# LATTICE GAUGE THEORY - A PROGRESS REPORT*

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Abstract

Some recent calculations in lattice gauge theory are reviewed. These include estimates of the heavy quark potential, the hadron spectrum, and scales of chiral symmetry breaking.

## 1. Introduction

Lattice gauge theory[1] provides an important nonperturbative approach to quantum gauge field theories and it promises to continue serving this purpose very productively in the years to come. In particular with the advent of Monte Carlo calculations with fermions, a much wider range of problems can now be investigated.[2-9] The next couple of years should witness a wealth of new quantitative and qualitative results which will hopefully help unravel the intricate dynamics of non-abelian gauge theories.

In today's talk I would like to give a progress report on some of the calculations that are currently being carried out on the lattice. Since time is limited, I will unfortunately not be able to touch upon a large part of the research in pure gauge theories. I apologize to the many physicists who have made important contributions to the field for not mentioning their work. To list just some of the topics that I will have to omit, they include:

Estimates of glueball masses
Studies of the restoration of rotational invariance
Experiments with different lattice actions
Investigations of the role of monopoles, vortices, etc.

The above work has contributed to our understanding of lattice gauge theory and its continuum limit and the topics that I will be discussing are built on the knowledge

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accumulated through such efforts.

First, I would like to describe a recent calculation of the heavy quark potential in Ref. 10 by John Stack. This will give an example of how continuum physics is extracted from the lattice theory by working in the scaling region. I will then go on to discuss Monte Carlo calculations with fermions. A major objective there is to obtain a QCD prediction for the hadron spectrum. Many groups have embarked on this ambitious program and initial results are encouraging.[2-8] However, these calculations should still be regarded as being at a preliminary stage. I do not believe that there is sufficient understanding of the approximations involved in order to be able to estimate errors reliably. Undoubtedly, future work will be devoted to gaining control over and improving on the approximations.

Fermion Monte Carlo techniques are also being used to investigate more general properties of gauge fields coupled to fermions. The spectrum calculations mentioned above indicate that chiral symmetry is broken spontaneously in non-abelian gauge theories with fermions in the fundamental representation. If one wants to go beyond the strong interactions to gauge theories of the electro-weak plus strong interactions, one must understand chiral symmetry breaking (or the lack thereof) in a more general setting. What is the scale of chiral symmetry breaking relative to the confinement scale? Is chiral symmetry breaking sensitive to the center of the gauge group? Does one see evidence for "tumbling" ideas? At the end of this talk I would like to describe some attempts to answer these questions using lattice techniques.[9]

Before turning to the specific topics, let me remind you of the basic features of lattice gauge theory. In the lattice approach one replaces continuum space-time by a hypercubic discrete lattice. The gauge degrees of freedom are unitary matrices $U(x,x+\mu)$ that reside on the links between neighboring sites $x$ and $x+\mu$. The U-matrices can be viewed as the lattice analogues of the path ordered non-abelian phase.

$$P\cdot\exp\left[ig\int_x^{x+\mu} A_\mu d\ell\right] \to e^{iagA_\mu} \sim U(x,x+\mu) \quad . \tag{1.1}$$

The matter degrees of freedom live on lattice sites. One defines, for instance, four component spinors $\psi(x)$ and $\bar{\psi}(x)$ at each site $x$. The lattice action splits up into two parts

$$S = S_G + S_F \quad . \tag{1.2}$$

The pure gauge part $S_G$ is the Wilson action

$$S_G = \frac{1}{g^2}\sum_p \mathrm{tr}\left\{2 - \left[U_p + U_p^\dagger\right]\right\} \tag{1.3}$$

where $\sum_p$ is the sum over unoriented plaquettes and $U_p$ is the product of four U's on links bordering the plaquette. For the fermionic part $S_F$, one takes

$$S_F = m \sum_x \bar{\psi}(x)\psi(x) + \frac{1}{2} \sum_x \sum_\mu \bar{\psi}(x)\gamma_\mu \Big[U(x,x+\mu)\psi(x+\mu) - U^\dagger(x-\mu,x)\psi(x-\mu)\Big] + \frac{r}{2} \sum_x \sum_\mu \bar{\psi}(x)\Big[2\psi(x) - U(x,x+\mu)\psi(x+\mu) - U^\dagger(x-\mu,x)\psi(x-\mu)\Big] \quad (1.4)$$

