NUCLEAR COLLECTIVE MOTIONS IN SEMICLASSICAL TDHF

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Abstract

Using the Wigner transform of the one-body density matrix we extract a collective hamiltonian from the TDHF energy functional for slow nuclear collective motions. For a rotating nucleus and for small vibrations we find solutions of the Vlasov equation which allow an explicit evaluation of currents, inertial parameters and eigenfrequencies. The extension to generalized scaling modes is analysed to show that the TDHF-Fluid-Dynamical chain of equations can be exactly truncated. Quantum effects, one-body damping mechanisms and the elasticity limit are finally discussed.

1. Introduction

The fluid-dynamics approach seems to be a powerful tool to understand collective motions in nuclei in terms of macroscopic quantities as local density, current and pressure tensor. The microscopic foundation of nuclear fluid dynamics is the mean field theory (TDHF): his validity lies on the fact that, in low energy excitations of nuclei, the long mean free path regime prevents local equilibration; instead, dynamycal distortions of the local momentum distribution would have a crucial role. The starting point is the TDHF equation for the Wigner transform $f(\underline{r},\underline{p},t)$ of the one body density matrix, which is the quantum-mechanical analog of the classical distribution function[1])

$$\dot{f}(\underline{r},\underline{p},t)= \frac{2}{\hbar} \sin \left(\frac{\hbar}{2} \Lambda_{12}\right) h^{(1)}(\underline{r},\underline{p},t)\; f^{(2)}(\underline{r},\underline{p},t) \qquad (1.1)$$

where $h=\frac{p^2}{2m} + w(\underline{r},\underline{p},t)$ is the Wigner transform of the self-consistent HF hamiltonian, $\Lambda_{12}=\underline{\nabla}_r^{(1)}\underline{\nabla}_p^{(2)}-\underline{\nabla}_p^{(1)}\underline{\nabla}_r^{(2)}$ and the differential operator is intended to act on the corresponding function: $\underline{\nabla}^{(i)} \rightarrow g^{(i)}$.

From eq.(1.1) one can generate in the usual way the chain of fluid-dynamical equations which express the conservation laws of mass, momentum, energy and so on.

Since

$$\frac{2}{\hbar} \sin \left(\frac{\hbar}{2} \Lambda_{12}\right) h^{(1)} f^{(2)} = \{h,f\} - \frac{h^2}{24}\Lambda_{12}^3\, h^{(1)} f^{(2)} + \;\ldots\ldots \qquad (1.2)$$

neglecting the second and higher order terms in $\hbar$ one obtains the semiclassical limit of TDHF, the well known Vlasov equation.

Moreover we remark that , due to the structure of the Λ_{12} operator, we do not have explicit quantum terms up to the second $\underline{p}$-moment dynamical equation derived from eq.(1.1), allowing a Galilei invariant non locality for the HF potential. This means that if we are able to truncate the chain at that order we get a completely classical set of equations, quantum effects being in the initial conditions and in the truncation procedure. In particular in the extreme case of a harmonic oscillator single particle potential the quantum terms are exactly dropped out and the TDHF dynamics becomes wholly classical.

This paper is devoted to a search for analytical solutions of the fluid dynamical equations for a wide class of nuclear collective motions.

2. Slow collective motions: adiabatic expansion

Taking in mind the remarks of the previous section we start our analysis from the Vlasov equation $\dot{f}=\{h,f\}$. In the case of small collective velocities

$$\omega\cdot A<<v_F \tag{2.1}$$

where ω,A are respectively the collective frequencies and amplitudes and v_F is the Fermi velocity, we can follow an adiabatic expansion which leads in a natural way to a collective Hamilton dynamics [2,3]

$$f(\underline{r},\underline{p})=f_o(\underline{r},\underline{p})+\{f_o,\chi\}+\tfrac{1}{2}\{\chi,\{\chi,f_o\}\}+\ ..=f_o+f_1+f_2+\ldots.. \tag{2.2}$$

with f_o,χ $\underline{p}$-even functions in the phase space; f_o being a time-dependent equilibrium distribution and χ proportional to the collective velocity field. Correspondingly the transform of the HF potential can be expanded

$$h(\underline{r},\underline{p})=\frac{p^2}{2m}+w(\underline{r},\underline{p})=h_o(\underline{r},\underline{p})+w_1(\underline{r},\underline{p})+w_2(\underline{r},\underline{p})\ +\ \ldots \tag{2.3}$$

The quantities with odd (even) labels are odd (even) with respect to the $\underline{p}\rightarrow-\underline{p}$ transformation.

