# VACUUM ENERGY IN THE BAG MODEL

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## Abstract

The vacuum energy of the Yang-Mills field is examined for the conditions of the bag model. The dominance of high frequency effects results in a vacuum energy that decomposes naturally into a volume energy, a surface energy and higher shape energies. These quantities are identified with the parameters of the bag model. The imposition of confining boundary conditions for all frequencies is shown to be inconsistent since this would result in the bag constant and certain of the shape tensions being infinite. The manner in which the boundary conditions should be relaxed at high frequency is discussed. The most naive procedure for relaxing the boundary conditions, which is to apply confining conditions only on modes of frequency less than some cutoff frequency, results in a negative bag constant and surface tension and would render the vacuum unstable against the spontaneous breaking of Poincaré invariance. Consideration of the manner by which the interacting electromagnetic field avoids a similar instability suggests that a more realistic way to relax the boundary conditions on the bag surface is to endow the vacuum exterior to the bag with a frequency dependent dielectric constant and magnetic permeability.

## Introduction

The aim of this report is to examine the energy of the Yang-Mills vacuum under the conditions of the bag model and to show that a consideration of the shift in the vacuum energy of the field due to its confinement may yield important insight into the nature of the vacuum state for non-Abelian gauge theories.

The bag model [1] has achieved reasonable phenomenological success in the explanation of hadron spectroscopy. This model pictures the quantum chromodynamic vacuum as coexisting in two phases. One of these, the 'ordinary' vacuum, is impenetrable to colour while the other, corresponding to the interior of the hadron, is such that the gluon fields that are the carriers of colour are able to propagate freely. These two phases are taken to be separated by a sharp boundary on the interior of which the gluon fields satisfy confining boundary conditions

$$\underline{n} \cdot \underline{E}^a = 0 , \qquad \underline{n} \times \underline{B}^a = 0 \quad . \tag{1}$$

The model was first formulated by the MIT school who proposed that the two phases of the vacuum be taken to differ by an amount of energy $\mathcal{B}$ per unit volume. Good phenomenology results if the energy of the bag is taken to be given by

$$\mathcal{E} = \mathcal{B}V - \frac{Z}{R} \qquad (2)$$

with V the volume of the bag and R its radius. A variant of the model, due to the Budapest school, adds to the right hand side of (2) the effect of surface tension so that

$$\mathcal{E} = \mathcal{B}V + \mathcal{E}^S S - \frac{Z}{R} \qquad (3)$$

with S the surface area of the bag and $\mathcal{E}^S$ a surface tension. Fitting the data with the MIT expression (2) yields the values $Z \simeq 1.8$ and $\mathcal{B}^{\frac{1}{4}} \simeq 0.145$ GeV. The Budapest energy (3) achieves a similarly good fit with the same value of Z and a variety of pairs of values for $\mathcal{B}$ and $\mathcal{E}^S$.

The origin of the Z/R term is not well understood though a contribution to Z of approximately 0.75 is explained as a center of mass effect [2]. It has been suggested [3] that the remainder of this term represents the change in the vacuum energy due to the confinement of the field. The inspiration for this suggestion would seem to be the result obtained first by Boyer [4] and subsequently by several authors for the change in the energy of the electromagnetic vacuum due to the introduction of a perfectly conducting spherical shell. A recent calculation by Milton, deRaad and Schwinger [5] yields the value

$$\mathcal{E}_{MDS} = \frac{\hbar c}{2R}(0.09235) \quad . \qquad (4)$$

The primary aim of this report is to assert that:
(i) The change in the vacuum energy occasioned by the confining surface is radically different in its effect from the simple 1/R dependence that has been suggested on the basis of (4) and
(ii) that the vacuum energy resides near the boundary. This has the effect that, for a sharp boundary, the vacuum energy possesses a geometrical expansion and may be expressed in the form

$$\mathcal{E}_{vac} = \mathcal{E}^V V + \mathcal{E}^S S + \mathcal{E}^C \int dS(\kappa_1 + \kappa_2) + \mathcal{E}^C_I \int dS(\kappa_1 - \kappa_2)^2 + \\ + \mathcal{E}^C_{II} \int dS\kappa_1\kappa_2 + \cdots \qquad (5)$$

where $\kappa_1$ and $\kappa_2$ denote the principal curvatures of the surface and the coefficients $\mathcal{E}^S, \mathcal{E}^C, \cdots$ are shape tensions (the first of these is the surface tension) which are independent of the geometrical configuration of the surface. The fact that the vacuum energy decomposes naturally in this way suggests strongly that the coefficients in the geometrical expansion should be identified with the parameters of the bag model. On the basis of this identification we find

(iii) that it is inconsistent to apply confining boundary conditions for all frequencies since, as a result of high frequency effects, the bag constant and the surface tension turn out to be not only infinite but of the wrong sign and hence would be such as to render the vacuum unstable against the spontaneous breaking of Poincaré invariance.

