A microscopic calculation of potentials and mass parameters for heavy-ion reactions

P.-H. HEENEN [+]

Physique Nucléaire Théorique CP 229, Campus de la Plaine U.L.B., Bd. du Triomphe , B 1050 Brussels, Belgium

H. FLOCARD and D. VAUTHERIN

Division de Physique Théorique [x], Institut de Physique Nucléaire, F - 91406 Orsay, Cedex , France.

Many methods are used to compute the potential energy of a heavy-ion system as a function of the interdistance R between the ions. However, the dependance on R of the mass parameter M associated with the kinetic energy is generally neglegted, M being assumed to be equal to the reduced mass. The adiabatic time-dependent Hartree-Fock method (1) - ATDHF - provides a framework to derive the potential and the mass parameter in a consistent way. The use of ATDHF for the study of a heavy-ion collision is greatly simplified by the fact that the interdistance between the ions is the most important collective variable. It is therefore sufficient to solve the ATDHF equations along a fixed path. This simplification decreases the amount of numerical work. It also permits to derive a potential energy directly comparable with the real part of the experimental optical potentials.

In this communication, we review the potentials and mass parameters calculated for the $^{16}O + ^{16}O$ and $^{40}Ca + ^{40}Ca$ systems (2) by ATDHF . We shall show that the fusion cross sections calculated with these potentials and mass parameters reproduce very well the experimental data.

The collective path of the collision process is generated by means of a set of constrained Hartree-Fock calculations :

$$\left(-\frac{\hbar^2}{2m}\Delta + U(r) - \lambda\,\hat{Q}(r)\right)\varphi_i(r,\lambda) = \varepsilon_i(\lambda)\,\varphi_i(r,\lambda)$$

where Q(r) is a quadrupole constraint related to the distance between the ions by :

$$\frac{1}{2}\,A\,R^2(\lambda) = \langle \Phi(\lambda)|\,\hat{Q}\,|\Phi(\lambda)\rangle$$

The Slater determinant $\Phi(\lambda)$ is composed of the individual wave functions $\varphi_i(r,\lambda)$

With the BKN force (3), the Hartree-Fock potential U(r) is given by :

$$U(r) = \frac{3}{4}t_0\,\rho(r) + \frac{3}{16}t_3\,\rho^2(r) + \int d\bar{r}'\,\rho(\bar{r}')\,\frac{a V_0 \exp-\frac{|\bar{r}-\bar{r}'|}{a} + \frac{\rho^2}{4}}{|\bar{r}-\bar{r}'|}$$

[+] Chercheur agréé FNRS

[x] Laboratoire associé au CNRS

Each orbitals being assumed to be occupied by two neutrons and two protons, the density ρ (r) is given by :

$$\rho(\lambda, r) = 4 \sum_i |\varphi_i(\lambda, r)|^2$$

The collective potential is the expectation value of the many-body hamiltonian with respect to $\Phi(\lambda)$

$$V(R) = \langle \Phi(\lambda) | H | \Phi(\lambda) \rangle$$

while the mass parameter is the polarizability with respect of the operator:

$$\hat{P}(\lambda) = i\hbar \left[\rho(\lambda), \frac{\partial \rho}{\partial \lambda} \right]$$

The calculation of this polarizability requires the solution of another set of Hartree-Fock equations with a second constraint :

$$\left(-\frac{\hbar^2}{2m} \Delta + U(r) - \lambda \hat{Q} - \mu \hat{P} \right) \varphi_i(r, \lambda, \mu) = \varepsilon_i \varphi_i$$

The mass parameter M(R) is then given by :

$$M(R) = \left(\frac{\partial}{\partial \mu} \langle \Phi(\lambda, \mu) | \hat{P} | \Phi(\lambda, \mu) \rangle \right)_{\mu=0}$$

where $\Phi(\lambda, \mu)$ is the Slater build from the $\varphi_i(r, \lambda, \mu)$

The solutions of the Hartree-Fock equation were performed in the coordinate space, assuming reflection symmetries of the individual wave functions with respect to the three planes x = 0, y = 0 and z = 0.

The potential and mass parameter are shown in fig. 1 for the $^{16}O + ^{16}O$ system and in fig. 2 for the $^{40}Ca + ^{40}Ca$ system. The barrier radii are in good agreement with the radii of the optical potentials deduced from elastic scattering data (4), (5). However, the barrier radius of $^{40}Ca + ^{40}Ca$ (R_B = 10.4 fm) is larger than the values determined from various phenomenological potentials (6). This larger value is due to the very rapid formation of the neck between the ^{40}Ca nuclei, lowering the total energy of the system. The kink found in the potential energy is also due to this very rapid formation of the neck.

