ASPECTS OF GRAND UNIFIED MODELS WITH SOFTLY BROKEN SUPERSYMMETRY

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Supersymmetric theories have an outstanding property of no-renormalization for F-terms. This may give an important key to the solution of the gauge hierarchy problem in the grand unified theories (GUTs).

If nature chooses a supersymmetric theory, the supersymmetry must be broken at low energies spontaneously or explicitly. In spontaneously broken supersymmetric theories, however, there is a severe constraint on masses of component fields in a given supermultiplet, and in order to construct a realistic model, we must introduce disgusting complexity in the model[1)].

On the other hand in the explicit breaking scheme, it is possible to construct a realistic model with minimal set of supermultiplets[2)]. The minimal supersymmetric SU(3)×SU(2)×U(1) model contains the following supermultiplets:

					SU(3)	SU(2)	Y/2
vector multiplets							
	V_1 :	V_1^μ ,	λ_1 ,	D_1	($\underline{1}$,	$\underline{1}$,	0)
	V_2 :	V_2^μ ,	λ_2 ,	D_2	($\underline{1}$,	$\underline{3}$,	0)
	V_3 :	V_3^μ ,	λ_3 ,	D_3	($\underline{8}$,	$\underline{1}$,	0)
left-handed Higgs multiplets							
	H_1 :	A_1 ,	ψ_1 ,	F_1	($\underline{1}$,	$\underline{2}$,	-1/2)
	H_2 :	A_2 ,	ψ_2 ,	F_2	($\underline{1}$,	$\underline{2}$,	1/2)
left-handed matter multiplets							
	ℓ_r :	$A(\ell_r)$,	$\psi(\ell_r)$,	$F(\ell_r)$	($\underline{1}$,	$\underline{2}$,	-1/2)
	$\bar{e}_r$:	$A(\bar{e}_r)$,	$\psi(\bar{e}_r)$,	$F(\bar{e}_r)$	($\underline{1}$,	$\underline{1}$,	1)
	q_r :	$A(q_r)$,	$\psi(q_r)$,	$F(q_r)$	($\underline{3}$,	$\underline{2}$,	1/6)
	$\bar{u}_r$:	$A(\bar{u}_r)$,	$\psi(\bar{u}_r)$,	$F(\bar{u}_r)$	($\underline{3}^*$,	$\underline{1}$,	-2/3)
	$\bar{d}_r$:	$A(\bar{d}_r)$,	$\psi(\bar{d}_r)$,	$F(\bar{d}_r)$	($\underline{3}^*$,	$\underline{1}$,	1/3)

where r(=1,2,3) are generation indices.

In this scheme, gauge fermion masses (M_1, M_2 and M_3 for λ_1, λ_2 and λ_3 respectively) and all mass terms of the scalar components of matter multiplets ($m^2(\phi)$ for

$A(\phi)$ with $\phi = \ell_r, \bar{e}_r, q_r, \bar{u}_r, \bar{d}_r$) and those of Higgs multiplets (m_1^2 for A_1, m_2^2 for A_2 and m_3^2 for A_1-A_2 mixing terms) are freely adjustable parameters of the theory.

If the explicit breaking scheme is the case in nature, the low energy phenomena give considerable constraints on the values of these breaking parameters as we will see below.

All $m^2(\phi)$ for $\phi = \ell_r, \bar{e}_r, q_r, \bar{u}_r, \bar{d}_r$ must be large ($\sim O((10^2 \text{ GeV})^2)$) and positive. If any one of them (except ℓ_r) are negative, $U(1)^{EM}$ or color SU(3) conservation breaks down.

