Supersymmetric Dipole Mechanism and Vacuum Energy

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Much attention has been paid recently to supersymmetric theories in the hope of resolving the gauge hierarchy problem in grand unified theories. Quadratic divergences in elementary scalar theories make it necessary to adjust a parameter to 38 decimal precision to achieve the large hierarchy.[1] The remarkable non-renormalization theorem in supersymmetric theories may provide a natural resolution of this problem.[2]

One of major questions in this approach is how to break supersymmetry taking the following points into account:

i) Supersymmetry must be broken in realistic models,

ii) if supersymmetry is broken spontaneously at the tree level, the unrealistic mass formula $\sum_J (-1)^{2J}(2J+1)m_J^2 = 0$ always follows,

iii) Supersymmetry remains unbroken by perturbation if it is so at the tree level.

It then seems more realistic to accept the following explicit breakings which are soft in the sense that no quadratic divergences are generated.[3]

$$
\begin{aligned}
&\text{a)} \quad L_a = \mu_1^2(A^2 + B^2), \\
&\text{b)} \quad L_b = \mu_2^2(A^2 - B^2), \\
&\text{c)} \quad L_c = \mu_3\bar{\lambda}\lambda, \qquad\qquad (1)\\
&\text{d)} \quad L_d = \mu_4(A^3 - 3AB^2), \\
&\text{e)} \quad L_e = \mu_5^3 A,
\end{aligned}
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where A and B are the scalar and pseudoscalar fields of a chiral multiplet, respectively, while λ is the spinor field of a vector multiplet.

We shall show, however, that all these soft breakings can also be derived by the mechanism of spontaneous breaking and discuss the vacuum energy in this scheme,[4] which was originally proposed by Slavnov.[5]

Let L_0 be a supersymmetric Lagrangian of chiral multiplets and/or vector multiplets, which are called the matter fields. we also introduce left handed chiral superfields S_- and $\tilde{S}_-$, and consider the additional supersymmetric Lagrangian

$$\Delta L = \frac{1}{8}(\bar{D}D)^2(S_-^+\tilde{S}_-) - \frac{1}{2}(\bar{D}D)(\xi S_-^+\tilde{\Phi}_-^+ + \eta\tilde{S}_-) + \text{h.c.}, \tag{2}$$

where ξ and η are real constants and $\tilde{\Phi}_-$ is a left-handed chiral superfield made of the matter fields. Expressed in terms of component fields $S_- = (a_-, \alpha_-, f_-)$, $\tilde{S}_- = (\tilde{a}_-, \tilde{\alpha}_-, \tilde{f}_-)$ and $\tilde{\Phi}_- = (\tilde{A}_-, \tilde{\psi}_-, \tilde{F}_-)$, eq.(2) becomes

$$\begin{aligned}\Delta L &= \partial_\mu a_-^+\partial^\mu\tilde{a}_- + \bar{\alpha}_- i\not\partial\tilde{\alpha}_- + f_-^+\tilde{f}_- \\ &\quad + \xi\{\tilde{A}_-^+ f_-^+ - \bar{\alpha}_-\tilde{\psi}_-^c + \tilde{F}_-^+ a_-^+\} + \eta\tilde{f}_- + \text{h.c.},\end{aligned} \tag{3}$$

where $\tilde{\psi}_-^c = c\bar{\tilde{\psi}}_-^T$. The first line can be diagonalized by orthogonal transformations

$$\begin{pmatrix}\phi_1 & \psi_{1-} & f_1 \\ \phi_2 & \psi_{2-} & f_2\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1 \\ -1 & 1\end{pmatrix}\begin{pmatrix}a_- & \alpha_- & f_- \\ \tilde{a}_- & \tilde{\alpha}_- & \tilde{f}_-\end{pmatrix} \tag{4}$$

resulting in

$$\begin{aligned}\Delta L &= \partial_\mu\phi_1^+\cdot\partial^\mu\phi_1 - \partial_\mu\phi_2^+\cdot\partial^\mu\phi_2 + \bar{\psi}_{1-}i\not\partial\psi_{1-} - \bar{\psi}_{2-}i\not\partial\psi_{2-} + f_1^+f_1 - f_2^+f_2 \\ &\quad + \frac{\xi}{\sqrt{2}}[\tilde{A}_-^+(f_1^+ - f_2^+) - (\bar{\psi}_{1-} - \bar{\psi}_{2-})\tilde{\psi}_-^c + \tilde{F}_-^+(\phi_1^+ - \phi_2^+) + \text{h.c.}] \\ &\quad + \frac{\eta}{\sqrt{2}}(f_1 + f_2 + f_1^+ + f_2^+).\end{aligned} \tag{5}$$

From eq. (5) we derive the field equations

$$\Box\phi_1 = \Box\phi_2 = -\frac{\xi}{\sqrt{2}}\tilde{F}_-^+, \tag{6a}$$

$$i\partial\!\!\!/\psi_{1-} = i\partial\!\!\!/\psi_{2-} = \frac{\xi}{\sqrt{2}}\tilde{\psi}_-^c, \tag{6b}$$

$$f_1 = -\frac{\eta}{\sqrt{2}} - \frac{\xi}{\sqrt{2}}\tilde{A}_-^+,\ f_2 = \frac{\eta}{\sqrt{2}} - \frac{\xi}{\sqrt{2}}\tilde{A}_-^+. \tag{6c}$$

Clearly, the fields ϕ_1 and ψ_{1-} are fields of positive metric while ϕ_2 and ψ_{2-} are ghosts of negative metric, forming a supersymmetric set of dipole fields. We show that these dipole fields are unobservable yet break supersymmetry spontaneously. For this reason we call this scheme "supersymmetric dipole mechanism."

