ABSENCE OF PARTICLE CREATION AS AN EQUILIBRIUM CONDITION*

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1. INTRODUCTION

From the observed range of validity of the Newtonian approximation, the cosmological constant Λ is known to be very small. In units with $\hbar = c = 1$, one has $\Lambda < 10^{-56}\ \mathrm{cm}^{-2}$. In view of the fact that vacuum energy can contribute to the value of Λ,[1] especially in conjunction with symmetry breaking,[2] it is puzzling that the value of Λ is so small. Here I wish to recall and extend an old argument[3-5] which implies that the cosmological constant is zero or very small. The same hypothesis evidently implies also that the expansion of the universe is isotropic and that the early universe is dominated by relativistic particles and radiation.

The basic hypothesis is that the absence of particle creation, in particular, gravitons and minimally coupled massless scalar particles, is a kind of equilibrium condition toward which the evolution of the universe tends, and that the classical Einstein equations must be consistent with that condition. Here we are referring to particle creation as detected by a measuring instrument on one of the preferred geodesics of the expanding universe. In Ref. 5 (Section F), we argued that there must be a deep connection between the Einstein equations and the conditions for zero particle creation. We viewed the Einstein equations as macroscopic equations governing the large scale evolution of the universe. We asserted that the underlying (unknown) microscopic theory, from which the Einstein equations follow, must be such that "in an expansion of the universe in which a particular type of particle (i.e., relativistic or non-relativistic) is predominant, the expansion achieved after a long time will be such as to minimize the average creation rate of that particle" and "the reaction of the particle creation back on the gravitational field will modify the expansion in such a way as to reduce the creation rate." Investigations of particle creation in anisotropic[6-8] as well as isotropic[9] expansions support that hypothesis, but do not address the question of deriving the Einstein equations from an underlying microscopic theory. Candidates for such an underlying unified theory include induced gravity. Adler[10] has noted that the above hypothesis can serve within the context of that theory as a reason for the observed smallness of the cosmological constant.

Our hypothesis may well be a dynamical or statistical consequence of an underlying unified theory, perhaps resulting from a feedback mechanism involving particle creation, or reflecting an underlying equilibrium or consistency condition. Perhaps it is a stability condition on the true vacuum. In any event, we refer to the hypothesis, loosely speaking, as an equilibrium condition. Without being specific about an underlying theory, one can not address the question of the time in the early universe at which the suppression of particle creation is imposed. Possibilities are the Planck time or the times at which the grand unified theory phase transitions occur. It is not even necessary that the usual concepts of space-time or metric should be meaningful "before" the above equilibrium sets in. The underlying microscopic theory is assumed to be such that, at the time and scale when the classical Einstein equations are reasonably well defined, they are consistent with an expansion law in which the creation rate of gravitons and minimally coupled scalar particles is suppressed. In order to find the consequences of that hypothesis, we can regard the background metric and background matter as governed by the classical Einstein equations. Gravitons are taken to be the quantized gravitational wave perturbations of the background metric.

2. GRAVITATIONAL WAVE PERTURBATIONS

It is well known that in an anisotropically expanding universe, particle creation is in general more intense than in an isotropically expanding universe, and that the reaction back of the created particles is such as to bring about isotropic expansion.[6-8] There is no known case in which particle creation vanishes in an anisotropically expanding universe with nonvanishing Riemann tensor. The existence of such a case is all the less likely because we are demanding that the expansion be such that the graviton creation rate is zero at all times. (That excludes global interference effects resulting in no net graviton creation.) Therefore, we turn our attention to isotropic expansions.

The Robertson-Walker universes have line elements

$$ds^2 = -dt^2 + a^2(t)d\sigma^2, \tag{1}$$

where $d\sigma^2 = \tilde{g}_{ij}dx^i dx^j$ is the line element for a space of constant curvature. Define a quantity ε such that $\varepsilon = 1$, -1, or 0 if the spatial curvature is positive, negative, or zero, respectively.