Eq.(1.4) is often rewritten as:

$$S_F = \sum_x \bar{\psi}'(x)\psi'(x) + K \sum_x \sum_\mu \bar{\psi}'(x) \Big\{(\gamma_\mu - r)U(x,x+\mu)\psi'(x+\mu) - (\gamma_\mu + r)U^\dagger(x-\mu,x)\psi'(x-\mu)\Big\} \quad (1.5)$$

In going from (1.4) to (1.5) we have introduced rescaled spinors,

$$\psi'(x) \equiv \sqrt{\frac{1}{2K}}\, \psi(x) \quad (1.6)$$

and a new parameter,

$$K \equiv \frac{1}{2m + 8r} . \quad (1.7)$$

The connection with continuum physics is established by going to the scaling region, i.e., towards a continuous phase transition point of the lattice theory. In order to make contact with an asymptotically free continuum field theory, one is interested in the critical point at $g_{crit} = 0$. As one approaches $g \to 0$ the renormalization group tells us that physical dimensional quantities such as masses $M_i$ obey the following relationship:

$$M_i = \frac{C_i}{a}\left(b_o g^2\right)^{-b_1/2b_o^2} \exp\left[\frac{-1}{2b_o g^2}\right] \equiv C_i \Lambda_L . \quad (1.8)$$

In Eq.(1.8), $b_o, b_1$ are the coefficients of the two loop perturbative $\beta$-function and $C_i$ is a pure number. The identity symbol defines the quantity $\Lambda_L$. Ratios between $C_i$'s for different masses are calculable predictions of the theory. Physical distance-dependent quantities such as the interquark potential $V(R)$ obey

$$\xi \cdot V(R) = f(R/\xi) = \text{function only of } R/\xi \quad (1.9)$$

where,

$$\xi = \text{correlation length} = (\text{typical mass})^{-1} \tag{1.10}$$

Work on the pure gauge theory has verified that Eq.(1.8) is obeyed by quantities such as the string tension $\sigma$, the glueball mass $M_G$, and the deconfinement temperature $T_{dec}$. One now believes that the scaling region extends to relatively large values of $g^2$, namely

$$\begin{array}{lll} \beta \equiv 4/g^2 \gtrsim 2.2 & SU(2) \\ \beta \equiv 6/g^2 \gtrsim 5.5 & SU(3) \end{array} \quad . \tag{1.11}$$

## 2. The Heavy Quark Potential

An important quantity that can be calculated in the quarkless theory is the static potential between two external color sources. I would like to show you a recent result obtained in Ref. 10 for the gauge group SU(2) (the discrete 120 element icosahedral subgroup was actually used, but this should provide an excellent approximation to the full SU(2) group for the values of the coupling constant that were involved).[12,13] Using previous calculations of the string tension $\sigma$ to fix the scale,[11-13] the continuum heavy quark potential was obtained up to an overall additive constant (other calculations of V(R) have been reported in Ref. 14).

The static potential can be extracted from an evaluation of the Wilson loop around a rectangular contour $\Gamma$ of extension $R \times t$.

$$W(\Gamma) = \left\langle \mathrm{tr}\, P \exp\left[ig \oint_\Gamma A\, d\ell\right]\right\rangle \rightarrow \langle \mathrm{tr}(UU\ldots U)_\Gamma\rangle \quad . \tag{2.1}$$

For $t \gg R$ one has

$$W(\Gamma) \sim \exp\left[-t(V_o + V(R))\right] \quad . \tag{2.2}$$

$V_o$ represents the self-energy of the color sources. In Ref. 10, Wilson loops were evaluated for $R = a, 2a, 3a$ and $4a$ at several $\beta$-values, $2.2 \le \beta \le 3.1$. A correlation length $\xi$ was then introduced.