The Vlasov equation becomes a set of coupled equations in the even and odd parts of the distribution function

$$\dot{f}_o=\{h_o,f_1\}+\{w_1,f_o\} \tag{2.4.1}$$

$$\dot{f}_1=\{h_o,f_o\}+\{h_o,f_2\}+\{w_2,f_o\}+\{w_1,f_o\} \tag{2.4.2}$$

A main result of this analysis is that the HF total energy functional of the system can be split in two parts [4]

$$E_{TOT}=U(f_o)+K(\chi,f_o)+O(\chi^4) \tag{2.5}$$

with

$$U(f_\circ)=\int_{\underline{r}}\int_{\underline{p}}\left(\frac{p^2}{2m}+\frac{1}{2}w_\circ\right)f_\circ$$

$$K(\chi,f_\circ)=\int_{\underline{r}}\int_{\underline{p}}\left[h_\circ f_2+\tfrac{1}{2}w_1 f_1\right]=\tfrac{1}{2}\int_{\underline{r}}\int_{\underline{p}}\chi\frac{\partial f_\circ}{\partial t}$$

where U, function of the "coordinate" $f_\circ$ only, and K, quadratic in χ, are interpreted respectively as collective potential and kinetic energy. The form (2.5) for the semiclassical collective hamiltonian is the starting point to study potential and inertial parameters for collective modes[4]. It is also possible to support the interpretation of eq. (2.5) through a Hamilton formulation of the dynamical equations (2.4) with $f_\circ,\chi$ playing a role of collective canonical coordinates[5]. A natural consequence of a Hamilton form is that the resulting collective motion is undamped: consistently with the adiabatic assumption we do not have one-body dissipation terms in the collective dynamics. The variational approach has also a practical use: for a wide class of collective motions it represents a valid tool to determine the field $\chi(\underline{r},\underline{p})$ without solving the adiabatic aquations (2.4)[5].

This is also the main aim of the next section.

3. Generalized scaling modes (GSM)

In this class of collective motions all the nucleons are looked to move coherently in a common velocity field. The time-even part of the density matrix is given in terms of the static solution

$$\rho_\circ(t)=\exp(i\ K/\hbar)\rho_{static}\ \exp(-i\ K/\hbar) \qquad (3.1)$$

where K is a time-odd hermitian operator of the form

$$K=\frac{1}{2}\left[\underline{p}\cdot\underline{s}(\underline{r},t)+\underline{s}(\underline{r},t)\cdot\underline{p}\right] \tag{3.2}$$

with $\underline{s}(\underline{r},t)$ a vector field which characterizes the collective motion.

Eq.(3.1) supplies the external information we need on the collective path in order to avoid to solve the second adiabatic equation (2.4.2) . The time derivative gives

$$i\hbar\dot{\rho}_0 = \left[\rho_0, \dot{K}\right]$$

which in the Wigner transformed space, in the semiclassical limit, leads to

$$\frac{\partial f_0}{\partial t} = \{\underline{p}\cdot\underline{v}(\underline{r}), f_0\} \tag{3.3}$$

where we use the collective velocity field $\underline{v}(\underline{r},t) = -\dot{\underline{s}}(\underline{r},t)$.

From some properties of the Poisson brackets the collective kinetic energy assumes the form

$$K(\chi,f_0)=\frac{1}{2}\, m\int d\underline{r}\ \underline{v}(\underline{r})\cdot\underline{j}(\underline{r}) \tag{3.4}$$

where

$$\underline{j}(\underline{r})=\int_{\underline{p}} \frac{\underline{p}}{m}\, f(\underline{r},\underline{p}) = \int\frac{\underline{p}}{m}\{f_0,\chi\}$$

is the response current inside the system and in general is not parallel to $\underline{v}(\underline{r})$.

A second cosequence of eq. (3.3) is the relation

$$\{\chi,h_0\}=\underline{p}\cdot\underline{v}(\underline{r}) \tag{3.5}$$

obtained just comparing with the first adiabatic equation(2.4.1), in the case of local HF potential, $w_1(\underline{r},\underline{p})=0$. Eq. [3.5] can be used to find explicitly the χ field without solving the adiabatic equations.

Several cases of physical interest have the property (3.1). When

$\underline{s} = \alpha\, \underline{r} \times \underline{n}$ we describe a rotation through an angle α about the direction $\underline{n}$. In this case $\underline{v}(\underline{r}) = \underline{v}_{rigid}(\underline{r}) = \underline{\omega} \times \underline{r}$, with $\underline{\omega} = \dot{\alpha}\, \underline{n}$, angular velocity.

