NEW GENERAL RELATIVITY

- TRANSLATION GAUGE THEORY -

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1. Introduction

In 1956 Utiyama proposed to introduce the gravitational field as the gauge field of the Lorentz group.[1] He introduced 24 fields by generalizing 6 constant parameters ω_{ij} $(= - \omega_{ji})$ for homogeneous Lorentz transformations to arbitrary functions $\omega_{ij}(x)$. Later Kibble considered all the 10 parameters of the inhomogeneous Lorentz group (Poincaré group), and laid the basis for Poincaré gauge theory with 40 independent field variables.[2]

Hayashi and Nakano proposed to extend translations only, leaving the six parameters ω_{ij} constant.[3] In this translation gauge theory, 16 field variables $c_k{}^\mu$ are introduced as the gauge field, by requiring that the action integral be invariant under the group of extended translations and global Lorentz transformations. This invariance group is the simplest one that includes the group of general coordinate transformations

The underlying space-time of the translation gauge theory is the Weitzenböck space-time with absolute parallelism. The notion of absolute parallelism was first introduced into physics by Einstein, trying to unify gravitation and electromagnetism.[4] His attempt failed because there was no Schwarzschild solution in his field equation.[5] A purely gravitational theory based on the Weitzenböck space-time was revived by Møller,[6] and its Lagrangian formulation was given by Pellegrini and Plebanski.[7]

The theory of gravitation based on the Weitzenböck space-time was extensively studied by Hayashi and Shirafuji,[8] and it was given the name, new general relativity, since Einstein in 1928, after inventing general relativity, considered absolute parallelism for the first time, and since its main consequences were analogous to those of general relativity so far as macroscopic phenomena were concerned.

2. Fundamental particles and translation gauge group

We start from the action integral in special relativity for the fundamental particles of spin 1/2,

$$S_M = \int d^4x \; L_M(q, \partial_k q) \; , \tag{2.1}$$

which is invariant under Lorentz transformations,

$$\delta x^k = c^k + \omega^k{}_j x^j \; , \qquad (\omega_{kj} = - \omega_{jk}) \; , \tag{2.2a}$$

$$\delta q = (i/2)\omega_{ij} S^{ij} q \; , \qquad \delta(\partial_k q) = (i/2)\omega_{ij} S^{ij}(\partial_k q) + \omega_k{}^j(\partial_j q) \; , \tag{2.2b}$$

where S^{ij} are the Lorentz generators, and c^k and ω_{ij} are independent, constant 10 parameters. Here q collectively denotes quarks and leptons, and the Minkowski metric η_{ij} is given by diag(-1,+1,+1,+1).

We now extend translations to extended translations (namely, to general coordinate transformations) for which the parameters c^k are arbitrary functions of space-time points, and demand that the action integral should be invariant under general coordinate transformations and under global Lorentz transformations,

$$\delta x^\mu = \xi^\mu(x) \; , \qquad \delta q = (i/2)\omega_{ij} S^{ij} q \; , \tag{2.3}$$

where $\xi^\mu(x)$ are arbitrary four functions and ω_{ij} are constant 6 parameters as before. Since we are now treating general coordinate transformations, we use Greek letters for coordinate indices and distinguish them from Lorentz indices denoted by Latin letters.

To meet with the invariance requirement, we must define those quantities $D_k q$ which change under (2.3) in the same manner as $\partial_k q$ of (2.2b).[9] We define $D_k q$ by

$$D_k q = (\delta^\mu_k + c_k{}^\mu)\partial_\mu q \; , \tag{2.4}$$

then we get the following transformation rule for $c_k{}^\mu$;

$$\delta c_k{}^\mu = \partial_\nu \xi^\mu c_k{}^\nu + \omega_k{}^j c_j{}^\mu + \delta_k{}^\nu \partial_\nu \xi^\mu + \delta_j{}^\mu \omega_k{}^j \; . \tag{2.5}$$

The field $c_k{}^\mu$ is the gauge field associated with the translation gauge group. It transforms inhomogeneously under extended translations. The special relativistic limit is obtained by putting $c_k{}^\mu = 0$: When $c_k{}^\mu = 0$, Greek indices are equivalent to Latin ones, and the transformations (2.3) compatible with (2.5) are restricted by

$$\partial_j \xi^k + \omega_j{}^k = 0 \; , \tag{2.6}$$

from which we get (2.2a).