In order to obtain desirable symmetry breaking $SU(2)\times U(1) \to U(1)^{EM}$, Higgs scalars A_1 and A_2 must acquire non-vanishing vacuum expectation values through the minimization of the Higgs potential

$$V = \frac{g^2}{2}(A_1^\dagger \frac{\tau^a}{2} A_1 + A_2^\dagger \frac{\tau^a}{2} A_2)^2 + \frac{g'^2}{8}(A_1^\dagger A_1 - A_2^\dagger A_2)^2$$
$$+ m_1^2 A_1^\dagger A_1 + m_2^2 A_2^\dagger A_2 - m_3^2(A_1 A_2 + A_1^* A_2^*).$$

The existence of the Higgs vacuum requires the following conditions[3]:

$$m_1^2 + m_2^2 > 2|m_3^2|, \quad m_3^4 > m_1^2 m_2^2 ,$$

that is, m_3^2 must lie between the algebraic and geometrical average of m_1^2 and m_2^2. Since we expect that our SU(3)×SU(2)×U(1) model is embedded in some grand unified group G_{GUT}, we reject the possible existence of U(1)-D term in the lagrangian. Therefore if there are no soft-breaking effects, which implies $m_1^2 = m_2^2$ and $m_3^2 = 0$, SU(2)×U(1) does not break down.

The existence of the superpartners of leptons, quarks and gauge bosons induces the flavor changing neutral interactions such as $s + \bar{d} \to d + \bar{s}$ and $\mu \to e\gamma$. In order to suppress such effects, the following stringent conditions must be satisfied to validate the super-GIM mechanism[4]:

$$|m^2(q_1) - m^2(q_2)| \;/\; m^2(q_1) < O(10^{-3}),$$
$$|m^2(\ell_1) - m^2(\ell_2)| \;/\; m^2(\ell_1) < O(10^{-3}).$$

It is implausible to expect all these conditions are satisfied by accident. It may be desirable to find some kind of systematic treatment of breaking parameters which guarantees all the requirements. It may well be likely that all soft-breaking terms come from the single origin.

Here we examine the exciting possibility that at the unification energy scale ($\mu \cong M_X$), the theory is almost supersymmetric and the soft-breaking terms exist only in the G_{GUT} invariant mass terms of gauge fermions λ's[5]. At lower energies, all the other soft-breaking terms are generated through radiative corrections[6].

In order to clarify whether this "minimal" soft-breaking scheme really gives the required soft-breaking parameters at low energy ($\mu \cong M_W$), we must examine the renormalization group analysis.

The supersymmetric part of the lagrangian consists of the usual "kinetic" terms and the F-component of the super potential[7)]

$$W = f(\ell H_1 \bar{e}) + h(qH_1\bar{d}) + \tilde{h}(qH_2\bar{u}) + m(H_1H_2).$$

The soft-breaking terms are given by[8)]

$$\mathcal{L}_{break} = \{-M_1\lambda_1\lambda_1 - M_2\lambda_2\lambda_2 - M_3\lambda_3\lambda_3 + h.c.\} - \sum_{\phi=\ell,\bar{e},q,\bar{u},\bar{d}} m^2(\phi)A(\phi)^\dagger A(\phi)$$
$$- \Delta_1^2 A_1^\dagger A_1 - \Delta_2^2 A_2^\dagger A_2 + m\Delta_3(A_1A_2 + A_1^*A_2^*)$$
$$+ \{fm_f A(\ell)A_1A(\bar{e}) + hm_h A(q)A_1A(\bar{d}) + \tilde{h}m_{\tilde{h}}A(q)A_2A(\bar{u}) + h.c.\}.$$

The Higgs scalar mass terms are given as $m_1^2 = m^2 + \Delta_1^2$, $m_2^2 = m^2 + \Delta_2^2$, $m_3^2 = m\Delta_3$.

By examining the renormalization group equations for this minimal model, we see that all the supersymmetry breaking parameters are generated through radiative corrections starting from a boundary condition[5)]

$$m \sim O(10^2\ \mathrm{GeV})$$
$$M_1 = M_2 = M_3 \equiv M \sim O(10^2\ \mathrm{GeV})$$
$$m(\phi) = \Delta_1 = \Delta_2 = \Delta_3 = m_f = m_h = m_{\tilde{h}} = 0 \qquad \text{at } \mu = M_X.$$

The gauge fermion loop contributions give large positive masses of order 10^2 GeV to the scalar partners of leptons and quarks through the following renormalization group equation:

$$(4\pi)^2 \mu\frac{\partial}{\partial\mu} m^2(\phi) = -8 \sum_{i=SU(3),SU(2),U(1)} g_i^2 C_2(R)_i M_i^2 + [\text{Yukawa coupling}].$$