It is easy to see that $\phi_1 - \phi_2$ and $\psi_{1-} - \psi_{2-}$ are free fields:

$$\Box(\phi_1 - \phi_2) = i\partial\!\!\!/(\psi_{1-} - \psi_{2-}) = 0, \tag{7}$$

which allow us to impose the subsidiary conditions

$$(\phi_1 - \phi_2)^{(+)}|\text{phys}\rangle = (\psi_{1-} - \psi_{2-})^{(+)}|\text{phys}\rangle = 0. \tag{8}$$

The combinations $\phi_1 - \phi_2$ and $\psi_{1-} - \psi_{2-}$ turn out to be zero-norm fields, while $\phi_1 + \phi_2$ and $\psi_{1-} + \psi_{2-}$ are removed from physical states by eq. (8), and the net result is that they produce the breaking term spontaneously

$$-\xi\eta(\tilde{A}_- + \tilde{A}_-^+), \tag{9}$$

which is obtained by substituting (6c) into (5). Note that, although supersymmetry is broken spontaneously, there appear no physical Goldstino nor vacuum energy. This situation arises because one of the Goldstinos is of negative metric. We can also show that the mass splittings between bosons and fermions are generated without contradicting supercurrent conservation.

Eq. (8) gives the desired breaking terms L_e, L_b, L_d and L_c in

(1) by setting $\tilde{\Phi}_-$ = Φ_-, Φ_-^2, ϕ_-^3 and $\bar{\Psi}_{++}\Psi_{--}$, respectively, where Ψ_{++} and Ψ_{--} are the spinor superfields of the gauge field strengths. The term L_a can be obtained essentially in the same way by replacing the chiral multiplets S_- and $\tilde{S}_-$ by two vector multiplets V and $\tilde{V}$.[4)]

Now we wish to point out another peculiar feature of this model by choosing the Wess-Zumino model for the matter part and Φ_- for $\tilde{\Phi}_-$.

$$L_0 = \frac{1}{2}(\partial_\mu A)^2 + \frac{1}{2}(\partial_\mu B)^2 + \frac{1}{2}\bar{\psi}i\partial\!\!\!/\psi + \frac{1}{2}F^2 + \frac{1}{2}G^2$$

$$+ m(AF - BG - \frac{1}{2}\bar{\psi}\psi) + g(A^2F - B^2F - 2ABG - A\bar{\psi}\psi - B\bar{\psi}i\gamma_5\psi). \quad (9)$$

The additional Lagrangian gives a linear A term and thus the scalar field A acquires a nonvanishing vacuum expectation value a. The vacuum energy is

$$V(a) = \frac{1}{2}(m + ga)^2a^2 + \sqrt{2}\xi\eta\, a, \quad (10)$$

and a is determined by the minimization condition of this potential. Obviously V(a) may vanish or become negative, contrary to naive expectation. This is precisely due to the presence of negative-metric fields To see this, we have only to consider the linear part of the supercurrent

$$J_\mu = i\gamma_\mu[\frac{1}{\sqrt{2}}(m + ga)a\psi + (\frac{\eta}{\sqrt{2}} + \frac{\xi a}{2})\psi_1 + (\frac{\eta}{\sqrt{2}} - \frac{\xi a}{2})\psi_2]. \quad (11)$$

By using the canonical equal-time anticommutators of ψ, ψ_1 and ψ_2, we get

$$\frac{1}{4}\langle 0|\sum_\alpha\{S_\alpha, S_\alpha^+\}|0\rangle = \int d^3x[\{\frac{1}{\sqrt{2}}(m + ga)a\}^2 + (\frac{\eta}{\sqrt{2}} + \frac{\xi a}{2})^2 - (\frac{\eta}{\sqrt{2}} - \frac{\xi a}{2})^2], \quad (12)$$

where the negative sign in the last term is due to the negative metric of ψ_2. Eq. (12) agrees with (10), implying that the naive argument for positive vacuum energy must be modified in the presence of negative-metric fields.

In conclusion, we have shown that all the explicit soft breakings of supersymmetry may be regarded as spontaneous, providing with a theoretical basis for their softness. This model is a first nontrivial example with supersymmetry broken spontaneously but with not necessarily positive vacuum energy owing to an unobservable Goldstino of negative metric. Finally, from our results, we conjecture that the absence of quadratice divergences may lead to supersymmetry either unbroken or broken spontaneously. Some results in this direction have been obtained recently.[6)]

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