

## SUPERSYMMETRIC GRAND UNIFICATION

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### Abstract:

After a brief discussion of motivations and prototype models, we review recent studies of model building and of proton decay in supersymmetric grand unification. A new effect is mentioned for  $\Delta B \neq 0$  four-scalar interactions induced by an intermediate scale ( $10^{10} \sim 10^{12}$  GeV) supersymmetry breaking.

### I. Gauge hierarchy and naturalness

Up to the highest accelerator energies electroweak interactions are now adequately described by the  $SU(2) \times U(1)$  gauge model. Color  $SU(3)$  for strong interactions seems to be supported by all the available data too. This  $SU(3) \times SU(2) \times U(1)$  gauge model has been beautifully unified into grand unified theories (GUT) such as  $SU(5)$  [1]. GUT has achieved several nice points:

- i) The unique gauge coupling provides a true unification of electromagnetic, weak, and strong interactions.
- ii) Quantization of charge. The equality of charges of proton and positron is a mysterious accident in the  $SU(3) \times SU(2) \times U(1)$  gauge model, but it is explained by symmetry reasons in grand unified theories.
- iii) A number of quantitative successes such as  $\sin^2 \theta_W(M_W)$  and  $m_b/m_\tau$ .
- iv) Possibility of proton decay and of explaining the origin of the baryon number in the universe.

On the other hand GUT has left several important problems still unanswered:

- i) There are vastly different mass scales of gauge symmetry breaking for  $SU(2) \times U(1)$  and GUT (gauge hierarchy), e.g.  $M_W^2/M_{\text{GUT}}^2 \approx 10^{-26}$  in  $SU(5)$ .
- ii) How to explain fermion masses and generations?
- iii) How to incorporate gravity?

and so forth. Among them we now have some hope for a natural solution

of the gauge hierarchy problem.

Some time ago 'tHooft has clarified the concept of naturalness: If a theory contains small parameters, it should acquire a larger symmetry for vanishing values of the parameters<sup>[2]</sup>. Namely these parameters are protected from getting large values by the approximate symmetry. In that view gauge hierarchy becomes natural, if there is a larger symmetry in the limit of vanishing mass-squared for Higgs scalar which is responsible for the  $SU(2) \times U(1)$  symmetry breaking. For particles with spin one-half (one), chiral (local gauge) symmetry protects their masslessness. Since no symmetry is known which directly guarantees masslessness for spinless particles, we are led to two alternatives:

- i) Theories without elementary scalar particles (technicolor models)<sup>[3]</sup>.
- ii) Supersymmetry (SUSY)<sup>[4]</sup>. Higgs scalar can be guaranteed to be massless if SUSY relates it to a spin 1/2 fermion which is massless because of chiral symmetry.

## II. Supersymmetric grand unified models

### 1. $SU(5)$ with explicit soft breaking of SUSY<sup>[5], [6]</sup>

i) The standard  $SU(5)$  GUT has been successfully made supersymmetric. One fine tuning at the tree level is needed for light Higgs doublet, but is not disturbed by radiative corrections (nonrenormalization theorem<sup>[7]</sup>).

ii) Since the naturalness is not spoiled by explicit soft SUSY breaking of order

$\Delta m < \text{TeV}$ , superpartners of quarks and leptons can be given small masses ( $< \text{TeV}$ ).

iii) Renormalization-group analysis including many new particles showed phenomenologically acceptable values for  $\sin^2 \theta_W(M_W)$  and  $m_D/m_T$ <sup>[8]</sup>, but grand unification scale  $M_{\text{GUT}}$  tends to be larger than the nonsupersymmetric model.

## 2. $SU(3) \times SU(2) \times U(1) \times \hat{U}(1)$ models

i) Experimentally scalar partners of charged leptons are found to be heavier than  $16 \text{ GeV}^{[9]}$ . On the other hand spontaneous breakdown of SUSY gives a mass sum rule at the tree level<sup>[10]</sup>

$$\Sigma_{\text{Boson}}^2 - \Sigma_{\text{Fermion}}^2 = \sum_{U(1)} g \langle D \rangle \quad (1)$$

where the right-hand side is a measure of SUSY breaking, so-called D-terms associated to broken  $U(1)$  subgroups. There are only two such  $U(1)$  generators in the  $SU(3) \times SU(2) \times U(1)$  model: weak hypercharge  $Y$  and the third component  $I_3$  of  $SU(2)$ . Unfortunately both  $Y$  and  $I_3$  vanishes by summing over quarks (leptons).

$$\Sigma_{\text{scalar quark}}^2 - \Sigma_{\text{quark}}^2 = 0 \quad (2)$$

Therefore we are forced to enlarge the gauge group in order to have a scalar partner heavier than quarks and leptons<sup>[6]</sup>. The simplest of such possibilities is the  $SU(3) \times SU(2) \times U(1) \times \hat{U}(1)$  models<sup>[11],[12]</sup>.

ii) Irrespective of details of grand unification, one may look for a low energy SUSY model with  $SU(3) \times SU(2) \times U(1) \times \hat{U}(1)$  gauge symmetry. The model must satisfy:

- a) Anomaly cancellation for renormalizability.
- b)  $U(1)$  and  $\hat{U}(1)$  be traceless for the absence of quadratic divergences (this might be unnecessary according to ref. 7).
- c) Spontaneous breaking of supersymmetry.

