CONFINEMENT BY THICK MAGNETIC VORTICES

Tamiaki Yoneya
Institute of Physics, University of Tokyo
Komaba, Tokyo 153, Japan

Abstract: An argument is presented showing that the pure lattice gauge theories which are infrared unstable at the origin of the coupling constant are always in confining phase, provided that (i) the gauge group is compact and contains nontrivial center;(ii) Lagrangian does not contain long range interaction. The steps for an attempt at a proof of confinement are suggested.

1. Introduction

Although Monte Calro simulations on the lattice have provided several convincing evidences for believing in confinement in QCD, we are yet far from microscopic understanding of the mechanism of confinement in the continuum limit. In the lattice-regularized theory, the continuum theory corresponds to the critical region where the physical correlation length is much larger than the lattice spacing a.

In this talk, I wish to report some simple observations on the general structure of the large distance behavior of lattice gauge theories. I will first present a rigorous inequality [1] for magnetic flux free energies which holds in any lattice gauge models with local action and with compact gauge group containing nontrivial center. Then, I will show that if this inequality is supplemented by a simple assumption, which is apparently of kinetical nature, it leads to an interesting consequence for confinement in the large distance critical region.

2. Center transformation and magnetic vortices

The inequality is based on the center invariance of the gauge field Lagrangian. For definiteness, we take the standard Wilson action of pure gauge theory.

$$S = \frac{1}{g^2}\sum_P \mathrm{Tr}\,(U_P + U_P^\dagger)\ , \qquad U_P = U_{x\mu} U_{x+\hat{\mu}\nu} U^\dagger_{x+\hat{\nu}\mu} U^\dagger_{x\nu} \tag{1}$$

where $U_{x\mu}$ is the parallel transporter from the point x to a nearest neighbough point $x+\hat{\mu}$. The center transformation is defined by $U_{x\mu} \to \gamma_\mu U_{x\mu}$, γ_μ being an arbitrary element of the gauge group center C(G), under which the action is invariant. The center invariance is preserved even if the transformation is restricted to an infinitely extended d-1 dimensional plane-layer of parallel links. Furthermore, if a center transformation is performed in a finitely extended layer of parallel links, the action density changes its value only at the (d-2 dimensional) boundary of the layer, as in Fig. 1. The Wilson loop which winds around the boundary changes by the group center. It is appropriate to call the energetic object, which is produced at the boundary of the layer, the magnetic vortex (or simply vortex).

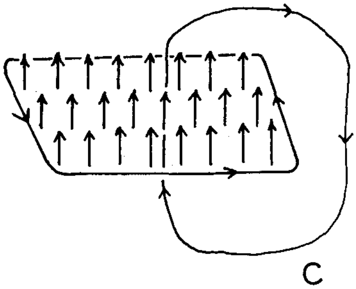

Fig. 1. The layer of parallel links and the vortex at its boundary. C is the Wilson loop winding around the vortex.

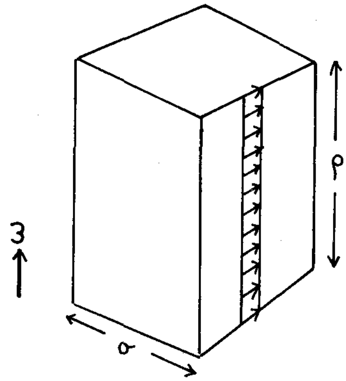

Fig. 2. The 3 dimensional rectangular box with periodic boundary condition in the 3rd direction. The strip of parallel links on the surface is the intersection with the layer of the center transformation.

To discuss the possible role of the vortices in the large distance behaviour, we have to consider the vortices with larger and larger thickness. For this purpose, we extract a finite hypertube which completely encloses the boundary of the layer of the center transformation. The center transformation then induces the change of the boundary condition on the surface of the hypertube. Instead of a hypertube, for simplicity, let us consider d-dimensional rectangular box in which the periodic boundary condition is imposed in its last d-2 directions. (See Fig. 2.) Let the sides of the box, which is denoted by $\Lambda(\rho,\sigma)$,be $(\sigma,\sigma,\rho,\cdots,\rho)$. The center transformation for the box is equivalent to a change of the boundary condition on the strip which is the intersection of the surface and the layer of the center transformation. The partition function for $\Lambda(\rho,\sigma)$ is defined

$$Z_{\Lambda(\rho,\sigma)}\{u\} = \int \prod_{U_{x\mu} \subset \Lambda(\rho,\sigma)} dU_{x\mu} \quad \exp \frac{1}{g^2} \sum_{P \subset \Lambda(\rho,\sigma)} \mathrm{Tr}\,(U_P + U_P^{+}) \tag{2}$$