The collective mode obtained just scaling the coordinates corresponds to a displacement field $\underline{s} = \alpha\, \underline{\nabla}\, \phi$ where $\alpha(t)$ is the scaling parameter and $\phi(\underline{r})$ is a real function which depends on the studied collective motion. We have an irrotational velocity field $\underline{v}_{scaling} = -\dot{\alpha}\, \underline{\nabla}\, \phi(\underline{r})$ and the eq. (3.5) can be solved to give a scaling field $\chi(\underline{r},\underline{p}) =$ $= - m\, \dot{\alpha}\, \phi\, (\underline{r})$ and a collective kinetic energy(4)

$$K = \tfrac{1}{2}\, m \int_{\underline{r}} \rho_{\circ}\, \underline{v}^{2}_{scaling} \tag{3.6}$$

with inertial parameter

$$M_{scaling}(\alpha) = m \int_{\underline{r}} \rho_{\circ}\, (\underline{r},\alpha)\, (\underline{\nabla}\phi)^{2} \tag{3.7}$$

4. Small amplitudes: semiclassical RPA

Scaling modes are meaningful for small amplitude oscillations. In this case the collective hamiltonian can be written in a harmonic form

$$E_{TOT} = \tfrac{1}{2}\, M\, \dot{\alpha}^{2} + \tfrac{1}{2}\, C\alpha^{2} \tag{4.1}$$

in terms of the scaling parameter $\alpha(t)$, and the frequencies are simply given by $\omega^2 = C/M$. The restoring parameter comes from second order variations of the collective potential

$$\tfrac{1}{2}\, C\, \alpha^{2} = \delta^{2}\, U(f_{\circ}) = \int_{\underline{r}} \int_{\underline{p}} h_{st}\, \delta^{2}\, f_{\circ} + \tfrac{1}{2} \int_{\underline{r}} \int_{\underline{p}} \delta w_{\circ}\, \delta\, f_{\circ} \tag{4.2}$$

where, from eq. (3.1),

$$f_\circ = f_{st} + \{f_{st},\xi\} + \frac{1}{2}\{\xi,\{\xi,f_{st}\}\} + \ldots = f_{st} + \delta f_\circ + \delta^2 f_\circ + \ldots.$$

with $\xi = \alpha \underline{p} \cdot \underline{\nabla}\phi$, and in correspondance the HF potential is $w_\circ = w_{st} + \delta w_\circ + \delta^2 w_\circ + \ldots..$

For isoscalar modes and a Skyrme -Hartree-Fock potential

$$w_{HF} = \frac{3}{4} t_\circ \rho + \frac{3}{16} t_3 \rho^2 + \beta(\rho p^2 - 2m\underline{j}\cdot\underline{p} + 2m\tau) + \gamma\nabla^2\rho + V_{coul} \tag{4.3}$$

with $\beta = \frac{3t_1+5t_2}{16}$, $\gamma = \frac{5t_2-9t_1}{32}$, we get with some algebra a stiffness

$$C = \int_{\underline{r}} \left\{ \frac{K_s}{m^*}\left[2(\nabla_i u_i)^2 - u_i\nabla^2 u_i\right] - \frac{3}{4} t_\circ \rho_{st} u_i \nabla_i\left[\rho_{st}\nabla_k u_k\right] - \frac{3}{8} t_3 \rho_{st} u_i \nabla_i\left[\rho_{st}^2 \nabla_k u_k\right] + \right.$$

$$\left. + \beta K_s \rho_{st}\left[7(\nabla_k u_k)^2 - 3u_i\nabla_i\nabla_k u_k\right] - \gamma\nabla_i(u_i\rho_{st})\left[u_i\nabla_i\nabla^2\rho_{st} - \nabla^2(\nabla_i(u_i\rho_{st}))\right] + \text{coulomb}\right\} \tag{4.4}$$

where $\underline{u}=\underline{\nabla}\phi$ and $k_s = \frac{1}{3}\int_{\underline{p}} p^2 f_{st}$ is proportional to the kinetic energy density in the static solution.

The form eq.(4.4) becomes very simple for low moltipoles[6]. For monopole ($\phi=r^2$) and quadrupole ($\phi=2z^2-x^2-y^2$) modes we get frequencies exactly of the same form as in the quantum sum rule approach of Bohigas, Lane and Martorell[7]: quantum effects are not present for low multipoles in the scaling mode. This is a quite general result as we"ll show in the next section.