Some comments are perhaps in order regarding the totally different appearances of expressions (4) and (5). In order to relate them we remark that for the case of pure electromagnetism, *i.e.* in the absence of interaction with say the Dirac field, the surface tension $\mathcal{E}^S$ vanishes while, for the case of thin shells, the first integral on the right hand side of (5) whose coefficient is the curvature tension $\mathcal{E}^C$ vanishes owing to a cancellation between the two sides of the shell. Furthermore the first of the integrals quadratic in the curvatures vanishes since for a sphere $\kappa_1 = \kappa_2$. This leaves us with the term whose coefficient is $\mathcal{E}^C_{II}$. For a sphere this term takes the value

$$\mathcal{E}^C_{II} \int dS\kappa_1\kappa_2 = 8\pi\ \mathcal{E}^C_{II} \qquad (6)$$

independent of the radius of the sphere (this integral is in fact a topological invariant and takes the same value for any surface topologically equivalent to a sphere). The remaining terms in (5) are the terms cubic in the curvatures which, again for the case of thin shells, cancel between the inside and the outside and a term which is cutoff independent, in the limit of large cutoff, and which corresponds to the energy (4) computed by M.D.S. The energy (6) however, although independent of the radius of the sphere, is not zero and in fact depends linearly on a cutoff. Thus the vacuum energy of a perfectly conducting spherical shell differs from the value (4), which seems to have been generally accepted, by a term which is independent of the radius of the shell. This possibility was known to Boyer who was scrupulous to point out that his calculation determines the derivative of the vacuum energy only up to an additive constant. Subsequent calculations have overlooked this term for a variety of technical reasons [6].

For the conditions appropriate to the bag model, however, it transpires that all the coefficients $\mathcal{E}^S$, $\mathcal{E}^C$, $\mathcal{E}^C_I$ and $\mathcal{E}^C_{II}$ are present. The coefficients $\mathcal{E}^C_I$ and $\mathcal{E}^C_{II}$ are present just as they are in the electromagnetic case and to lowest order in the Yang-Mills coupling g their values can be inferred by multiplying the corresponding electromagnetic quantities by eight. The curvature tension is present since the situation envisioned in the bag model is that of a cavity in an infinite medium rather than the vacuum in the presence of a thin shell hence there is no cancellation between the inside and outside. Perhaps the most striking difference between the Yang-Mills field and the electromagnetic field is the presence of a non-zero surface tension $\mathcal{E}^S$. The surface tension is brought about by the self interaction of the Yang-Mills field and since it depends cubically on a cutoff it has important effects. As for the volume energy, we expect on dimensional grounds that it should vary quartically with a cutoff. It is these last two terms those associated with $\mathcal{E}^S$ and $\mathcal{E}^V$ that we shall principally be concerned with here.

The following estimate [7] may be obtained for the surface tension of the Yang-Mills field due to its self interaction

$$\mathcal{E}^S_{YM} \simeq -\frac{1}{36\pi^4}\left(11 - \frac{2}{3}N_F\right) g^2\Lambda^3 \quad \text{(confining boundary conditions)} \tag{7}$$

where $N_F$ denotes the number of fermion species and $\Lambda$ a high frequency cutoff and in deriving this estimate we have worked to leading order in the coupling g and we have assumed that the fermion masses may be neglected in comparison with $\Lambda$.

