangle/\Omega = 1 - \tfrac{1}{2}\Sigma_\sigma \cos\psi_\sigma, \quad \gamma = \Sigma_\sigma \phi_\sigma$$
$$q = \langle Q\rangle/\Omega = -\tfrac{1}{2}\Sigma_\sigma \sigma\cdot\cos\psi_\sigma, \quad \delta = \Sigma_\sigma \sigma\cdot\phi_\sigma \tag{30}$$

then the equations of motion take a simple form:

$$(h\Omega/4)\,d\gamma/dt = \partial\mathcal{H}/\partial n, \quad (h\Omega/4)\,dn/dt = -\partial\mathcal{H}/\partial\gamma$$
$$(h\Omega/4)\,d\delta/dt = \partial\mathcal{H}/\partial q, \quad (h\Omega/4)\,dq/dt = -\partial\mathcal{H}/\partial\delta \tag{31}$$

The classical Hamiltonian $\mathcal{H}=\langle H\rangle$ is independent of γ, the total polar orientation of quasi-spins. Thus one recovers the conservation of the average number Ωn. In terms of the variables (30), the 1-form ω, (10), is given by

$$\omega = (\hbar\Omega/4)(n\,d\gamma + q\,d\delta) \tag{32}$$

in accordance with (31). On the two-dimensional invariant torus determined by a fixed energy and average number, there are two independent closed loops. From the quantization condition in the γ-direction one obtains

$$<\hat{N}> = \Omega n = \text{even integer} \tag{33}$$

which quantizes the average number of particles. From the condition on closed loops in the q-δ plane for fixed γ, we obtain the quantization of energy. Explicit dependence of $\mathcal{H}$ on q and δ shows that the period and the canonical action $\sim\oint q d\delta$ are given by the first and the third elliptic integrals, respectively. The quantized spectrum is plotted in Fig.1 together with the exact spectrum. Depending on the relative strengths of pairing (G) and particle-hole (X) forces, there appear four different types of orbits, from which one can classically locate a phase transition in the quantized spectrum.

The validity of the semiclassical approach may be further examined by looking at the wave functions. In the classical limit, the wave function is concentrated on the quantized invariant torus which is shown in the upper part of Fig.2. One can visualize a phase space structure of the exact eigenfunction $|m>$ by calculating

$$<\{\psi,\phi\}|m> = A(q,\delta)e^{iW(q,\delta)}e^{iN\gamma/4} \tag{34}$$

The γ-dependence of (34) is trivially reproduced by the semiclassical wave function (18). In the lower part of Fig.2 are shown the amplitudes $A(q,\delta)$ for a few cases. One can see a correspondence between the exact wave function and the classical invariant torus. One can

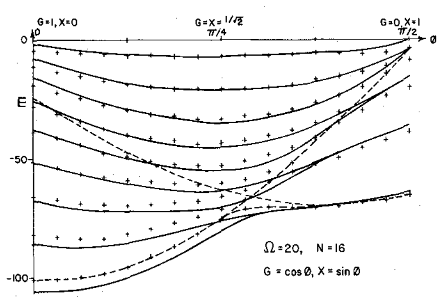

Fig.1 Exact(solid lines) and semiclassical(+) energy spectra of the R(4) model plotted against the relative strength of the pairing(G) and the particle-hole(X) forces. Dotted lines distinguish regions associated with the four different types of classical trajectories.

