A GAUGE INVARIANT RESUMMATION OF QUANTUM GRAVITY

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Abstract

Quantum gravity is expanded in powers of 1/D, where D is the number of dimensions. The extra dimensions are highly compactified. The expansion is gauge invariant. The leading term is equivalent to the iterated one loop matter corrections due to a free, massless scalar field without the $\frac{1}{6} R\phi^2$ term necessary for conformal invariance. The 1/D expansion is renormalizable. Flat space is found to be unstable under small fluctuations.

Despite many valiant efforts, the question of whether or not pure quantum gravity is a consistent theory remains unresolved. The standard expansion in powers of the dimensionless parameter GE = (Newtons constant) x (typical energy) encounters nonrenormalizable high energy divergences. No definitive conclusions can be drawn from this, however, since GE is not small at high energies and we cannot expect an expansion in GE to give a good estimate of high energy corrections.

What is needed is an expansion parameter that is small at high energies. Such a parameter has been suggested by Tomboulis.[1] Tomboulis considers gravity coupled to N matter fields, rescales Newton's constant, and then expands in powers of 1/N. The resulting effective action is, to leading order in 1/N, simply the classical Einstein action with one loop quantum matter corrections. For conformally invariant matter fields and certain choices of renormalization constants, it also turns out to be asymptotically free, renormalizable, and unitary with a Lee-Wick prescription. There are, however, several drawbacks to this approach:

(1) Since no graviton loops are included, it is not clear that we are learning anything about quantized gravity. We may have just swamped out the quantum gravitational effects by dominating the theory with matter fields. If one were to use the same approximation scheme for QCD, for example, one would conclude that it was neither asymptotically free nor confining.

(2) Qualitative features of the expansion depend on the type of matter fields (i.e. scalar or fermion), how they are coupled, and how

it is renormalized. Most choices lead to various types of instabilities. In particular, without conformal invariance, various difficulties arise from the spin zero degrees of freedom. Since conformal invariance is not an observed symmetry of the real world, this somewhat obscures the physical relevance of the expansion.

In view of the above, it would be nice to find a way to resum gravity itself--with no extra matter fields. We could then analyze the internal consistency of quantum gravity and would be spared the ambiguity associated with different choices of matter couplings.

Such a resummation is in fact possible. Pure quantum gravity contains a hidden expansion parameter that is small at all energies. That parameter is 1/D, the inverse number of dimensions. The fundamental fields of gravity are arranged in a DXD matrix. Just as in Yang Mills, Feynman diagrams contain factors of D that arise from traces over this matrix. With an appropriate rescaling of Newton's constant, S matrix elements can be expanded in a series of non-negative powers of 1/D, and the leading term can be explicitly evaluated.

Before proceeding further, however, we must define how the theory is to be extended to D dimensions. There are two inequivalent methods.

The first method is to simply take the standard D dimensional Einstein action on a D dimensional manifold. This theory is invariant under the full D dimensional diffeomorphism group. Extraction of the leading term in the 1/D expansion requires analysis of the D dependence of both the trace factors and of the phase space factors in the D dimensional Feynman integrations. This analysis can be found in Reference [2] and will not be discussed further here.

In this talk we consider a different approach. The extra dimensions are compactified to very small circles. Excitations of the metric along those dimensions are necessarily very short wavelength and very high energy. If the circles are made small enough, such excitations are negligible. The effective theory then consists of a D dimensional matrix of fields on a four dimensional manifold.

This theory is equivalent, via the Kaluza-Klein mechanism, to gravity coupled to (D-4) massless U(1) fields and (D-4)(D-3)/2 scalar fields. For the purpose of analyzing the large D behavior, however, it is not convenient to reexpress the theory in terms of these fields.

The Feynman rules for this theory are determined from the standard Einstein action with gauge fixing and ghost terms:

$$S = -\frac{1}{2K^2}\int d^4x\ \det{}^{-1/2}[G]\Big\{+\frac{1}{2}g^{\alpha\beta}\Big[tr[G,_{\beta}G^{-1},_{\alpha}]$$
$$+ tr[G,_{\beta}G^{-1}]tr[G,_{\alpha}G^{-1}]\Big]$$
$$+ 3g^{\mu\nu},_{\nu}tr[G^{-1},_{\mu}G] + \frac{1}{2}g^{A\beta},_{\nu}g^{B\nu},_{\beta}g_{AB}$$
$$+ \frac{1}{2\alpha}F_AF^A + \bar{\varepsilon}_A M^{AB}\varepsilon_A\Big\} \qquad (1)$$

where

$$(G)_{AB} = g_{AB}$$
$$A,B = 1,2,\ldots D$$
$$\alpha,\beta,\mu,\nu = 1,2,3,4.$$

$\frac{1}{2\alpha}F_AF^A$ is a gauge fixing term and $\bar{\varepsilon}_A M^{AB}\varepsilon_B$ the corresponding ghost action. K is the gravitational coupling. Where the indices label derivatives, they only run from 1 to 4 since the arguments of the fields are four dimensional. This has been indicated by the use of Greek indices. The invariance group of this action is

$$x_A \to x_A + \varepsilon_A(x_\mu)$$

under which

$$g_{AB}(x_\mu) \to g_{AB}(x_\mu) + \varepsilon_A(x_\mu);_B + \varepsilon_B(x_\mu);_A. \qquad (2)$$

This contains four dimensional coordinate transformations and constant translations in D dimensions. The expansion presented here is in the inverse dimensionality of this latter invariance group.