Much efforts have been devoted to build such a model<sup>[11]-[13]</sup>, but so far their models did not satisfy either one of the above three requirements. Recently we succeeded to construct a model with all three properties<sup>[14]</sup>. The result shows, however, a few annoying features too: a) Asymptotic nonfree ( $\beta$ -function for  $SU(3)$  is zero at one loop). b) Too many fields with the same quantum number. Therefore it appears difficult to embed the model into a simple group.

iii) A different viewpoint was proposed by a CERN group<sup>[15]</sup>. They take a mass scale in the Lagrangian ( $\hat{U}(1)$  D-term) to be of the order of the Planck mass  $M_{Pl}$ . Because of that they argued to disregard the nonrenormalizability due to anomalies. Their model contains a SUSY breaking mass scale  $\mu$  much larger than the electroweak mass scale  $M_W$ . However the effect almost decouples from our low energy world of quarks, leptons and Higgs doublet except effects of order  $\mu^2/M_{Pl}$ ,

which is identified as  $M_W$ .

### 3. Intermediate scale SUSY breaking

i) The mass sum rule (1) was the stumbling block of SUSY model building. Recently several people realized that radiative corrections violate the sum rule to give a desirable mass pattern at two loop order<sup>[16]</sup>. In this picture the electroweak mass scale arises as a radiative correction to the SUSY breaking at higher energy scale. In particular if particles affected by the SUSY breaking  $\mu$  at the tree level are themselves extremely heavy (of order  $M \sim M_{\text{GUT}}$ ), they approximately decouple from the low energy world. Therefore the effective SUSY breaking in quarks, leptons and Higgs doublet supermultiplets is of order  $\alpha\mu^2/M$ . Identifying  $\alpha\mu^2/M \sim M_W$  and  $M \sim M_{\text{GUT}}$  or  $M_{\text{Pl}}$ , one obtains the SUSY breaking mass scale  $\mu$  as  $10^{10} \sim 10^{12}$  GeV, intermediate between  $M$  and  $M_W$  (geometric hierarchy<sup>[17]</sup>).

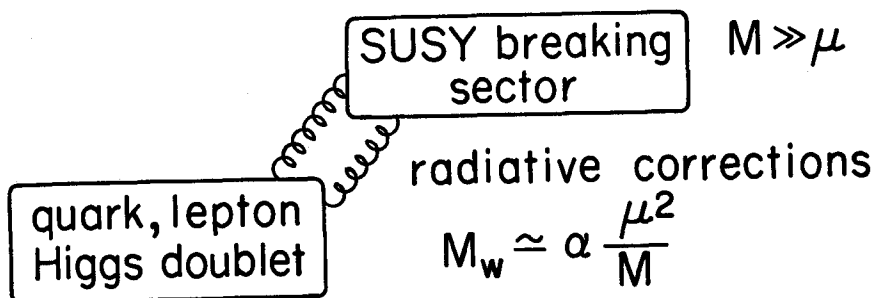


Fig. 1. Particles in the SUSY breaking sector are superheavy. Effective SUSY breaking in quark, lepton, and Higgs doublet is induced by radiative corrections.

ii) A realistic SU(5) model based on the above idea was constructed by Dine and Fischler<sup>[18]</sup> with  $M \sim M_{\text{GUT}}$ . Witten's mechanism<sup>[19]</sup> of generating large mass scale as a radiative correction was also implemented in a SU(5) model<sup>[17]</sup>, but was found to have several severe problems<sup>[20]</sup>. Polchinski and Susskind studied decoupling and worked out a systematic way to extract the effective low energy theory<sup>[21]</sup>. The resulting picture is a theory with explicit soft breakings of SUSY, which are derivable from and constrained by the underlying high energy theory.

iii) Most recently a very interesting type of models were proposed where an explicit SUSY breaking arises as an effect of embedding SUSY GUT models into supergravity. This will be discussed by R. Arnowitt in this conference.

### III. Proton decay in supersymmetric models

#### 1. Model independent analysis in SUSY models

i) Proton decay offers the most spectacular and important information on the grand unification. Possible baryon-number violating effective interactions are known to be constrained by low-energy symmetries such as  $SU(3) \times SU(2) \times U(1)$ <sup>[22]</sup>. Model-independent operator-analysis has also been done for SUSY and  $SU(3) \times SU(2) \times U(1)$  as the low-energy symmetry<sup>[23], [12]</sup>. One finds a dimension-four  $\Delta B \neq 0$  operator, but one can easily forbid it, for instance, by imposing a discrete symmetry, "matter parity", namely a sign change of quark and lepton superfields. On the other hand in SUSY models, dimension-five operators generally exist such as

$$\frac{1}{M} [q_- q_- q_- \ell_-]_F = \frac{1}{M} A_{q_-} A_{q_-} \psi_{q_-} \psi_{\ell_-} + \dots \quad (3)$$

where  $q_-$  and  $\ell_-$  are quark and lepton superfields and  $A_{q_-}$  and  $\psi_{q_-}$  are scalarquark and quark. The GUT mass scale is denoted by  $M$ . A typical term arises as an interaction between two scalars and two fermions (of quarks and a lepton) due to the baryon-number violating Higgs fermion exchange in SU(5) model for instance. If there are SUSY breaking Majorana masses for gauge fermions, the dominant contribution to proton decay comes from a loop diagram containing the dimension-five

operator<sup>[24]</sup>.