The Einstein equations with cosmological constant Λ are

$$R_{\mu\nu} - \Lambda g_{\mu\nu} = 8\pi G(T_{\mu\nu} - \frac{1}{2} g_{\mu\nu}T) , \tag{2}$$

where $T = T_\lambda{}^\lambda$. The above equation refers to the background metric and background matter, both regarded as classical. The background energy-momentum tensor is that of a perfect fluid

$$T_{\mu\nu} = (\rho+p)u_\mu u_\nu + pg_{\mu\nu} , \tag{3}$$

with four velocity u^μ. The equations governing gravitational wave perturbations on a Robertson-Walker background have been worked out by Lifshitz.[11] Let $\delta\rho = \delta p = \delta u^\mu = 0$ and let the symmetric metric perturbation $\delta g_{\mu\nu} = h_{\mu\nu}$ satisfy the conditions

$$u^\mu h_{\mu\nu} = 0 , \qquad h^{\mu\nu}{}_{;\nu} = 0 , \tag{4}$$

which are consistent in a Robertson-Walker space-time and also imply that

$$h_\mu{}^\mu = 0 . \tag{5}$$

(Indices are raised and lowered with the unperturbed metric and covariant differentiation is with respect to that metric.) Two components of $h_{\mu\nu}$ remain independent and correspond to the two polarizations of a gravitational wave.

In the co-moving coordinate system of Eq. (1), only the spatial components $h_i{}^j$ do not vanish. Perturbation of the above Einstein equations yields

$$a^{-3}\partial_t(a^3\partial_t h_i{}^j) - a^{-2}\tilde{g}^{\ell m}\tilde{\nabla}_\ell\tilde{\nabla}_m h_i{}^j + 2\varepsilon a^{-2}h_i{}^j = 0 , \tag{6}$$

where ∂_t denotes $\partial/\partial t$, $\tilde{\nabla}_\ell$ denotes covariant differentiation with respect to the spatial metric $\tilde{g}_{ij}$, and ε takes values 1, -1, or 0 corresponding to the sign of the spatial curvature. The perturbation equation is independent of the cosmological constant. Let $h_i{}^j = G_i{}^j(\vec{x})\psi(t)$ where the $G_i{}^j$ are tensor spherical harmonics (see Ref. 11) for $\varepsilon = \pm 1$ or plane waves for $\varepsilon = 0$. They satisfy

$$\tilde{g}^{\ell m}\tilde{\nabla}_\ell\tilde{\nabla}_m G_i{}^j = -k^2 G_i{}^j , \quad \tilde{\nabla}_j G_i{}^j = 0 , \quad \text{and} \quad G_i{}^i = 0 , \tag{7}$$

with eigenvalues k^2 given by

$$k^2 = \begin{cases} |\vec{k}|^2, & \varepsilon = 0, \quad 0 < |\vec{k}| < \infty \\ n^2-3, & \varepsilon = 1, \quad n = 3, 4, \ldots \\ q^2+3, & \varepsilon = -1, \quad 0 < q < \infty . \end{cases} \tag{8}$$

The function $\psi(t)$ satisfies the equation

$$a^{-1}d(a^3 d\psi/dt)/dt + (k^2+2\varepsilon)\psi = 0 . \tag{9}$$

This is essentially the same equation as is obeyed by the time-dependent part of a minimally coupled scalar field, so that the methods developed in Refs. 3-5 to study the production of scalar particles are applicable to graviton production. The equation is not conformally invariant. The production of gravitons in Robertson-Walker universes was studied for the case $\varepsilon = 0$ by Grishchuk[12] and for all three cases by Ford and Parker.[13]

3. CONDITIONS FOR ZERO PARTICLE PRODUCTION

In this section, we search for criteria that can be used to infer that the creation rate is zero for gravitons and minimally coupled massless scalar particles. As noted earlier, there evidently are no anisotropically expanding universes in which the particle creation rate vanishes. Even conformally invariant wave equations give rise to particle creation when the expansion is anisotropic. Therefore, the first condition that must hold is that the expansion be isotropic. In seeking other criteria, we can limit our considerations to isotropic expansions.