$$\xi \equiv .012\ \Lambda_L^{-1} = .012\ a \left[b_o g^2\right]^{b_1/2b_o^2} \exp\left[\frac{1}{2b_o g^2}\right] \quad . \tag{2.3}$$

The coefficient .012 was chosen so that $\xi$ is given by $\xi = 1/\sqrt{\sigma}$, if one uses the string tension $\sigma$ measured in Refs. 11, 12, and 13. Although $R/a$ takes on only four values by combining data at different $\beta$, one can work at many more values of

$X \equiv R/\xi$. As mentioned before, in the scaling region all the data points for $\xi \cdot V(R)$ versus $R/\xi$ should fall on a single curve. The result is shown in Fig. 1 and one sees that scaling holds quite well.

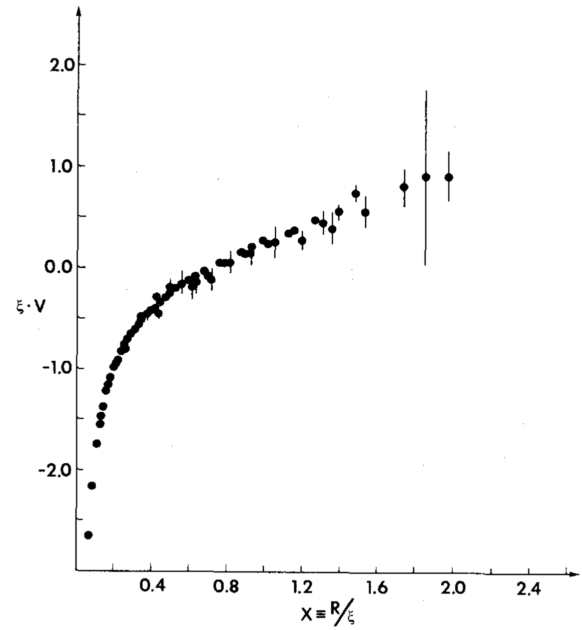


Fig. 1. $\xi \cdot V(R)$ versus $X \equiv R/\xi$. $\xi$ is the correlation length defined in Eq.(2.3).

In order to obtain Fig. 1, one had to subtract $V_o(\beta)$ for each $\beta$ separately. I refer to the original paper for details on how this was accomplished. Fig. 1 still contains a single overall additive constant that remains undetermined.

Although Fig. 1 represents the heavy quark potential for SU(2) and not for SU(3), it is still tempting to try to compare with phenomenological charmonium potentials. I have made a rough comparison in Fig. 2. $\xi^{-1}$ (or $\sqrt{\sigma}$) has been set to 400 MeV. This corresponds to renormalizing the theory such that the $R \gg 1$ behavior of $V(R)$ reproduces the experimentally measured Regge slope. Once the scale has been set one can express $R$ and $V(R)$ in physical units, fm and GeV respectively. The full curve in Fig. 2 is a phenomenological potential taken from Ref. 15 multiplied by 9/16, the ratio of the fundamental representation quadratic Casimirs of SU(2) and SU(3). This factor 9/16 will hopefully take into account the bulk of the changes necessary in going from SU(3) to SU(2). Keeping in mind that the Monte Carlo results can be shifted by an overall additive constant, one sees that the agreement between phenomenology and quarkless QCD is fairly good.