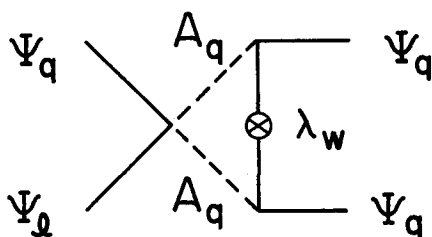


Fig. 2. A loop diagram with the dimension-five operator and the SUSY breaking Majorana mass for gauge fermion  $\lambda_w$ .

A detailed analysis gave the rate consistent with the present experimental bound and predicted the dominance of decay modes with neutrinos and higher generations<sup>[25]</sup>.

Since proton decay due to the dimension-five operators is rather close to the experimental bound, one may wish to construct models which forbid these operators. Discrete symmetries, R-invariance (a global symmetry)<sup>[23]</sup>, and local U(1) symmetry<sup>[12]</sup> were proposed for such a purpose. In this case SUSY restricts the dimension-six four-fermion operators to those of mixed chirality which can be experimentally verified from lepton-polarization measurement.

2.  $\Delta B \neq 0$  four-scalar interaction induced by intermediate scale SUSY breaking

In the case of intermediate scale SUSY breaking, there are addi-

tional important operators for proton decay. Using the method of ref. 21, we performed a systematic operator-analysis with a new superfield  $c_{\pm}$  responsible for the SUSY breaking [26]. More precisely the F-component of  $c_{\pm}$  develops a vacuum expectation value  $\mu^2$  intermediate between  $M \sim M_{\text{GUT}}$  and  $M_W$ , and hence  $c_{\pm}$  must be singlet of  $SU(3) \times SU(2) \times U(1)$ . For instance we obtain new operators of dimension six which give four-scalar interactions after SUSY breaking such as

$$\frac{1}{M^2} [q_{-} q_{-} \ell_{-} c_{-}]_F = \frac{\mu^2}{M^2} A_{q_{-}} A_{q_{-}} A_{q_{-}} A_{\ell_{-}} \quad (4)$$

where  $q_{-}$  ( $\ell_{-}$ ) and  $A_{q_{-}}$  ( $A_{\ell_{-}}$ ) denote left-handed quark (lepton) superfield and its scalar component. It contributes to proton decay through two-loop diagrams with Majorana masses of order  $M_W$  for gauge fermions.

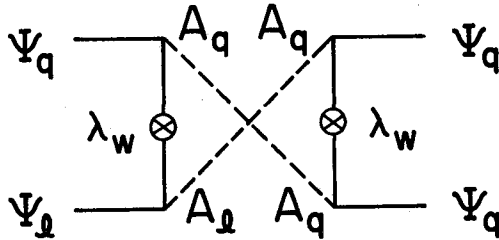


Fig. 3. A two-loop diagram with the SUSY breaking four-scalar interaction and the SUSY breaking Majorana mass for gauge fermion  $\lambda_W$ .

Since  $M_W \approx \mu^2/M$  in the geometric hierarchy picture, the contribution of the four-scalar interaction to proton decay is of the same order as the dimension-five SUSY operator (apart from powers of coup-

lings etc.):  $(\mu/M)^2 \cdot (1/M_W^2) \approx (1/M) \cdot (1/M_W) \approx 1/\mu^2$ . In certain models SUSY breaking Majorana masses for gauge fermions are negligible [27] and make the above four-scalar interactions and the dimension-five SUSY operators unimportant for proton decay. In that case more important operator is another type of four-scalar interactions with mixed chirality such as

$$\frac{1}{M^4} [q_- q_- u_+ e_+ c_+ c_-]_D = \left(\frac{\mu}{M}\right)^4 A_{q_-} A_{q_-} A_{u_+} A_{e_+} \quad (5)$$

where  $u_+$  ( $e_+$ ) and  $A_{u_+}$  ( $A_{e_+}$ ) are right-handed u-quark (electron) superfield and its scalar component. The operator can contribute to proton decay through two-loop diagrams without the Majorana masses for gauge fermions. In the geometric hierarchy picture its contribution to the proton decay is of the same order as the dimension six SUSY operator  $(\mu/M)^4 \cdot (1/M_W^4) \approx 1/M^2$ . Therefore the new  $\Delta B \neq 0$  four-scalar interactions can contribute to proton decay with comparable order of magnitude as the supersymmetric  $\Delta B \neq 0$  operators for both cases with or without significant Majorana mass for gauge fermions.

Existence of a type of four-scalar interactions in a model was also noted recently in ref. 28, but a systematic operator-analysis of proton decay in intermediate scale SUSY breaking is found in ref. 26.

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