where $\{u\}$ represents the boundary value of the parallel transporter matrices on the surface $\partial\Lambda(\rho,\sigma)$. In (2), the summation and the product are extended only over plaquettes and links which are completely contained inside $\Lambda(\rho,\sigma)$. Then, the vortex free energy corresponding to a center element γ is given by

$$F^{(\gamma)}_{\Lambda(\rho,\sigma)}\{u\} = -\ln\left[Z_{\Lambda(\rho,\sigma)}\{u_\gamma\} \,/\, Z_{\Lambda(\rho,\sigma)}\{u\}\right] \tag{3}$$

where $\{u_\gamma\}$ denotes the center-transformed boundary value. We now define [2]

$$<\hat{\gamma}> = \operatorname*{Max}_{\{u\}} \left[\sum_{\gamma \in C(G)} \gamma \exp - F^{(\gamma)}_{\Lambda(\rho,\sigma)}\{u\} \Big/ \sum_{\gamma \in C(G)} \exp - F^{(\gamma)}_{\Lambda(\rho,\sigma)}\{u\}\right] \tag{4}$$

If the gauge group is compact and the Lagrangian does not contain long range interaction, the following inequality [1] can be proven

$$\langle \hat{\gamma} \rangle_{\Lambda(\rho, n\sigma)} \leq \left[\langle \hat{\gamma} \rangle_{\Lambda(\rho,\sigma)} \right]^{n^2} . \tag{5}$$

This interrelates the vortices with different thickness. Let us next discuss the possible implications of this simple inequality to confinement problem. For this purpose, the properties of $\langle \hat{\gamma} \rangle_{\Lambda(\rho,\sigma)}$ in the large scale limit is crucial.

3. Large scale behaviour of vortex free energy and confinement

If the length of the vortex is much larger than its thickness, we expect the following behaviour for the free energy

$$\hat{F}^{(\gamma)}_{\Lambda(\rho,\sigma)} \equiv \underset{\{u\}}{\mathrm{Max}}\ F^{(\gamma)}_{\Lambda(\rho,\sigma)}\{u\} \simeq \rho^{d-2} f(\sigma) , \quad r \ (\equiv \sigma/\rho) \ll 1 . \tag{6}$$

The expected behaviour of the free energy density $f(\sigma)$ for large σ is the following [3], [4]:

(i) confinement phase	$f(\sigma)$	$\sim$ const. $\exp -\alpha\sigma^2$,	(7a)
(ii) Higgs phase	$f(\sigma)$	$\sim$ const. ,	(7b)
(iii) massless phase (d=4)	$f(\sigma)$	$\sim$ const.$/\sigma^2$.	(7c)

There is no rigorous justification for these properties (d$\geqslant$3). But any reasonable approximation schemes appropriate to each phase predict such behaviours. For instance, the strong coupling expansion predicts (7a) with α being the string tension. Among these properties, that the maximum vortex free energy is proportional to ρ^{d-2} for large ρ seems generally valid independently of different phases. Unfortunately, however, we have no rigorous control over how large ρ must be for the uniform validity of (7). To take into this plausible property account, we adopt the following working <u>hypothesis</u> (or <u>conjecture</u>): In compact pure lattice gauge theories, there exists a constant $k = k(\rho_0, \sigma_0, r)$ for any ρ, σ and r satisfying

$$\rho \gg \rho_0 , \quad \sigma \gg \sigma_0 , \quad r = \sigma/\rho \ll 1$$

such that

$$\langle \hat{\gamma} \rangle_{\Lambda(2\rho,\sigma)} \leq k \ \langle \hat{\gamma} \rangle_{\Lambda(\rho,\sigma)} . \tag{8}$$

When C(G) = Z(2), $\gamma = \pm 1$ and $\langle \hat{\gamma} \rangle_{\Lambda(\rho,\sigma)} = \tanh \hat{F}^{(-1)}_{\Lambda(\rho,\sigma)}$.