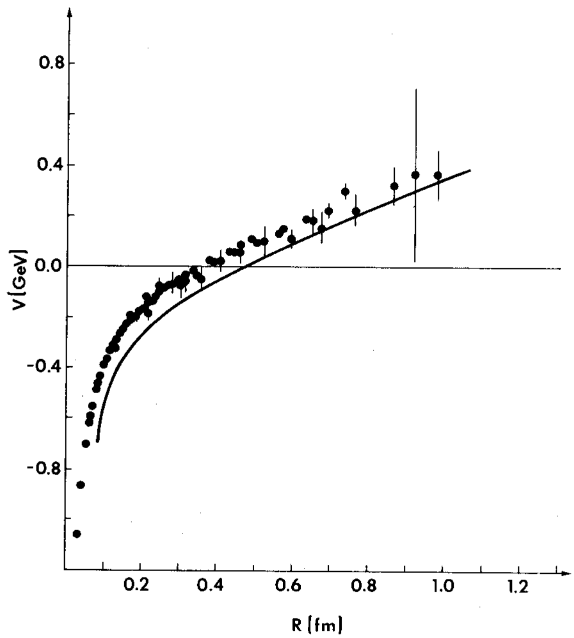


Fig. 2. Same as Fig. 1 with $\xi^{-1}$ set to 400 MeV. The full curve is a phenomenological potential from Ref. 15 converted approximately from SU(3) to SU(2).

## 3. Lattice Fermions and Spectrum Calculations

Recent advances in lattice gauge theory have led to the first round of Monte Carlo spectrum calculations for the full non-abelian gauge theory with quarks.[2-8]

One must now work with the full action $S_G + S_F$. In order to be able to perform Monte Carlo calculations, one formally integrates out the fermionic variables and ends up with an effective action,

$$S_{eff} = S_G - \mathrm{tr}\ \ell n(\Delta[U]) \quad . \tag{3.1}$$

$\Delta[U]$ is the lattice Euclidean Dirac operator defined in Eq.(1.4) or (1.5). In most of the 4D fermion Monte Carlo calculations to date (except for Refs. 6 and 7) one has ignored the second term in (3.1), namely one sets $\det(\Delta[U]) \to 1$. This amounts to neglecting virtual quark loops. In the loopless approximation (also called the quenched approximation) there is no feedback from the fermions on the gauge degrees of freedom. One can first evaluate the quark propagator $G(x,U)$ in a fixed background gauge field configuration. One then builds physical, gauge invariant quantities such as meson or baryon propagators and averages over $U$ with weight $dU\, e^{-S_G}$.

These calculations require in addition to a pure gauge Monte Carlo also efficient numerical matrix inversion methods to invert $\Delta[U]$ and obtain $G(x,U)$. Most calculations in 4D gauge theories have utilized the Gauss-Seidel method although other methods have also been proposed.[16,17] Hadron masses are extracted by writing the meson or baryon propagators $D(x)$ as

$$D(x)\Big|_{\substack{\text{averaged over}\\ \text{spatial positions}}} = \sum_n A_n e^{-m_n t} . \tag{3.2}$$

One then hopes that it is possible to go to sufficiently large $t$ so that only the lightest state contributes to (3.2). Periodic boundary conditions actually restrict the allowed values of $t$ to be $\leqslant N_t/2$, where $N_t$ is the number of lattice sites in the temporal direction.

As mentioned in the Introduction, the first estimates of the hadron spectrum are in reasonable agreement with experiment. Considering that these are first principles calculations of the spectrum of a very complicated theory, this is very encouraging news. On the other hand people are still struggling to understand and bring down the error bars. I will not attempt to show you table after table of hadron masses as they are quoted in the many references. Different groups have worked with different lattice sizes, at different values of $\beta$ and used different fermion methods. They have also averaged over different numbers of $U$ configurations and have different criteria for extrapolating to the physical values of $m$ or $K$ (it turns out one can never work exactly at $K_{phys}$ or $m_{phys}$ since the matrix inversion methods break down there). In order to be able to compare various references one must first understand the dependence of the final result on the many variables in the calculation. The next round of fermion Monte Carlo spectrum calculations must answer the following questions:

1. How do mass estimates change when one varies $N_s$ and/or $N_t$?
2. Has one averaged over a sufficient number of independent gauge field configurations?
2. Do hadron masses scale (i.e., obey Eq.(1.8))?
3. Do we understand how to extrapolate to physical values of $m$ or $K$?
4. Are there matrix inversion methods that enable us to work closer to $m_{phys}$ or $K_{phys}$?
5. How do virtual fermion loops affect the final results?