As a preliminary step, it is helpful to recall the conformally coupled scalar field. It is well known that in a Robertson-Walker universe an unaccelerated observer having no event horizon will find that such particles are not created.[3-5,14] That permits one to unambiguously identify positive frequency solutions of the wave equation. For the conformal scalar field one has

$$-\nabla^{\mu}\nabla_{\mu}\phi + (R/6)\phi = 0 \ , \tag{10}$$

where R is the scalar curvature. In the Robertson-Walker metric of Eq. (1), one can write $\phi = G(\vec{x})\psi(t)$, where

$$\tilde{g}^{\ell m}\tilde{\nabla}_{\ell}\tilde{\nabla}_{m}G = -K^2 G \tag{11}$$

and

$$K^2 = \begin{cases} |\vec{k}|^2, & \varepsilon = 0 \ , \quad 0 < |\vec{k}| < \infty \\ n^2-1, & \varepsilon = 1 \ , \quad n = 1,2, \ldots \\ q^2+1, & \varepsilon = -1, \quad 0 < q < \infty \ . \end{cases} \tag{12}$$

There the time-dependent part of the field satisfies

$$a^{-3}d(a^3 d\psi/dt)/dt + [a^{-2}K^2 + (R/6)]\psi = 0 \ . \tag{13}$$

Using the expression for the scalar curvature,

$$R = 6(a^{-2}\dot{a}^2 + a^{-1}\ddot{a} + \varepsilon a^{-2}) \tag{14}$$

one finds that a solution of Eq. (13) is

$$\psi_K(t) = a^{-1}(t)\exp[-i\int^t dt' a^{-1}(t')(K^2+\varepsilon)^{1/2}] \ . \tag{15}$$

As no particle creation is occurring and this solution is clearly positive frequency for sufficiently large K, we will identify it as a purely positive frequency solution. An instrument measuring the particle creation rate couples to the field ϕ and is not directly influenced by the coefficient of R appearing in the wave equation (10). Therefore, for a massless scalar field with a different coupling to the scalar curvature, such as the minimally coupled field, we assume that the creation rate is zero for particles of that field if and only if a purely positive frequency solution of the form of Eq. (15) exists. The same condition was used by us (for $\varepsilon = 0$) in Ref. 3-5.

We proceed in the same way for gravitons. If one adds a term $(R/6)h_i{}^j$ to Eq. (6), the equation satisfied by the time-dependent part of $h_i{}^j$ is

$$a^{-3}d(a^3 d\psi/dt)/dt + [a^{-2}k^2 + 2a^{-2}\varepsilon + (R/6)]\psi = 0 \ . \tag{16}$$

This has the purely positive frequency solution

$$\psi_k(t) = a^{-1}(t)\exp[-i\int^t dt'a^{-1}(t')(k^2+3\varepsilon)^{1/2}] \ . \tag{17}$$

An instrument measuring the graviton creation rate will not be directly sensitive to the coefficient of R in Eq. (16). Therefore, for the actual graviton equation, (9), we assume that the creation rate is zero if and only if a purely positive frequency solution of the form of Eq. (17) exists.

The special cases in which the metric of Eq. (1) describes a flat space-time are given by $\varepsilon = 0$, $a(t) =$ constant and $\varepsilon = -1$, $a(t) = t$. In those cases, the above criteria give zero particle creation. Another much studied example is the linearly expanding spatially flat universe ($\varepsilon = 0$, $a(t) = t$).[15,16,17] The above criteria imply that there is particle creation in that case, in agreement with the other methods. Finally, in the spatially curved static universes ($\varepsilon = \pm 1$, $a(t) =$ constant), our criteria yield different results depending on the coupling to the curvature. There would be no creation of conformally coupled massless scalar particles, but there would be creation of minimally coupled scalar particles and gravitons. It is certainly conceivable that spatial curvature can create particles in a globally static space-time. On the other hand, it is possible that the above criteria for zero particle creation rate may have to be generalized to include those static universes. However, until it becomes clear that such a generalization is needed and exactly how it should be done, we will assume that Eqs. (15) and (17) are the necessary and sufficient criteria for zero particle creation rate of massless scalar particles and gravitons in Robertson-Walker universes.