The transformation law of the translation gauge field given by (2.5) is rather complicated. The field $b_k{}^\mu$ defined by

$$b_k{}^\mu = \delta_k{}^\mu + c_k{}^\mu \tag{2.7}$$

obeys much simpler transformation law,

$$\delta b_k^{\ \mu} = \partial_\nu \xi^\mu b_k^{\ \nu} + \omega_k^{\ j} b_j^{\ \mu} . \tag{2.8}$$

Also, we define $b^k_{\ \mu}$ by

$$b^k_{\ \mu} b_k^{\ \nu} = \delta_\mu^{\ \nu} , \qquad b^k_{\ \mu} b_j^{\ \mu} = \delta_j^{\ k} . \tag{2.9}$$

The invariant action integral is now given by

$$S_M = \int b\, d^4x\, L_M(q, D_k q) \tag{2.10}$$

with

$$b = \det(b^k_{\ \mu}) , \tag{2.11}$$

because b changes like $\delta b = - b\partial_\mu \xi^\mu$, and bd^4x gives invariant volume element.

3. Gravitational field equation

We shall construct a gravitational Lagrangian density in vacuum

$$S_G = \int b\, d^4x\, L_G \tag{3.1}$$

by the following basic postulates: (1) Invariance under the group of extended translations and global Lorentz transformations, (2) L_G be quadratic in the translation gauge field strength,

$$T_{ijk} = b_j^{\ \mu} b_k^{\ \nu} (\partial_\nu b_{i\mu} - \partial_\mu b_{i\nu}) , \tag{3.2}$$

and (3) L_G be invariant under the parity operation. The most general gravitational Lagrangian density L_G can then be represented as

$$L_G = \alpha(t_{ijk} t^{ijk}) + \beta(v_i v^i) + \gamma(a_i a^i) \tag{3.3}$$

with α, β and γ three unknown parameters with dimension of $(\text{mass})^2$, where t_{ijk}, v_i and a_i are the three irreducible parts of T_{ijk};

$$t_{ijk} = (1/2)(T_{ijk} + T_{jik}) + (1/6)(\eta_{ki} v_j + \eta_{kj} v_i) - (1/3)\eta_{ij} v_k , \tag{3.4a}$$

$$v_i = T^k_{\ ki} , \tag{3.4b}$$

$$a_i = (1/6)\varepsilon_{ijkm} T^{jkm} . \tag{3.4c}$$

Taking the variation of the total action integral,

$$S = S_G + S_M , \tag{3.5}$$

with respect to $b^k_{\ \mu}$, we get the following gravitational field equation,

$$2D^k \tilde{F}_{ijk} + 2v^k \tilde{F}_{ijk} + 2\tilde{H}_{ij} - \eta_{ij} L_G = T_{ij} , \tag{3.6}$$

where

$$\tilde{F}_{ijk} = \alpha(t_{ijk} - t_{ikj}) + \beta(\eta_{ij}v_k - \eta_{ik}v_j) - (\gamma/3)\epsilon_{ijkm}a^m \ , \tag{3.7a}$$

$$\tilde{H}_{ij} = T_{mni}\tilde{F}^{mn}{}_j - (1/2)T_{jmn}\tilde{F}_i{}^{mn} \ . \tag{3.7b}$$

Here T_{ij} is the energy-momentum tensor defined by

$$T_{ij} = (1/b)b_{j\nu}[\delta(bL_M/\delta b^i{}_\nu] \ , \tag{3.8}$$

which reduces to the canonical energy-momentum tensor

$$T_{ij} = - [\partial L_M/\partial(\partial^j q)]\partial_i q + \eta_{ij}L_M \tag{3.9}$$

in the special relativistic limit, $c_k{}^\mu = 0$.