Isolating the large D behavior in terms of $h^{\mu\nu} = g^{\mu\nu} - \eta^{\mu\nu}$ is awkward because of the double trace term in (1). It is not described by any simple set of diagrams, and in fact gets a contribution from every diagram.

This difficulty is circumvented by using an exponential parametrization:

$$g^{AB} = e^{2K\Omega/D}(e^{K\phi})^{AB}. \qquad (3)$$

ϕ is a traceless DxD matrix and Ω is a scalar. The factor of 1/D in front of Ω is necessary to ensure that Ω has a D independent propagator. The action may now be written:

$$S = \int d^4x\ e^{-K\Omega(1-2/D)}\Big\{\frac{1}{4}tr[\phi,_{\alpha}\phi,_{\beta}](e^{K\phi})^{\alpha\beta} + \Omega,_{\alpha}\Omega,_{\beta}(e^{K\phi})^{\alpha\beta}(-1+\frac{7}{D}+\frac{2}{D^2})$$
$$+ \frac{1}{K}(e^{K\phi})^{\alpha\beta},_{\alpha}\Omega,_{\beta}(6-\frac{1}{D}) - \frac{1}{2K^2}(\frac{1}{2}(e^{K\phi})^{A\beta},_{\mu}(e^{K\phi})^{\mu B},_{\beta}(e^{K\phi})_{AB}$$
$$+ \frac{1}{2\alpha}F_AF^A + \bar{\varepsilon}_A M^{AB}\varepsilon_B)\Big\}$$

The large D behavior can now be examined by rescaling the coupling:

$$K \to K/D \tag{5}$$

and counting powers of D in Feynman diagrams. Alternately, one may note that, because of the trace, the first term in (4) is order D while the subsequent terms are order one. The ghosts are the fermions (fundamental multiplet) of this theory and can't contribute to the large D limit. This has the pleasant consequence that <u>the large D limit is gauge invariant</u>.

The quantum contribution to the large D limit thus comes from fluctuations of the first, trace term of (4). Since

$$\frac{1}{4} e^{-K\Omega(1-2/D)} (e^{K\phi})^{\alpha\beta} \mathrm{tr}[\phi,_{\alpha}\phi,_{\beta}] = \frac{1}{4} \sqrt{-g}\, g^{\alpha\beta}\, \mathrm{tr}[\phi,_{\alpha}\phi,_{\beta}] \tag{6}$$

we arrive at the following conclusion: <u>The 1/D expansion of quantum gravity is equivalent to the 1/N expansion of gravity coupled to N free, massless scalars, where $N = D^2$.</u>

This result might seem obvious in view of the fact that the theory is equivalent, via the Kaluza-Klein mechanism, to gravity coupled to (D-4)(D-3)/2 scalars and (D-4) U(1) fields. As we have seen here, however, the 1/D expansion arranges the fields in such a way that it is not just the scalar fields that contribute to the large D limit. This analysis shows that contributions remain when D=4 and the "extra" scalar fields are not there. The expansion should remain valid at D=4, where it describes pure quantum gravity.

The leading 1/N corrections for quantum gravity coupled to N massless, free scalar fields has been discussed in various forms in the literature. The resummed propagator has a $1/p^4$ behavior at high energies which allows the theory to be renormalized with an R^2 type counterterm. Because there is no conformal invariance, however, difficulties arise from the spin zero modes. This can be seen by evaluating the energy of static, spatially varying perturbations of flat space. This energy is equal, by standard arguments, to minus the effective action and can be obtained from general formulae computed by Hartle and Horowitz.[3] For our case, it is given by:

$$E[h_{\alpha\beta}] = \frac{K^2}{4} \int \frac{d^3p}{(2\pi)^3} \left[\vec{p}^{\,2} |h^{TT}_{\alpha\beta}|^2 - \frac{45}{32}\left(\frac{768\pi^2}{K^2 \ln \vec{p}^{\,2}/\mu^2} - \vec{p}^{\,2} \right) |h^T|^2 \right] \left[1 + \frac{K^2 \vec{p}^{\,2} \ln \vec{p}^{\,2}/\mu^2}{1920\pi^2} \right]$$

where $h^T = P^{\alpha\beta} h_{\alpha\beta}$ is the trace part of $h_{\alpha\beta}$ and $h^{TT}_{\alpha\beta} = P_{\alpha}{}^{\gamma} h_{\gamma\delta} P^{\delta}{}_{\beta} - \frac{1}{2} h^T_{\alpha\beta}$ is the traceless part of $h_{\alpha\beta}$ in momentum space. It is readily seen that the energy can be decreased by fluctuations in h^T. This means that flat space is unstable and is not the ground state expectation value of the metric.

Several different conclusions may be inferred from this:

(1) The Einstein action is not a fundamental action but an effective action and should not be quantized. The sickness we found is a result of incorrect quantization.

(2) Quantum gravity needs matter fields for consistency (e.g. supergravity).

(3) Quantum gravity is a good theory. The instabilities are just telling us that we have the wrong ground state.

(4) Quantum gravity is a good theory but the 1/D expansion is bad at D=4.

A final conclusion awaits further analysis.

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References

1. E. Tomboulis, Phys. Lett. 70B (1977) 361, and Phys. Lett. 97B (1980) 77.
2. A. Strominger, Phys. Rev. D24 (1981) 3082.
3. J.B. Hartle and G. Horowitz, Phys. Rev. D24 (1981) 257.