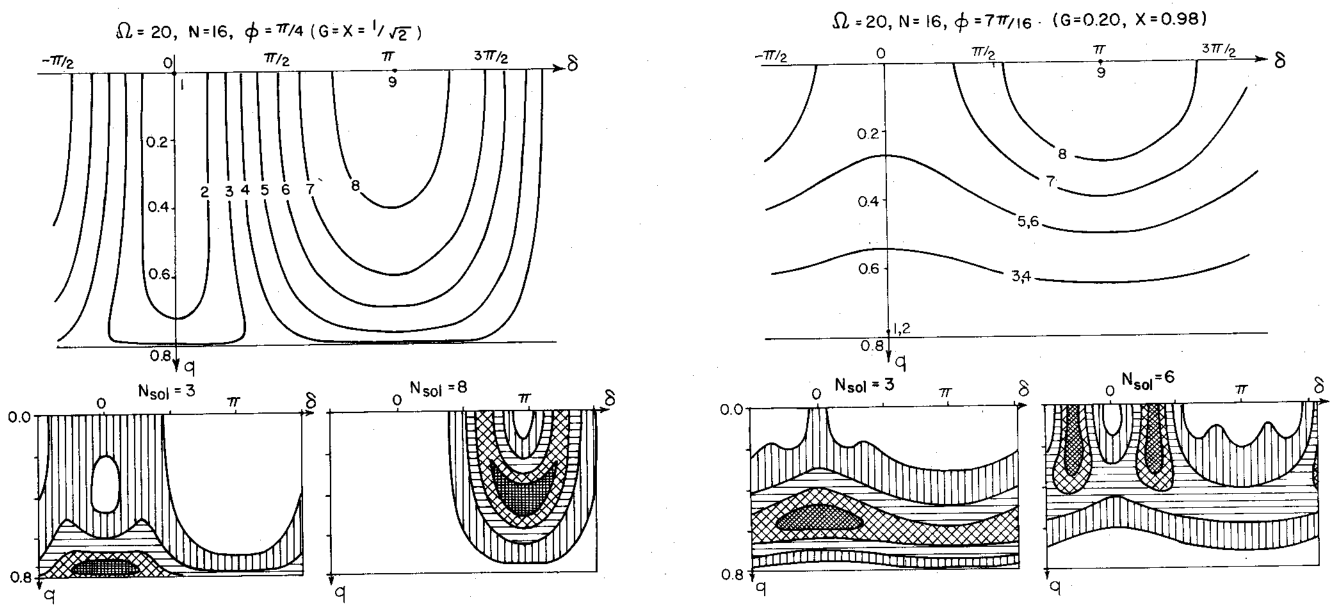

Fig.2 (Upper part) Projection of the classically quantized tori on the q-δ plane for $\phi=\pi/4$ and for $\phi=7\pi/16$ ($G=\cos\phi$, $X=\sin\phi$). On each line is given the solution number N_{sol}(=m) from the lowest one. The figure is symmetrical about the δ-axis and only the positive-q part is shown. (Lower part) Selected phase space picture of the amplitude $A(q,\delta)$ of the exact solution. Darker regions indicate a concentration of the wave function. Wave functions for the other solutions are also peaked roughly around the corresponding quantized tori.

also trace a change of the structure of the exact wave function similar to the phase transition appearing in the classical orbits. This change roughly agrees with the dotted lines in Fig.1.

Another possible check of the semiclassical method may be given by the matrix elements of observables. Because of a symmetry of the Hamiltonian, the 'Q-moment' $\langle m|Q|m\rangle$ of the eigenstates vanishes exactly. One can, instead, calculate the matrix element $\langle m|Q^2|m\rangle$ which gives the sum-rule value of Q-transition for the m-th excited state. In the semiclassical method one can calculate the quantity from the value, $\partial E_\lambda/\partial\lambda|_{\lambda=0}$, where E_λ is the quantized energy of the Hamiltonian $H+\lambda Q^2$. The result is shown in Table I. Except for the quantized states close to a separatrix, the main features of the exact result are reproduced.

Table I

	m	1	2	3	4	5	6	7	8	9
G=0.92	E	26.2	75.9	118.9	152.2	169.4	163.9	134.7	88.1	30.8
X=0.38	S	20	66	108	136	152	140	110	68	20
G=0.71	E	64.0	144.0	161.5	157.5	142.6	120.1	91.8	58.4	20.2
X=0.71	S	20	114	138	136	124	106	82	54	20
G=0.56	E	207.5	216.1	77.8	123.3	105.3	93.6	73.0	47.1	16.5
X=0.83	S	240	240	80	102	100	90	70	46	20

Calculated values of $\langle m|Q^2|m\rangle$ for the m-th quantized state. E=exact and S=semiclassical. $\Omega=20$, N=16.

REFERENCES

1) M.V.Berry and K.E.Mount, Rep.Prog.Phys.35(1972)315.
2) H.Reinhardt, Nucl.Phys.A346(1980)1.
3) S.Levit, J.W.Negele and Z.Paltiel, Phys.Rev.C21(1980)1603.
4) H.Kuratsuji and T.Suzuki, Phys.Lett.92B(1980)19.
5) J.Blaizot and H.Orland, Phys.Rev.C24(1981)1740.
6) H.Kuratsuji and T.Suzuki, J.Math.Phys.21(1980)472.
7) H.Kuratsuji, Phys.Lett.103B(1981)79, Phys.Lett.108B(1982)367.
8) T.Suzuki, preprint NBI-82-22.
9) A.M.Perelomov, Comm.Math.Phys.26(1972)222, Sov.Phys.Uspekhi 20(1977)703.
10) A.K.Kerman and S.E.Koonin, Ann.Phys.100(1976)332.
11) M.V.Berry, in 'Topics in Nonlinear Dynamics', AIP Conf.Proceedings No.46(American Institute of Physics, N.Y., 1978) p.16.
12) I.C.Percival, Adv.Chem.Phys.36(1977)1.
13) M.C.Gutzwiller, in 'Path Integrals', eds. G.J.Papadopoulos and J.J.Devereese (Plenum, N.Y., 1978) p.163.
14) A.Einstein, Verh.Deut.Phys.Ges.19(1917)82.
15) K.Matsuyanagi, Prog.Theor.Phys.67(1982), in press. See also, R.Piepenbring et al., Nucl.Phys.A348(1980)77.
16) Y.Mizobuchi, Prog.Theor.Phys.65(1981)1450.