4. IMPLICATIONS OF ZERO PARTICLE CREATION

Suppose now that the classical background metric and background matter obey the Einstein equations given in Eq. (2). The requirement that gravitons are not created places constraints on the form of the Einstein equations and on the equation of state of the matter. (We will deal with gravitons, but it should be understood that minimally coupled massless scalar particles would give the same results.)

The absence of graviton production requires that Eq. (9) for $\psi(t)$ has a solution of the form in Eq. (17). Substitution of (17) into (9) yields the condition that

$$a^{-2}\dot{a}^2 + a^{-1}\ddot{a} + a^{-2}\varepsilon = 0 \ , \tag{18}$$

or

$$R = 0 \ . \tag{19}$$

With the background energy-momentum tensor given by Eq. (3), contraction of the Einstein equations yields

$$R - 4\Lambda + 8\pi G(3p-\rho) = 0 \ , \tag{20}$$

or with Eq. (19),

the behavior of a(t) is essentially arbitrary.

The energy-momentum tensor

$$T^{(\phi)}_{\mu\nu} = \partial_\mu\phi\partial_\nu\phi - \frac{1}{2} g_{\mu\nu}\partial_\alpha\phi\partial^\alpha\phi + \xi[g_{\mu\nu}\nabla_\alpha\nabla^\alpha(\phi^2) - \nabla_\mu\nabla_\nu(\phi^2) + G_{\mu\nu}\phi^2] \tag{28}$$

satisfies the local conservation law $\nabla_\mu T^{(\phi)\mu\nu} = 0$. Furthermore, in the "in" and "out" regions, the factor multiplying ξ is a four-divergence. For $\xi = 1/6$, $T^{(\phi)}_{\mu\nu}$ is the "improved" energy-momentum tensor of Callan, Coleman, and Jackiw.[18] Let us take $\xi = 1/6$.

In a Robertson-Walker metric of Eq. (1), it can be shown that[14]

$$\rho^{(\phi)}(t_2)a^4(t_2) - \rho^{(\phi)}(t_1)a^4(t_1) = g(t_2) - g(t_1) , \tag{29}$$

where

$$g(t) = 6(4\pi^2)^{-2}[B(\dot{a}^4 + 2\varepsilon\dot{a}^2) + C(-a^2\dot{a}\dddot{a} - a\,\dot{a}^2\ddot{a} + \frac{1}{2} a^2\ddot{a}^2 + \frac{3}{2}\dot{a}^4 + \varepsilon\dot{a}^2)] \tag{30}$$

and $\rho^{(\phi)} = \langle T^{(\phi)00}\rangle$, the expectation value being taken in a state having the symmetries of the space-time. In Eq. (30), B = -1/360 and C = 1/180. Let t_1 be in the "in" region and t_2 be in the "out" region. Then clearly $g(t_1) = g(t_2)$, and

$$\rho^{(\phi)}(t_2)a^4(t_2) = \rho^{(\phi)}(t_1)a^4(t_1) . \tag{31}$$

The energy density of the particles present initially is merely red-shifted, with no real particles being created by the expansion. As the behavior of a(t) in the interpolating regions is arbitrary, one concludes that the creation rate of minimally coupled massless particles must be zero during the radiation dominated stage of the expansion. Although interpolating regions and "in" and "out" regions were used to derive that result, it must be valid when there are no such regions, in agreement with our earlier method.

6. CONCLUSIONS

We have shown that if one requires as an equilibrium condition that the creation rate for gravitons or minimally coupled massless scalar particles vanishes in the early universe, then the expansion of the universe must be isotropic, the cosmological constant must be zero, and the early universe must be dominated by relativistic particles and radiation. Such an equilibrium does not necessarily imply that gravitons will be absent, since they may have been created prior to equilibrium.

The linearized equation for gravitational wave perturbations was used. In higher order, the presence of self interactions among gravitons may give a small graviton creation rate when R = 0, but one would nevertheless expect the graviton creation rate to be near its minimum in the radiation dominated universe. In the case of a self interacting scalar field which has been studied,[19,20] it is interesting that the explicit term shown in Ref. 19 to cause particle creation vanishes when R = 0.

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