Stochastic Quantization and Gribov Problem

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§1. Introduction

In my talk I would like to report a modest progress in the Gribov problem[(1)] in the framework of stochastic quantization method.

According to Paris: and Wu[(2)], the Langevin equation for a gauge field A_μ has a form:

$$\frac{\partial A_\mu}{\partial t} + \frac{\delta S_{cl}}{\delta A_\mu} = \eta_\mu \quad ,$$

$$S_{cl} = 1/4 \int d^4x \, \mathrm{tr}\,(F_{\mu\nu}^2) \; , \tag{1}$$

where η_μ is the white noise. The advantage of the stochastic quantization will be the unnecessity of the gauge fixing. So our natural question will be: Is the stochastic quantization method with no gauge fixing equivalent to the well-established quantized gauge theory with gauge fixing ? In order to answer such a problem it is natural to consider an intermediate step: stochastic quantization with gauge fixing. The Langevin equation (1) can be deformed by t -dependent gauge transformation $U(t)$ as

$$\frac{\partial A_\mu}{\partial t} + \frac{\delta S_{cl}}{\delta A_\mu} - D_\mu V = U\eta_\mu U^{-1},$$

$$V = -i\, \partial U/\partial t \cdot U^{-1} \quad . \tag{2}$$

As far as gauge invariant quantities are concerned, Eq. (2) gives the same results as Eq. (2) does.

Baulieu and Zwanziger[(3)] claimed to find V such that the probability density $P(t)$ approaches the Faddeev-Popov measure:

$$P = \int dc\, d\bar{c}\; e^{-S_{tot}} \; ,$$

where $S_{tot} = S_{cl}$ + gauge fixing and Faddeev-Popov ghost terms and

c and $\bar{c}$ are the ghost fields. That is, the Faddeev-Popov measure P is a static solution of the Fokker-Planck equation associated with the Langevin equation (2), if we choose the functional V as

$$V^a(x) = -P^{-1}\int dy \int dB\, dc\, d\bar{c}\; c^a(x)\, \frac{\delta}{\delta A^b_\nu(y)}\left[\frac{\delta K}{\delta A^b_\nu(y)}\, e^{-S_{tot}}\right], \qquad (3)$$

where K is such that

BRS transform of K = gauge fixing and Faddeev-Popov terms,

A popular choice of K will be

$$K = \mathrm{tr}\int dx\, \left\{ -\frac{\alpha}{2} B\bar{c} + F(A)\bar{c} \right\} \qquad (4)$$

with F being a gauge fixing function (e.g. $F = \partial^\mu A_\mu$)[4].

§2. Singular Langevin Equation and Gribov Problem

The attention should be paid to the factor P^{-1} in front of the expression (3). As Gribov pointed out Faddeev-Popov measure has zeros. Therefore the Langevin equation has singularities in the drift force. The Brownian motion of gauge field { $A(t)$ } may or may not cross the Gribov boundary $\partial\Omega$ where $P = 0$, depending on the sign and strength of the drift force. Let us consider the time development $P[A]$ itself regarding it as a functional of the stochastic variable $A(t)$.

Namely,

$$\frac{dP[A]}{dt} = \int dx\, \frac{\delta P[A]}{\delta A^a_\mu}\, \dot{A}^a_\mu$$

$$\doteq P^{-1}\int dx\, \frac{\delta P[A]}{\delta A^a_\mu(x)}\, g^a_\mu(x) + \int dx\, \frac{\delta P[A]}{\delta A^a_\mu(x)}\, \eta^a_\mu(x) \qquad \text{near } \partial\Omega,$$

where g_μ is a regular functional: $P D_\mu V$. Defining

$$\tilde{P} = P \Big/ \sqrt{\int dx\, (\delta P/\delta A)^2} \quad \text{and} \quad \tilde{\eta} = \eta \Big/ \sqrt{\int dx\, (\delta P/\delta A)^2},$$

we obtain

$$\frac{d\tilde{P}}{dt} = \frac{c}{\tilde{P}} + \tilde{\eta}(t), \qquad (5)$$

where

$$C = \int dx \frac{\delta P}{\delta A^a_\mu} P D^{ab}_\mu v^b \bigg/ \int dx \left(\frac{\delta P}{\delta A^a_\mu} \right)^2 . \tag{6}$$

Equation (5) actually gives a projection of the Brownian motion to the normal direction to the Gribov boundary. According to the theorem by Feller[(5)], the Brownian motion will never reach the boundary $\partial\Omega$ if the repulsive drift force is strong enough: $C \geq 1$. Otherwise it will go through.

In the case of "entrance" $C \geq 1$, the Langevin equation by Baulieu and Zwanziger gives the equilibrium distribution Gribov suggested; the path-integral should be limitted within the Gribov region,

$$\int_\Omega [dA]\, P[A] . \tag{7}$$

In the other cases ($|C| < 1$, $C < -1$), we must impose the boundary condition at $\partial\Omega$ to solve the Fokker-Planck equation, which implies the non-equivalence of (1) and (2). Unfortunately we have not yet succeeded in the evaluation of C for the covariant gauge. (We obtained a trivial result $C=0$ for the axial gauge.)

References

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