As a simple example, let us consider a spherically symmetric case where the field $b_k{}^\mu$ takes a diagonal form,

$$\begin{aligned} &b_{(0)}{}^0 = A(r,t) \ , \qquad && b_{(0)}{}^\alpha = 0 = b_{(a)}{}^0 \ , \\ &b_{(a)}{}^\alpha = B(r,t)\delta_a{}^\alpha \ , && (a,\ \alpha = 1,\ 2,\ 3) \end{aligned} \tag{3.10}$$

with A and B two unknown functions of t and $r = (x^\alpha x^\alpha)^{1/2}$, where we have enclosed Latin indices by parentheses. In this case the gravitational field equation <u>in vacuum</u> (namely, $T_{ij} = 0$) can be solved exactly: The functions A and B should be <u>time-independent</u>, and are given by

$$\begin{aligned} A(r) &= (1 - \frac{GM}{pr})^{-p/2}(1 + \frac{GM}{qr})^{q/2} \ , \\ B(r) &= (1 - \frac{GM}{pr})^{(p-2)/2}(1 + \frac{GM}{qr})^{-(q+2)/2} \ , \end{aligned} \tag{3.11}$$

where p and q are defined by

$$p = \frac{2}{1-5\epsilon}\{[(1-\epsilon)(1-4\epsilon)]^{1/2} - 2\epsilon\} \ , \qquad q = \frac{2}{1-5\epsilon}\{[(1-\epsilon)(1-4\epsilon)]^{1/2} + 2\epsilon\} \tag{3.12}$$

with

$$\epsilon = (\alpha+\beta)/(\alpha+4\beta) \ . \tag{3.13}$$

Here GM is an integration constant: G is Newton's gravitational constant and M can be interpreted as the gravitational mass of the central gravitating body. We notice that when the parameter ϵ is vanishing, this solution gives the Schwarzschild metric written in the isotropic coordinates with the metric tensor $g_{\mu\nu}$ defined from $b_k{}^\mu$ according to (4.1) below.

4. Geometry of space-time structure

The set of four vectors $\underline{b} = \{b_k{}^\mu\}$ given by (2.7) defines a global set of orthonormal frame with respect to the metric $\underline{g}$ with the metric tensor $g_{\mu\nu}$ given by

$$g_{\mu\nu} = b^k{}_\mu b_{k\nu} \ . \tag{4.1}$$

The spinor fields q of the fundamental particles of spin 1/2 are defined by referring to this orthonormal frame. The operator $D_k = b_k{}^\mu \partial_\mu$ of (2.4) is the covariant derivative with respect to the absolute parallelism which takes b as the parallel vector fields. This absolute parallelism defines the nonsymmetric affine connection Γ,

$$\Gamma^\lambda_{\mu\nu} = b_k{}^\lambda \partial_\nu b^k{}_\mu \ . \tag{4.2}$$

The torsion tensor of this connection is given by

$$T^\lambda{}_{\mu\nu} = \Gamma^\lambda_{\mu\nu} - \Gamma^\lambda_{\nu\mu} = b_k{}^\lambda(\partial_\nu b^k{}_\mu - \partial_\mu b^k{}_\nu) \ , \tag{4.3}$$

and coincides with the translation gauge field strength of (3.2).

Accordingly, the translation gauge theory can be interpreted as a gravitational theory based on the Weitzenböck space-time with absolute parallelism. The translation gauge field defines the parallel vector fields by (2.7), and the translation gauge field strength represents the torsion tensor. In the Weitzenböck space-time, the curvature tensor defined by the connection (4.2) is identically vanishing, and the torsion tensor describes the non-Minkowskian structure of space-time. This situation can be contrasted with that in the Riemann space-time characterized by the curvature tensor alone.

The Riemann-Cartan space-time has the curvature tensor and the torsion tensor, and it is the underlying space-time of Poincaré gauge theory. From this space-time follow two interesting space-time models: One is the Riemann space-time with the curvature tensor alone and the other is the Weitzenböck space-time with the torsion tensor alone. General relativity is a gravitational theory based on the Riemann space-time, while the translation gauge theory is based on the Weitzenböck space-time. Both theories can indeed be formulated as special limiting cases of Poincaré gauge theory.