Most of the points raised above are technical ones. There are also more basic questions concerning lattice fermions that still need to be checked. Just as one must verify scaling and try to test the restoration of Lorentz (rotational) invariance in as many quantities as possible, a sensible continuum limit of theories with fermions must exhibit all continuous chiral symmetries for $m = 0$.

It is well known that there are many difficulties with lattice fermions and chiral symmetry. Let me discuss separately the cases $r = 0$ and $r \neq 0$.

For $r = 0$, $S_F$ of Eq.(1.4) becomes the "naive" lattice fermion action. It describes 16 species (flavors) of Dirac fermions. For finite lattice spacing, the $m = 0$ theory does not have the full $U(16) \times U(16)$ flavor symmetry of the continuum model. It has only a subset of the continuous axial and vector symmetries. The remaining continuous symmetries are supposed to be restored in the $a \to 0$ limit. However even at finite lattice spacing, there are enough discrete symmetries to prevent quark bilinear counter terms from developing. This is the main advantage of the $r = 0$ formulation.

One can actually reduce the number of flavors from 16 to 4 by spin diagonalizing the fermionic action. A convenient way to achieve this is to perform the following canonical transformation.[18]

$$\begin{aligned} \psi(x) &\to T(x)\ \psi(x) \equiv \chi(x) \\ \overline{\psi}(x) &\to \overline{\psi}(x)\ T^{\dagger}(x) \equiv \overline{\chi}(x) \end{aligned} \tag{3.3}$$

where

$$T(x) \equiv \gamma_0^{x_0} \gamma_1^{x_1} \gamma_2^{x_2} \gamma_3^{x_3} . \tag{3.4}$$

Then

$$\begin{aligned} S_F' = m \sum_x \overline{\chi}(x)\,\chi(x) + \frac{1}{2} \sum_x \sum_\mu \overline{\chi}(x)\,\eta_\mu(x)\,(U(x,x+\mu)\,\chi(x+\mu) \\ - U^{\dagger}(x-\mu)\,\chi(x-\mu)) \end{aligned} \tag{3.5}$$

where

$$\eta_0(x) = 1, \quad \eta_1(x) = (-1)^{x_0}, \quad \eta_2(x) = (-1)^{x_0+x_1}, \quad \eta_3(x) = (-1)^{x_0+x_1+x_2} . \tag{3.6}$$

Although the $\chi(x)$'s start out as four component spinors, Eq.(3.5) shows that the different components decouple and one can work with single component fermionic variables. This leads to the "staggered" fermion method.

The Euclidean staggered fermion action describes 4 flavors. At finite lattice spacing the $m = 0$ theory has only one continuous axial symmetry (nonsinglet) in addition to the $U(1)$ vector symmetry. The full $U(4) \times U(4)$ symmetry is restored only in the $a \to 0$ limit (up to anomalies). Again discrete symmetries prevent mass counter terms from developing.

The staggered fermion method has been used to evaluate $\langle\overline{\psi}\psi\rangle_{m=0}$ in the quenched approximation.[2,3,9] There were clear indications that $\langle\overline{\psi}\psi\rangle \neq 0$ both at strong coupling and into the scaling region. Since staggered fermions have one continuous symmetry that is spontaneously broken by $\langle\overline{\psi}\psi\rangle \neq 0$, one should find a Goldstone boson even at strong coupling. This particle will be one out of the 15 pions expected in the 4 flavor continuum theory. This massless state has been observed in Refs. 2, 3 and 7. The real challenge now is to show that the remaining 14 pions also become massless as one approaches the continuum limit. This will verify that the full $SU(4) \times SU(4) \times U(1)$ flavor symmetry is being restored, realized however in the Nambu-Goldstone mode. Finally at the same time that the 14 pions are becoming massless, one would like to observe that the flavor singlet meson, the $\eta'$, remains massive. In order to achieve this however one must put back quark annihilation graphs. Although both the naive and the staggered Euclidean fermions are believed to have the correct $U(1)$ anomaly in weak coupling perturbation theory,[19] it is not known how easily the anomaly will show up in a Monte Carlo calculation.