A similar estimate, which proves useful for the purpose of comparison, may be derived for the electromagnetic field for the case of perfect conductor boundary conditions we have

$$\mathcal{E}^S_{EM} \simeq -\frac{e^2\Lambda^3}{216\pi^4} \quad \text{(perfect conductor boundary conditions)} \tag{8}$$

The negative surface tension (7) would seem to indicate an instability that would lead to bag fragmentation. Equally serious is the result of computing the bag constant $\mathcal{B}$ (if the confining conditions (1) are taken literally then this is just the volume energy $\mathcal{E}^V$) from the non-linear boundary condition

$$\mathcal{B} = -\frac{1}{4} F_{\mu\nu}{}^a F^{\mu\nu a} + i \frac{\partial}{\partial x} \bar{\psi}\psi \tag{9}$$

which dictates the response of the bag wall to the gluon pressure. We may estimate $\mathcal{B}$ by taking a vacuum expectation value of this equation for the case of a plane boundary. The contribution of the second term on the right hand side of this equation is small in comparison with that of the first term if the fermion masses are small compared with the cutoff, neglecting this term we find

$$\mathcal{B}_{YM} = -\frac{\Lambda^4}{\pi^2} \quad . \tag{10}$$

The fact that $\mathcal{B}$ and $\mathcal{E}^S$ turn out to be negative indicates that the vacuum is unstable against dissolving into foam. Clearly something is seriously amiss.

Let us pursue our *reductio ad absurdum* a little further since the manner in which the interacting electromagnetic field avoids a similar instability indicates, I believe, the resolution to the difficulty. Consider for definiteness the result of taking a large box on the surface of which we impose the confining conditions (1) and subdividing the box into smaller ones by creating new surfaces on which the field is also subject to the same boundary conditions. Since the surface tension (7) is negative this process is energetically favourable. Along with the new surfaces we will create edges and corners where these surfaces intersect. This turns out to be energetically favourable also.

It is significant that the interacting electromagnetic field subject to perfect conductor boundary conditions would be subject to a similar instability. Of course charges and currents are required to enforce perfect conductor boundary conditions but these are available to the interacting field which will create electron-positron pairs from the vacuum if it is energetically favourable to do so. The point is that if the field were to create particle-antiparticle pairs in an attempt to create a perfectly conducting surface which, if possible, would be energetically advantageous then the created particles would form not a perfect conductor but rather a medium akin to an electron gas the electromagnetic properties of which is described by a *dielectric constant*. This is an important point since, as a consequence of the analyticity properties enjoyed by the dielectric constant, the *surface tension due to a dielectric boundary is always positive* thereby restoring the stability of the vacuum against partitioning. The positivity of the surface tension for a dielectric boundary derives ultimately from the fact that when account is taken of the energy of the sources required to enforce the boundary conditions the sum of the field energy proper and the energy of the sources is always positive.

Dielectric boundary conditions also resolve the difficulty associated with the sign of the bag constant at least to zeroth order in the coupling. This can be seen either by appealing to the appropriate generalization of (9) *viz*

$$\frac{1}{2} [\langle \underline{E} \cdot \underline{D} - \underline{B} \cdot \underline{H} \rangle] + i \frac{\partial}{\partial x} \bar{\psi}\psi = \mathcal{B} - \mathcal{E}^S(\kappa_1 + \kappa_2) + \cdots , \quad (11)$$

where the square bracket denotes the discontinuity of the enclosed quantity across the interface, or by the following elementary argument.

The dispersion relation for modes outside the bag is $k^2 = \mu\varepsilon\omega^2$, but since we require $\mu\varepsilon = 1$ to preserve Lorentz invariance this relation is just $k^2 = \omega^2$ which is the same as the dispersion relation inside. It follows that the volume energies inside and outside are equal and hence that

$$\mathcal{B} = \mathcal{E}^V_{in} - \mathcal{E}^V_{out} = 0$$

at least to zeroth order in the coupling. Clearly the calculation of $\mathcal{B}$ should be pursued to higher order. It is an important point, however, that the coefficients $\mathcal{B}$ and $\mathcal{E}^S$ can be calculated either from the boundary condition (10) or directly from the energy density and that the results agree. This is not the case if confining boundary conditions are employed.

In conclusion: we have shown that confining boundary conditions cannot be applied for all frequencies since otherwise the bag constant and the surface tension would be infinite and we have suggested a way in which these boundary conditions might be relaxed at high frequencies by supposing that the external vacuum can be viewed as a dielectric medium. These boundary conditions suffer from the serious deficiency that in all probability they fail to confine. It is an interesting question whether it is possible to find boundary conditions that both confine and yield physically acceptable values for the bag constant and the surface tension.

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