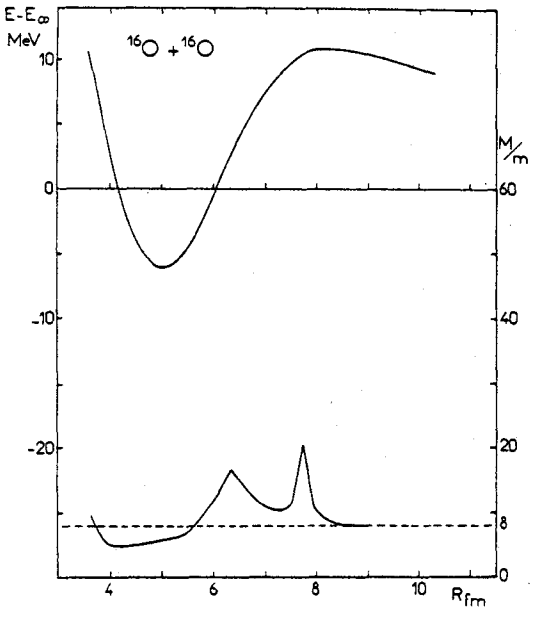
The mass parameters of both systems exhibit large peaks in the region where the ions are touching. In the case of the BKN interaction, the ATDHF mass parameter is equal to the cranking one :

$$M(R) = 2\hbar^2 \sum_{i=1}^{A} \sum_{j>A} \frac{|\langle \varphi_j(\lambda, r) | \frac{\partial}{\partial R} | \varphi_i(\lambda, r) \rangle|^2}{\varepsilon_j - \varepsilon_i}$$

One sees that M(R) has a peak either when an unoccupied level comes closed to an occupied one or when the individual wave-functions change rapidly. No crossings occur in the region where the mass increases. In the case of the $^{16}O + ^{16}O$ system, it is possible to relate each peak with the rearrangement of an individual wave function due to the Pauli principle. The most external peak is due to the individual wave functions with the largest elongation along the collision axis. The peaks are no more

Fig. 1

Potential and mass parameter obtained for the $^{16}O + ^{16}O$ system.



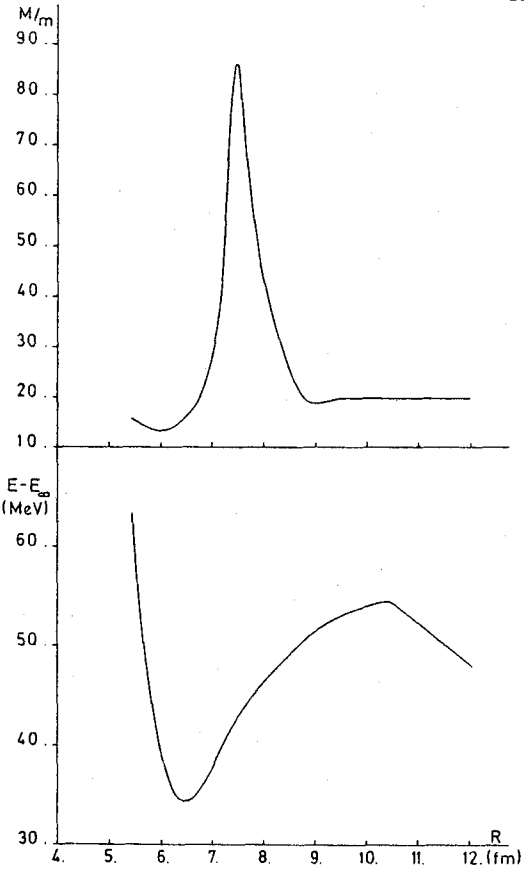


Fig. 2

Potential and mass parameter obtained for the $^{40}Ca + ^{40}Ca$ system.

resolved in the $^{40}Ca + ^{40}Ca$ system, the number of individual states being too large. To show the effect of the mass parameter on observable quantities, we have calculated the fusion cross section of $^{40}Ca + ^{40}Ca$ for which experimental data exist over a large energy range above the barrier (6). We have used a barrier penetration model proposed by Descouvemont et al. (7) in which the flux transmitted down to a critical radius R_c is assumed to be completely absorbed. To calculate the transmission coefficient down to R_c, the collective Bohr Hamiltonian :

$$H(R, \dot{R}) = \frac{1}{2} M(R)\, \dot{R}^2 + V(R)$$

must be requantized. We used the Pauli prescription, leading to the Schrödinger equation :

$$\left\{ -\left[\left(\frac{\hbar^2}{2M(R)} \right)^{\frac{1}{4}} \frac{\partial}{\partial R} \left(\frac{\hbar^2}{2M(R)} \right)^{\frac{1}{4}} \right]^2 + V(R) + \frac{\hbar^2}{2M_o} \frac{\ell(\ell+1)}{R^2} \right\} \Psi_\ell(R) = E\, \Psi_\ell(R)$$

where M_o is the reduced mass of the system.