5. Small oscillations in generalized scaling: elasticity equation

In this section we release any semiclassical assumption. The fluid TDHF chain can be exactly truncated for small amplitude GSM.

In the Wigner space, at the first order in the $\underline{s}$ field, the $\underline{p}$-even part of the distribution function from eq.(3.1) satisfies the equation

$$\delta f_\circ = \frac{2}{\hbar} \sin \frac{\hbar}{2} \Lambda_{12} \quad f_{st}^{(1)} (\underline{p}.\underline{s})^{(2)} \tag{5.1}$$

which gives a transition density of the form $\delta\rho = \int_{\underline{p}} \delta f_\circ = \nabla_i(\rho_{st} s_i)$

Through the continuity equation we have in correspondance a current

$\underline{j} = \rho_{st}\,\underline{v}$, with $\underline{v} = -\dot{\underline{s}}$

The first p-moment of the Wigner-TDHF eq.(1.1), linearized for small oscillations, is a Euler-like equation of the form

$$m \frac{\partial}{\partial t}\,\underline{j} + \underline{\nabla}\,\delta\overleftrightarrow{\tau} + \delta\rho\underline{\nabla}w_{st} + \rho_{st}\underline{\nabla}\delta w = 0 \qquad (5.2)$$

where $\delta\tau_{ij}$ is the transition kinetic energy tensor and the HF potential $w=w_{st}+\delta w$ for simplicity has been assumed to be local.

Because the tensor $\delta\tau_{ij}$ can be written, from (5.1),

$$\delta\tau_{ij}(\underline{r},t) = \int_{\underline{p}} \frac{p_i p_j}{m}\,\delta f_\circ(\underline{r},\underline{p},t) =$$

$$\nabla_k(s_k\tau^{st}_{ij}) + \tau^{st}_{kj}\nabla_i s_k + \tau^{st}_{ik}\nabla_j s_k - \frac{\hbar^2}{4m}\nabla_k(\rho_{st}\nabla_i\nabla_j s_k) \qquad (5.3)$$

only in terms of the $\underline{s}$-field and the static kinetic energy tensor, we see that the fluid-dynamical chain is exactly truncated at the first p-moment eq.(5.2) which now becomes a Schrodinger-like equation for the vector field $\underline{s}(\underline{r},t)$. With appropriate boundary conditions we can find solutions and eigenvalues which give respectively transition densities, flow patterns and frequencies of the various multipole oscillations.

The only quantum effect comes from the last term of the transition kinetic energy eq.(5.3): all the solutions with $\nabla_i\nabla_j s_k=0$ give rise to a completely classical response. This is the case of the simple Tassie scaling model, discussed in the previous section, for isoscalar monopole and quadrupole resonances.

Remaining in the scaling picture $\underline{s}(\underline{r})=\underline{\nabla}\phi(\underline{r})$, the eq.(5.2) supplies a general way to find the scalar field $\phi(\underline{r})$ for any multipole. A particular case was studied with a variational approach by Serr et al.[8] Moreover, if we specialize our HF potential in order to get the semiclassical energy density functional used by Holzwarth et al.[9], we easily get from eq. (5.2) the same fluid dynamical equations. This discussion shows that our result, derived in a direct way from a linearized TDHF theory is very general: 1) we can reach a fluid dynamical description of giant resonances with clear microscopic foundations; 2) we can use realistic interactions such as Skyrme-type forces.

Finally we would like to remark that assuming an isotropic static momentum distribution $\tau_{ij}^{st} = \frac{p_F^2}{5m} \rho_{st} \delta_{ij}$ and a uniform model density, eq.(5.2) takes the form of a Lamé equation for the displacement field in an elastic medium [10]

$$m\rho \frac{\partial^2}{\partial t^2} s_i = \lambda \nabla_i \underline{\nabla} \cdot \underline{s} + \mu \nabla_j (\nabla_i s_j + \nabla_j s_i) \qquad (5.4)$$

with shear modulus $\mu = \frac{p_F^2}{5m} \rho_{st}$ and longitudinal modulus $\lambda = \frac{\partial w_{st}}{\partial \rho} \rho_{st}^2 + \frac{p_F^2}{5m} \rho_{st}$. This means that there is no damping and the widths cannot be described in generalized scaling approach. Actually it can be shown that the heat flow tensor (third $\underline{p}$-moment of the distribution function) is exactly zero for GS modes: the coherence of the particle motion prevents the developing of the Landau damping, which is the only dissipation present in TDHF.

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