Hence, if (6) is valid, we have (8) with $k \sim 2^{d-2}$. Compactness of gauge group is required because only for compact gauge group one can prove that $\lim_{\rho\to\infty} \rho^{-(d-2)} \hat{F}_{\Lambda(\rho,\sigma)}$

have finite limit. Under this hypothesis, we prove

Lemma : Under the hypothesis (8), if there exists an allowed set of ρ, σ such that

$$< \hat{\gamma} >_{\Lambda(\rho,\sigma)} \quad < \quad k^{-1} \tag{9}$$

then,

$$< \hat{\gamma} >_{\Lambda(\lambda\rho,\lambda\sigma)} \quad \leq \text{const. } \exp - \alpha (\lambda\sigma)^2$$

for sufficiently large λ , with

$$\alpha \geq \quad - \sigma^{-2} \ln k < \hat{\gamma} >_{\Lambda(\rho,\sigma)} \quad . \tag{10}$$

Proof : Combining the inequality (7) and our hypothesis (8) we have

$$< \hat{\gamma} >_{\Lambda(2\rho,2\sigma)} \leq k \left(\quad < \hat{\gamma} >_{\Lambda(\rho,\sigma)} \right)^4 \quad .$$

Repeating this inequality n times (see Fig. 3), we have for large n,

$$< \hat{\gamma} >_{\Lambda(2^n\rho, 2^n\sigma)} \leq k \left[k < \hat{\gamma} >_{\Lambda(\rho,\sigma)} \right]^{4^n} \quad .$$

By setting $\lambda = 2^n$, one arrives at our claim.

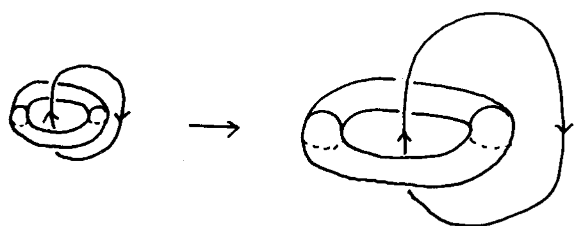

Fig. 3. Uniform dilatation of the vortex and the Wilson loop.

This lemma means that under the assumption (6), the vortex free energy vanishes exponentially with λ^2 provided that the vortex free energy is sufficiently small so that (9) is satisfied for some ρ and σ. Hence, (9) is sufficient for confinement. Clearly, this condition is always satisfied when the bare coupling constant is sufficiently large. Furthermore, even if the bare coupling constant is arbitrarily small, it is possible that (9) is satisfied at sufficiently large scale because of the renormalization effect. We also note that the area-law decay is a rather universal property of gauge systems in the sense that if $f(\sigma)$ is known to decay faster than $\sigma^{-(d-2)}$, it automatically decays exponentially with the cross section. The Mack-Petkova inequality [2] then implies that the tension given by (10) is a lower bound for the exact string tension.

In theories which are not scale invariant, $\hat{F}_{N(\lambda\rho,\lambda\sigma)}$ would either vanishes or diverges in the limit $\lambda \to \infty$. (We can show [1] that $f(\sigma)$, if it exists, is monotonically decreasing function of σ .) On the other hand, at the scale of cutoff, the inverse free energy (which is dimensionless) is proportional to the bare coupling constant. Hence, at general scale λ , the inverse vortex free energy can serve as an effective coupling constant. For the effective coupling constant in the U(N) model, we can indeed prove the following inequality [1] as suggested from the perturbation theory,

$$g^2_{eff}(\lambda) \geqslant \lambda^{4-d} g^2 \quad . \tag{11}$$

In asymptotically free theories, or more generally, in theories which are infrared unstable at the origin of the coupling constant, the effective coupling constant should increase at large distances. ((11) shows that this happens at least if $d < 4$ in the U(N) model.) Thus the only possibility seems to be that the maximum vortex free energy vanishes as $\lambda \to \infty$. In summary, our conclusion is that the pure lattice gauge theories which are infrared unstable at the origin of the coupling constant are always in confinig phase with linear confinig potential, provided that (i) Lagrangian is local; (ii)the center of gauge group is nontrivial. The crucial assumption in our argument was that the proportionality of the vortex free energy to its length is valid uniformly for any ρ, σ if r is sufficiently small.

Clearly, this conclusion is almost equivalent to the old infrared slavery conjecture. An exact proof, if any, of confinement is now reduced to establish this fundamental assumption and the infrared instablity in nonperturbative fashion. Finally, it should be remarked that our discussion is not affected by the presence of matter fields if the matter fields are in the adjoint representation of the gauge group.

References

1 T. Yoneya, Nucl. Phys. B205 [FS](1982)130.

2 G. Mack and V. B. Petkova, Ann. Phys. 125 (1980)117.

3 See G. Mack, "Properties of lattice gauge theory models at low temperature", in Progress in Gauge Theories, eds. G. 'tHooft et al (Plenum Press, New York, 1980).

4 G. 'tHooft, Nucl. Phys. B153 (1979)141.