The translation and the Lorentz gauge field strengths of Poincaré gauge theory represent the torsion and the curvature of the underlying Riemann-Cartan space-time, respectively. We assume that the gravitational Lagrangian density be linear and quadratic in the field strengths. The curvature tensor has one linear invariant and 6 quadratic invariants, while the torsion tensor has 3 quadratic invariants. The most general gravitational Lagrangian density of Poincaré gauge theory is then given by[10)]

$$L_G = a(\text{lenear invariant}) + (\alpha,\beta,\gamma)(\text{3 quadratic invariants of the torsion tensor}) + (a_1,\ldots,a_6)(\text{6 quadratic invariants of the curvature tensor}),$$

where the four parameters, a, α, β, and γ, are of dimension $(\text{mass})^2$, and the remaining six parameters, $a_1,\ldots,a_6$, are dimensionless. It has been shown that Poincaré gauge theory reduces to general relativity and to new general relativity (namely, to the translation gauge theory) in the limits, (i) $\alpha\to\infty$, $\beta\to\infty$, $\gamma\to\infty$, and (ii) $a_i\to\infty$, respectively.[11] In the first limit (i), the torsion tensor is vanishing, and the above Lagrangian density reduces to the following quadratic Lagrangian density in general relativity,

$$L_G = aR(\{\}) - bR_{\mu\nu}(\{\})R^{\mu\nu}(\{\}) + cR(\{\})^2 \tag{4.4}$$

with two dimensionless constants b and c related linearly to a_i's. In the second limit (ii), the curvature is vanishing, and the Lagrangian density reduces to (3.3).

5. Comparison with experiments

(A) The equivalence principle

The world line of the fundamental particles of spin 1/2 and of light rays propagating in vacuum is the geodesics of the metric $\underline{g}$ of (4.1), as can be shown by taking the short wave-length limit of the Dirac equation and the Maxwell equation. A macroscopic system such as a planet or a test particle can be described to a good approximation by the macroscopic energy-momentum tensor, which is obtained from the microscopic energy-momentum tensor by averaging in space and in time. When the spin direction of constituent particles is randomly distributed, the antisymmetric part of the microscopic energy-momentum tensor cancels out in the averaging process because of the Tetrode formula,

$$T^{[ij]} = (1/2b)\partial_\nu(bS^{ij\nu}) \tag{5.1}$$

with $S^{ij\nu}$ the spin tensor. It can then be shown from the gravitational field equation (3.6) that the macroscopic, symmetric energy-momentum tensor obeys the conservation law,

$$\nabla_\nu T^{\mu\nu} = 0 \ , \tag{5.2}$$

where ∇_ν is the covariant derivative with respect to the Christoffel symbol. The world line of macroscopic bodies such as planets and test bodies is then the geodesics of the metric $\underline{g}$.

Thus, as far as the effects due to intrinsic spin are negligibly small, the equivalence principle is valid in new general relativity. Violation of the equivalence principle occurs only in the microscopic world: For example, the precession of spin in the torsion field.

(B) Comparison with solar-system experiments

We demand that the gravitational field equation reproduces the correct Newtonian limit: This gives the following condition of the parameters,

$$\alpha\kappa + 4(\beta\kappa) + 9(\alpha\kappa)(\beta\kappa) = 0 \ , \tag{5.3}$$

where κ is Einstein's gravitational constant, $\kappa = 8\pi G$. Notice that $\alpha\kappa$ and $\beta\kappa$ are dimensionless, and by virtue of (3.13) and (5.3) they are expressed by

$$\alpha\kappa = -1/3(1-\varepsilon) , \qquad \beta\kappa = 1/3(1-4\varepsilon) . \tag{5.4}$$

From the exact solution (3.11) we see that Eddington-Robertson's post-Newtonian parameters defined by

$$ds^2 = (1 - 2(\frac{GM}{r}) + 2c(\frac{GM}{r})^2 + \ldots)dt^2 - (1 + 2d(\frac{GM}{r}) + \ldots)dx^\alpha dx^\alpha \tag{5.5}$$

are given by

$$c = 1 - \varepsilon/2 , \qquad d = 1 - 2\varepsilon . \tag{5.6}$$

(We notice that Eddington-Robertson's parameters c and d defined above are usually denoted by β and γ, respectively.)