The fusion cross sections calculated with the reduced mass and with the ATDHF mass are compared with experiment in fig. 3. It can be seen that the cross section

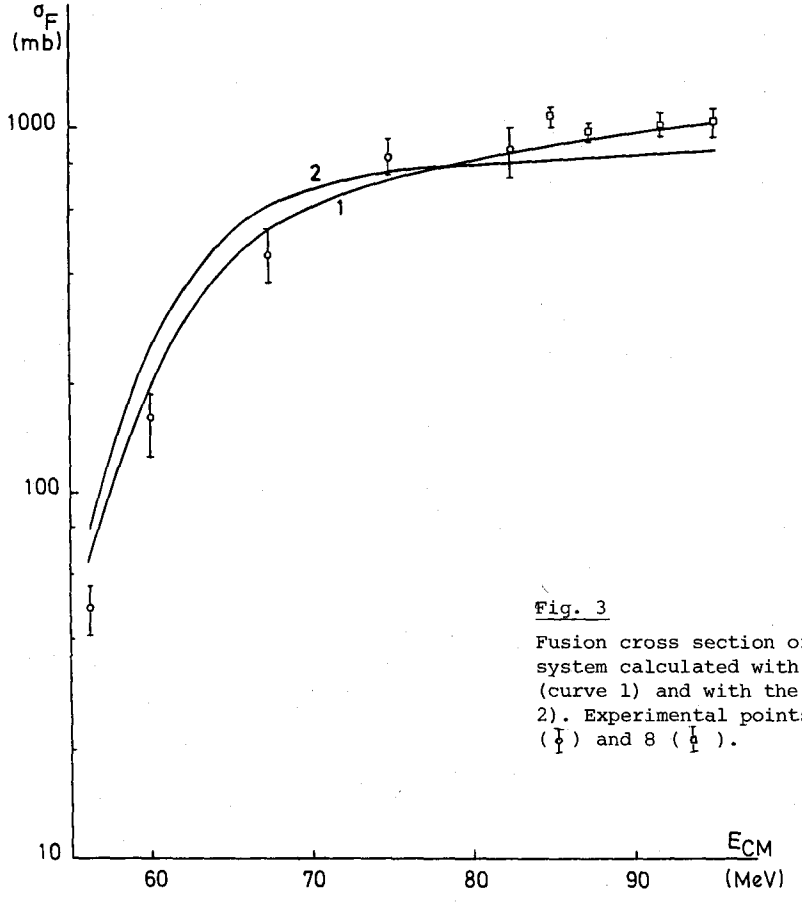


Fig. 3

Fusion cross section of the $^{40}Ca + ^{40}Ca$ system calculated with the ATDHF mass (curve 1) and with the reduced mass (curve 2). Experimental points are from ref. 6 ($\frac{1}{1}$) and 8 ($\frac{1}{1}$).

obtained with a constant mass is in reasonable agreement with experiment. However, it overestimates the cross section in the vicinity of the Coulomb barrier. The peak in the mass parameter decreases the probability of penetrating the barrier at low energy. It brings the fusion cross section calculated with the ATDHF mass in closer agreement with experiment.

This result indicates that the effect of the effective mass is not negligible for the fusion of two nuclei as heavy as ^{40}Ca. For collision more peripherical than fusion like elastic or inelastic scattering, the effect of the mass parameter is probably much weaker for $^{40}Ca + ^{40}Ca$ than for $^{16}O + ^{16}O$. As can be seen from figs 1 and 2 , the peaks of the $^{16}O + ^{16}O$ mass are located at relatively larger interdistances than the peak of the $^{40}Ca + ^{40}Ca$ mass. It is therefore probable that the effect of M(R) is more weakened by absorption for heavy systems than for light systems.

References

1) M. BARANGER and M. VENERONI Ann. of Phys. 114 (1978) 123
2) H. FLOCARD, P.-H. HEENEN and D. VAUTHERIN, Nucl. Phys. A339 (1980) 336 and to be published
3) P. BONCHE, S.E. KOONIN and J.W. NEGELE, Phys. Rev. C13 (1976) 1226
4) A. GOBBI et al. Phys. Rev. C7 (1973) 30
5) M. RICHTER et al. Nucl. Phys. A278 (1977) 163
6) E. TOMASI et al. Nucl. Phys. A373 (1982) 341
7) P. DESCOUVEMONT, D. BAYE and P.-H. HEENEN to appear in Z. Phys.
8) H. DOUBRE et al. Phys. Lett. 73B (1978) 135.