VARIATIONAL METHODS IN THE MASTER FIELD FORMULATION FOR QCD_{3+1}

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ABSTRACT

Master fields are formulated for finite N-QCD_{3+1}. They satisfy classical Yang-Mills equations with an infinite number of internal indices and an infinite number of constraints. Master fields and constraints on them in the large N limit are derived from the finite N master fields and constraints using vacuum dominance among color singlet states. The large N constraints can be explicitly solved and the solutions involve arbitrary functions which are used as trial functions in variational calculations.

I. FINITE N MASTER FIELDS

In this letter we will report the outline of our previous works[1,2] on master field methods for QCD in 4-dimensional Minkowski space (QCD_{3+1}).

We choose the axial gauge ($A_3 = 0$) and SU(N) for the color group. Then the dynamical variables are the 1 and 2 components of matrix operator gluon fields, $(\hat{A}_\alpha)_{ab}(x)$ ($\alpha = 1, 2$; $a, b = 1, \cdots, N$), and the conjugate momenta, $(\hat{\Pi}_\alpha)_{ab}(x)$, which satisfy usual canonical commutation relations. The operator Hamiltonian, $\hat{H}$, is given by

$$\hat{H} = \int d^3x \; 1/2 \; \mathrm{Tr}[\hat{\Pi}_\alpha^2(x) + (\partial_3\hat{A}_\alpha(x))^2 + \hat{F}_{12}^2(x)]$$
$$+ 1/4 \int d^3x \; d^3y |x_3 - y_3| \delta^2(x_\alpha - y_\alpha) \; \mathrm{Tr}[(\hat{\nabla}_\alpha \hat{\Pi}_\alpha)(x)(\hat{\nabla}_\beta \hat{\Pi}_\beta)(y)], \quad (1)$$

where Tr represents a trace over color indices and all operator products are color ordered products (COP): adjoint operator products which are made from $\hat{A}_\alpha$ and $\hat{\Pi}_\alpha$ such that the order of the operators and the order of the matrix products in the color space coincides. An example of COP is given by

$$(\hat{A}_1 \hat{\Pi}_2 \hat{\Pi}_3 \hat{A}_4)_{ab} \equiv (\hat{A}_{\alpha_1})_{ac_1}(x_1)(\hat{\Pi}_{\alpha_2})_{c_1c_2}(x_2)$$
$$\times (\hat{\Pi}_{\alpha_3})_{c_2c_3}(x_3)(\hat{A}_{\alpha_4})_{c_3b}(x_4). \quad (2)$$

Since the Hamiltonian (1) and the total momentum operator are invariant under global color SU(N) transformations, the energy-momentum eigenstates can be classified by the irreducible representations of the color SU(N) group.

Finite N master fields, $A_\alpha(x)$ and $\Pi_\alpha(x)$, are defined as reduced matrix elements of $\hat{A}_\alpha(x)$ and $\hat{\Pi}_\alpha(x)$ between all singlet and adjoint energy-momentum eigenstates such as [1,2,3]

$$\langle s|(\hat{A}_\alpha)_{ab}(x)|g, cd\rangle = \sqrt{N/(N^2-1)}$$
$$\times (\delta_{ad}\delta_{bc} - 1/N\,\delta_{ab}\delta_{cd})(A_\alpha(x))(s\,;g), \quad (3)$$

where s(g) represents the quantum number of a singlet (adjoint) energy-momentum eigenstates, $|s\rangle$ ($|g, ab\rangle$). The master fields, $(A_\alpha(x))(n;n')$ and $(\Pi_\alpha(x))(n;n')$, can be treated as C-number matrix fields with matrix indices n (= s or g). Since COP creates only singlet or adjoint states from singlet and adjoint states (when COP is applied to an adjoint state one of the indices of the adjoint state and the neighboring index of COP should be contracted), any matrix element of COP between singlet and adjoint states can be expressed as a matrix product of the master fields such as

$$\langle s|(\hat{A}_1 \hat{\Pi}_2 \hat{\Pi}_3 \hat{A}_4)_{ab}|g, cd\rangle = \sqrt{N/(N^2-1)}$$
$$\times (\delta_{ad}\delta_{bc} - 1/N\,\delta_{ab}\delta_{cd})(A_1 \Pi_2 \Pi_3 A_4)(s\,;g), \quad (4)$$

where a matrix notation is used for products of the master fields;

$$(A_1 \Pi_2)(s\,;\,g) = \sum_{n=s',g'} (A_1)(s\,;\,n)(\Pi_2)(n\,;\,g). \tag{5}$$

Taking matrix elements of the operator equations of motion between the singlet and adjoint states, it can be shown that the finite N master fields satisfy the classical Yang-Mills equations with an infinite number of internal indices, n (= s or g). The canonical commutation relations can be inverted into an infinite number of constraints. They are derived by considering the difference between two traces of any COP and the induced COP by the cyclic permutation. An example of the constraints is given by

$$\begin{aligned}
&(A_1 A_2 \Pi_3 A_4 A_5 A_6)(s_1\,;\,s_2) - (A_6 A_1 A_2 \Pi_3 A_4 A_5)(s_1\,;\,s_2) \\
&= -iN\delta(3,\ 6)\{\sum_{s_3} (A_1 A_2)(s_1\,;\,s_3)(A_4 A_5)(s_3\,;\,s_2) \\
&\qquad - 1/N^2\ (A_1 A_2 A_4 A_5)(s_1\,;\,s_2)\}.
\end{aligned} \tag{6}$$

There is another set of constraints which fix an integer value of N and correspond to the Mandelstam constraints[4] for the Wilson loops.

II. THE LARGE N LIMIT

In the large N limit contributions from the vacuum dominate over those from other excited singlet states[4,5]. Then the large N master fields can be obtained from the finite N master fields by reducing an infinite number of the singlet indices to a single index which corresponds to the vacuum. They also satisfy the classical Yang-Mills equations. Large N constraints are obtained from the constraints corresponding to the commutation relations by replacing the sum over intermidiate singlet states with the vacuum index. In the large N limit Mandelstam-like constraints can be neglected.

The large N constraints can be explicitly solved and the solutions involve arbitrary functions which should be determined by using dynamical equations. To solve the large N-QCD_{3+1} exactly, the solutions should involve an infinite number of arbitrary functions, which gives us great difficulty. Therefore we will use solutions with a finite number of arbitrary functions and determine them by minimizing the vacuum energy. The simplest solution involves a 1-particle wave function as arbitrary functions. The variational calculation, in which the simplest solution is used as a trial function,[1] gives a logalismic potential between quarks and anti-quarks along the Z-axis under straightforward cut off. More realistic trial functions were given in Ref. 2. The Lorentz invariance and the gauge invariance of our method will be discussed within the master field formulation in covariant gauges in a forthcoming paper.

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