For $r \neq 0$, one obtains the Wilson fermion formulation which has been used in most spectrum calculations.[2,4,5,6] The moment $r$ is nonzero, 15 of the 16 species encountered in the naive fermion method acquire large masses as $a \to 0$. The Wilson formulation avoids the "doubling" problem. However the term in $S_F$ proportional to $r$ breaks chiral symmetry completely. There is no symmetry to prevent mass terms from developing. With this method one must fix the parameter $K$ of Eq.(1.7) by some prescription before being able to extract hadron masses. One has traditionally fixed $K$ so that the pion mass comes out right. In particular if one adjusts $K$ to $K_c$ such that the pion becomes massless, one argues that the theory has been finetuned to its chirally symmetric point. I believe it is important to devise as many independent tests as possible that one is indeed dealing with a chirally symmetric theory at $K = K_c$.

The Wilson method has also been shown to have the correct $U(1)$ anomaly.[20] There are in principle no obstacles in obtaining a correct pion-eta splitting once annihilation graphs have been taken into account. Some work in this direction has already been carried out in Ref. 2.

## 4. Scales of Chiral Symmetry Breaking

As the last topic, I would like to describe some further studies of chiral symmetry breaking using lattice techniques. My collaborators, J. Kogut, Steve Shenker, D. Sinclair, M. Stone, W. Wyld and I are investigating chiral properties of theories with quarks in different representations of the gauge group. We hope that such studies will shed new light on the mechanism responsible for chiral symmetry breaking.

According to the picture that we have of chiral symmetry breaking, the vacuum of QCD should be unstable with respect to the formation of quark-antiquark pair condensates. Once a condensate has formed chirality of the vacuum becomes indefinite and it is possible to have nonzero vacuum expectation values of operators such as $\bar{\psi}\psi$. This picture tells us that in order for chiral symmetry breaking to occur, at the very least pairs must bind. Attractive, maybe relatively strong, binding forces must exist. It is, however, not at all clear whether long range confining forces are necessary. In fact, initial models in 4D of chiral symmetry breaking were theories with very short range interactions.[21] If an effective four fermion interaction theory is ever a good guide to what goes on in QCD, maybe chiral symmetry breaking is insensitive to the long distance behavior of the theory.

The question raised above can be studied by considering theories with quarks in N-ality zero representations (e.g., the adjoint representation). Such quarks do not experience a confining force at large distances since they can be screened by gluons. Consequently if chiral symmetry breaking is sensitive to the force law at large distances one would expect the chiral properties of theories with adjoint fermions to be very different from in models with fundamental representation fermions. It could also be that adjoint quarks prefer to bind with glue degrees of freedom rather than form pairs.

My collaborators and I have performed Monte Carlo evaluations of $\langle\bar{\psi}\psi\rangle_{\ell}$ for several representations $\ell$ of SU(2). We have used the staggered fermion formulation (Eq.(3.5)) and work within the quenched approximation described in the previous section. We find that quarks in screenable representations also lead to chiral symmetry breaking. We conclude from this that chiral symmetry breaking is independent of confinement and it occurs in general, (depending on the representation) at shorter distances than confinement.

Having established that chiral symmetry breaking is not associated with confinement, one might ask the following questions: Is it possible to introduce disparate length scales into a theory with a single gauge group? Does one see evidence for "tumbling"?[22]

The authors of Ref. 22 have pointed out that in an asymptotically free theory one can obtain a large hierarchy in length scales by allowing different types of condensates to form sequentially. We have attempted to use fermion Monte Carlo calculations to estimate the relevant scales of chiral symmetry breaking for different representations. We find that indeed a hierarchy of scales emerges. Let me be a little bit more specific about what is meant by the "relevant scale of chiral symmetry breaking".