Since the Yukawa coupling contributions are negligibly small for the scalars of the first and second generations, those with the same SU(3)×SU(2)×U(1) quantum numbers are almost degenerate. Therefore in our scheme the super-GIM mechanism works well to suppress the dangerous flavor changing neutral interactions. Combining with the renormalization group equations for gauge fermion masses

$$(4\pi)^2\mu\frac{\partial}{\partial\mu}M_1 = 22g'^2M_1, \quad (4\pi)^2\mu\frac{\partial}{\partial\mu}M_2 = 2g^2M_2, \quad (4\pi)^2\mu\frac{\partial}{\partial\mu}M_3 = -6g_c^2M_3,$$

we get the following mass relations at $\mu = M_W$:

$$M_2/M_1 = 2.01, \qquad M_3/M_1 = 7.19,$$
$$m(\ell_r)/M_1 = 1.78, \qquad m(\bar{e}_r)/M_1 = 0.95,$$
$$m(q_r)/M_1 = 6.60, \qquad m(\bar{u}_r)/M_1 = 6.40,$$
$$m(\bar{d}_r)/M_1 = 6.38, \qquad\qquad (r = 1,2)$$

where $M_X = 2.37 \times 10^{16}$ GeV and $\alpha_G^{-1} = 4\pi/g^2(\mu = M_X) = 24.1$ are used.

In order to obtain desired Higgs vacuum, which is characterized by the conditions $m_1^2 + m_2^2 > |m_3^2|$ and $m_3^4 > m_1^2 m_2^2$ at $\mu \cong M_W$, parameters m, M and Yukawa couplings at $\mu = M_X$ must be in the appropriate domain. If all Yukawa couplings are negligibly small, Higgs scalars A_1 and A_2 receive the same renormalization effects and the relation $m_1^2 = m_2^2$ follows. Therefore the breakdown of SU(2)×U(1) does not occur. It is indispensable in our minimal scheme that at least one of the Yukawa couplings is large enough to generate sizable mass difference between m_1^2 and m_2^2. The only candidate which can be freely large is the top quark Yukawa coupling $\tilde{h}$. Therefore the occurrence of the spontaneous breakdown $SU(2)\times U(1) \to U(1)^{EM}$ requires the existence of the lower bound of $\tilde{h}$. The detailed calculation shows $\tilde{h}(\mu = M_X) \gtrsim 0.095$. This lower bound turns out to be that of top quark mass,

$$m_t \simeq \langle A_2 \rangle \tilde{h}(\mu = M_W) \gtrsim 60 \text{ GeV}.$$

Matter scalars acquire additional mass terms through vacuum expectation values $\langle A_1 \rangle$ and $\langle A_2 \rangle$. Especially their couplings to the D-components of the gauge multiplets give the following mass terms,

$$M_W^2 \cos 2\theta \; A^\dagger [I_3 - Y(\tan^2\theta_W)/2] A$$

where $\theta = \cot^{-1}(\langle A_1 \rangle / \langle A_2 \rangle)$. Their contributions are not always positive. The most dangerous is that for the lightest scalars $A(\bar{e}_r)$ (r=1,2). Their physical masses are given by

$$m^2(\bar{e}_r) - M_W^2 \cos 2\theta \; \tan^2\theta_W.$$

Since $m^2(\bar{e}_r) = (0.95)^2 M_1^2$, the positivity of this physical mass requires the lower bound of M_1. The detailed computation gives

$$M_1 \gtrsim 30 \text{ GeV}.$$

In conclusion, our "minimal" soft-breaking scheme in the minimal supersymmetric SU(3)×SU(2)×U(1) model embedded in the standard GUT group works well. The spontaneous breakdown $SU(2)\times U(1) \to U(1)^{EM}$ occurs through radiative corrections. The super-GIM mechanism works well to naturally suppress the dangerous flavor changing neutral interactions. In order for our scheme to work, following constraints must be satisfied:

$$m_t \gtrsim 60 \text{ GeV},$$

$$M_1 = M_2/2.01 = M_3/7.19 \geq 30 \text{ GeV}.$$

References

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