According to the solar-system experiments, the parameter ε should be severely restricted;

$$\varepsilon = -0.004 \pm 0.004 , \tag{5.7}$$

and therefore, we have for $\alpha\kappa$ and $\beta\kappa$ the following experimental values,

$$\alpha\kappa = -1/3 + (0.001 \pm 0.001) , \qquad \beta\kappa = 1/3 + (-0.005 \pm 0.005) . \tag{5.8}$$

6. Model with $\alpha+\beta=0$

As we have seen in the previous section, the parameter ε is severely restricted by solar-system experiments. In view of this, we shall now assume that the parameter ε is exactly vanishing. The parameters α and β are then given by

$$\alpha = -1/3\kappa , \qquad \beta = 1/3\kappa . \tag{6.1}$$

The gravitational Lagrangian density L_G of (3.3) becomes

$$\begin{aligned} L_G &= -(1/3\kappa)(t_{ijk}t^{ijk} - v_i v^i) + \gamma(a_i a^i) \\ &= (1/2\kappa)R(\{\}) + (9/4\lambda)(a_i a^i) + (\text{a total derivative}) , \end{aligned} \tag{6.2}$$

where $R(\{\})$ is the Riemann-Christoffel scalar curvature defined by the metric tensor of (4.1), and $\lambda = 9\kappa/(4\gamma\kappa - 3)$.

The gravitational field equation of the present case is still complicated compared with the Einstein equation of general relativity, and very little is known about its solutions. We briefly mention the results for spherically symmetric and axially symmetric solutions in vacuum.

(1) Spherically symmetric solution in vacuum:

Irrespectively of whether the source is time-independent or not, the axial-vector part of the torsion tensor a_i should be vanishing, and $b_k{}^\mu$ obeys the Einstein equation in vacuum. According to the Birkhoff theorem, the metric should then be given by the Schwarzschild solution. Therefore, the parallel vector fields $b_k{}^\mu$ are obtained from the special solution (3.10-11) with $\varepsilon = 0$ by a local Lorentz transformation which preserves the condition $a_i = 0$.

(2) Stationary, axially symmetric solutions in vacuum:

Fukui and Hayashi derived a class of axially symmetric solutions in vacuum starting from axially symmetric solutions of the Einstein equation in vacuum (such as the Kerr solution or the Tomimatsu-Sato solutions).[12] In their solutions in vacuum, the axial-vector part of the torsion tensor, a_i, is non-vanishing.

The Lagrangian density of (6.2) is invariant under a restricted class of local Lorentz transformations, namely under those local Lorentz transformations which leave the axial-vector part of the torsion tensor a^μ invariant. The Dirac equation is also invariant under these local Lorentz transformations. The gravitational field equation derived from (6.2), however, is not covariant under such local Lorentz transformations. This fact casts some doubts on the internal consistentcy of the present model with $\alpha+\beta = 0$.[13)] At present we have not definite answer to this problem.

In the weak field situations with $|c_k{}^\mu| \ll 1$, we can expand the field equation into power series of $c_k{}^\mu$ and can keep only lowest order terms. We need not distinguish Greek indices from Latin indices, and can decompose the translation gauge field into symmetric and antisymmetric parts,

$$c_{ij} = -(1/2)h_{ij} - A_{ij} \tag{6.3}$$

with $h_{ij} = h_{ji}$ and $A_{ij} = -A_{ji}$. Notice that the symmetric field h_{ij} is just the weak field correction to the metric tensor: $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ with $h_{\mu\nu} = \delta_\mu{}^i\delta_\nu{}^j h_{ij}$. In this approximation the gravitational field equation is decoupled into its symmetric and antisymmetric parts,

$$\Box h_{ij} - \partial^k(\partial_i h_{jk} + \partial_j h_{ik}) + \eta_{ij}\partial_m\partial_n h^{mn} - (\eta_{ij}\Box - \partial_i\partial_j)h^k{}_k = -2\kappa T_{(ij)} , \tag{6.4a}$$

$$\Box A_{ij} - \partial^k(\partial_i A_{jk} - \partial_j A_{ik}) = -\lambda T_{[ij]} . \tag{6.4b}$$