Consider a system at nonzero real temperature T. As T increases one expects all symmetries that are spontaneously broken at zero temperature to eventually be restored. This should also be true of chiral symmetry breaking (the simplest picture of such a phase transition would have the pairs in the condensate unbind as the system becomes too hot). Thus if one observes the order parameter $\langle\bar{\psi}\psi\rangle$ at different temperatures one should see,

$$\langle\bar{\psi}\psi\rangle \begin{cases} \neq 0 & T < T_c \\ = 0 & T > T_c \end{cases} \quad . \tag{4.1}$$

The critical temperature $T_c$ sets the relevant scale for chiral symmetry breaking. We find that if one compares two representations such that their quadratic Casimirs $C_2(\ell)$ obey

$$C_2(\ell_2) > C_2(\ell_1) \tag{4.2a}$$

then

$$T_c(\ell_2) > T_c(\ell_1) \tag{4.2b}$$

which is consistent with tumbling ideas.

It is also interesting to compare $T_c$ for the fundamental representation with the deconfining temperature $T_{dec}$ of the quarkless theory.[23-26] One finds

$$T_{dec} \leqslant T_c(\text{fundamental}) \ . \tag{4.3}$$

In Fig. 3 we compare our curve for $T_c$(fundamental), denoted $T_F$, with a curve for $T_{dec}$ which is taken from Ref. 23. Since temperature is a physical quantity with dimension of mass, it obeys the renormalization group relation Eq.(1.8) once one has entered the scaling region $(4/g^2 \geqslant 2.2)$.

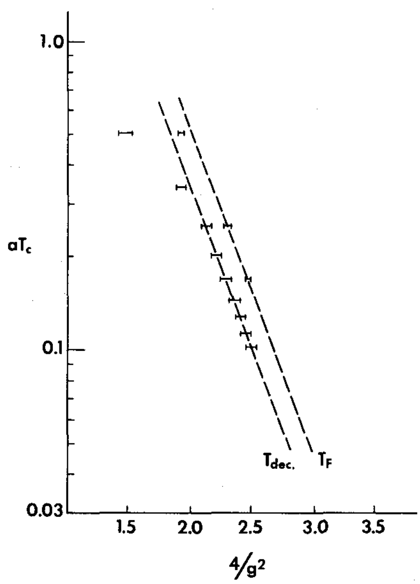


Fig. 3. Comparison of the chiral symmetry restoration temperature $T_F$ for fundamental quarks with the deconfining temperature $T_{dec}$. $T_{dec}$ is taken from Ref. 23.

From Fig. 3 one reads off the continuum result

$$T_F/T_{dec} \sim 1.6 \pm .2 \ . \tag{4.4}$$

(Other estimates of $T_{dec}$ [24-26] tend to place it slightly higher than in Ref. 23, so $T_{dec}$ and $T_F$ could be closer than indicated by Eq.(4.4).)

We also have data for the representation $\ell = 1$ (adjoint), $\ell = \frac{3}{2}$ and $\ell = 2$ and they are consistent with Eq.(4.2) with large ratios between the $T_c$'s. We are currently working very hard to nail down $T_c$ more precisely for the adjoint representation. We are finding that $T_A/T_F$ is at least 7 or larger. However, the error bars are still too big and uncertain to enable us to quote a reliable number. Needless to say, many of the reservations and difficulties with fermion Monte Carlo calculations that were mentioned in the previous section also apply here. We are trying to understand possible sources of systematic errors in our calculations. For instance previously when we had less data on different sized lattices, our analysis gave $T_A/T_F \sim 56$. We now believe that this number will be reduced when finite size effects are properly taken into account. In a forthcoming paper we will give a detailed discussion of finite size effects and other aspects of our calculations. We will also report on results for SU(3) and U(1) gauge theories.

## 5. Concluding Remarks

I have tried to give you a flavor of some current work on the lattice. Most of the projects are very time consuming (and often also very CPU time consuming) long term endeavors. We may still have some ways to go before we can be truly satisfied with the accuracy of the results. However it is gratifying that more and more questions are starting to lend themselves to the lattice analysis and that this nonperturbative method is producing a continuous flow of new results and ideas.

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