From (6.4a) and (6.4b) follow the conservation laws,

$$\partial_j T^{(ij)} = 0 , \qquad \partial_j T^{[ij]} = 0 . \tag{6.5}$$

The symmetric part (6.4a) is just the linearized Einstein equation, and describes the massless graviton field of spin 2. The antisymmetric part, on the other hand, represents a massless field of spin 0 interacting with the intrinsic spin of the fundamental particles. If the parameter λ is positive, the energy of this spinless field is positive-definite. So we shall assume λ is positive.

The symmetric field h_{ij} gives rise to the universal attraction (namely, gravitation) between the fundamental particles. The antisymmetric field A_{ij} induces a universal spin-spin interaction which can be described by the interaction Hamiltonian,[14]

$$\begin{aligned} H_{\text{spin-spin}} &= (\lambda/8\pi)\vec{S}_A\cdot[\vec{\nabla}\times(\vec{\nabla}\times\vec{S}_B)]\frac{1}{r} \\ &= (\lambda/8\pi)[(8\pi/3)(\vec{S}_A\cdot\vec{S}_B)\delta^3(\vec{x}) - r^{-3}\{(\vec{S}_A\cdot\vec{S}_B) - 3r^{-2}(\vec{S}_A\cdot\vec{x})(\vec{S}_B\cdot\vec{x})\}] \;, \end{aligned} \tag{6.6}$$

where $\vec{S}_A$ and $\vec{S}_B$ are the spin vectors of spin 1/2 particles A and B, respectively. This interaction is of the same form as that between two magnetic moments, and is expected to contribute to the hyperfine splitting of energy levels in atoms and muoniums. The theoretical values for the hyperfine splitting based on Q.E.D. are in very good agreement with the experimental values both for atoms and muoniums. So we get the following upper bound for the parameter λ,

$$\lambda/4\pi \lesssim 3\times10^{-4}\ (\text{GeV})^{-2} \;. \tag{6.7}$$

7. Conclusion

The translation gauge theory (or new general relativity) is a gravitational theory on the Weitzenböck space-time with absolute parallelism. The Weitzenböck space-time is a special case of the Riemann-Cartan space-time with a curvature and a torsion. Analogously to this, the translation gauge theory follows from Poincaré gauge theory in the limit, $a_i \to \infty$. Roughly speaking, the parameters a_i measure the inverse of the coupling strength of the Lorentz gauge field, so the limit $a_i\to\infty$ is the "zero coupling" limit of the Lorentz gauge field.

The translation gauge theory passes all the experimental tests so far carried out if the parameters α and β are finely tuned so that

$$|(\alpha+\beta)/(\alpha+4\beta) \equiv \varepsilon| \lesssim 0.004 \;. \tag{7.1}$$

This suggests us to assume

$$\alpha + \beta = 0 \;.$$

The theoretical basis for this choice has not yet been fully understood, however. We shall compare the main consequences of the present theory with those of general relativity in the following Table I.

Table I

	General relativity	New general relativity
Space-time	Riemann space-time	Weitzenböck space-time
Connection	Levi-Civita connection	Non-symmetric affine connection

(Table I continued)

Basic structure	Metric tensor	Parallel vector fields
Gravitation	Riemann-Christoffel curvature tensor	Torsion tensor
Transformation group	General coordinate transformation group (Local Lorentz group)	General coordinate transformation group Global Lorentz group
The Birkhoff theorem	Yes	Yes
Isotropic, gravitational field	The Schwarzschild solution	The Schwarzschild solution
Newtonian approximation	Yes	Yes
Weak field approximation	Symmetric field; massless and spin 2	Symmetric field; massless and spin 2 Antisymmetric field; massless and spin 0
Theory	Macroscopic	Microscopic
Equivalence principle	Yes	Yes, for macroscopic phenomena No, for microscopic phenomena

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