NOSONOMY OF AN INVERTED HIERARCHY MODEL

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This is a report of an unsuccessful attempt to build a realistic Grand Unified model based on Witten's[1] inverted hierarchy mechanism. The work was a collaboration with Vadim Kaplunovsky[2]. Witten's phenomenon is a common property of supersymmetric gauge theories in which supersymmetry (SUSY) is spontaneously broken by the vacuum expectation value of an F term.

Details of the mechanism:

1) $\frac{\partial W}{\partial \phi_i} \neq 0$ for all ϕ_i and $W = C_{ijk}\phi_i\phi_j\phi_k + C_{ij}\phi_i\phi_j + C_i\phi_i$ imply that the potential $V = \left|\frac{\partial W}{\partial \phi_i}\right|^2$ has a line of degenerate minima extending to ∞. Call this flat direction X.
2) The one loop radiative corrections to the potential V(X) for large X have the form
   $M^4 C \ell n X/M$ (M is the Lagrangian mass scale).
   In a gauge theory C can be negative. Renormalization group analysis[2] shows that in many cases V(X) develops a minimum at $X = 0(Me^{1/g^2})$ with g a small coupling and M the SUSY breaking scale.
3) Since $\frac{\partial^2 W}{\partial X^2} = 0$ the partner of X is the Goldstone fermion.
4) When X>>M some particles get mass of 0(X). These are precisely those particles which couple to X (to order $\frac{M}{X}$). The large part of the mass matrix is
   $$\langle X \rangle C_{xij}\phi_i\phi_j$$

So the couplings of particles of mass $\leq$M to the Goldstone fermion are 0(M/X). This implies that supersymmetry breaking in the spectum has the pattern

Supersymmetric mass	breaking
0(X)	0(M)
0(M)	$0(M^2/X)$
$\leq 0(M^2/X)$	$0(M^2/X)$

so only particles of mass $\leq 0(M^2/X)$ can be identified with the real world.

The model:

Fields A,Y in 24 of SU_5

$\underline{X},Z$ in 1 of SU_5

$H,\bar{H}$ in $5,\bar{5}$ of SU_5

+ quarks-lepton superfields

$$W = \lambda \mathrm{tr}A^2Y + \lambda'(\mathrm{tr}(A^2)-M^2)X + \beta\bar{H}AH + \gamma Z\bar{H}H$$

+ quark lepton couplings to $H+\bar{H}$

When X, Y get large expectation values all chiral superfields except $A_{1,1}$, $Y_{8,1}$, $Y_{1,3}$, X, Z, H, $\bar{H}$+quarks and leptons get masses of $0(X)$ (indices refer to $SU_3\times SU_2$ transformation properties). Also

$$SU_5 \to SU_3\times SU_2\times U_1$$

Coset gauge boson masses are $0(X)$. $A_{1,1}$ gets mass of $0(M)$. The other masses are $0(M^2/X)$ or 0.

Diseases of the Model

I. Renormalization

With 3 generations of quarks and leptons color is not asymptotically free above 10 TeV and becomes strong before the GUT scale is reached. (2 loop terms $\triangleq$ 1 loop)

Most realistic model has 2 generations but

$$\alpha_{e.m} \sim \frac{1}{137}$$

$$\Lambda_{QCD} \sim 100 \text{ MeV}$$

$$\Rightarrow M_{GUT} = 10^{24} \text{ GeV}$$

Best we can do with $M_{GUT} \leq 10^{18}$ GeV is

$$M_{GUT} = 10^{18} \text{ GeV}$$

$$\Lambda_{QCD} \sim 100 \text{ MeV}$$

$$\alpha_{e.m.} \sim \frac{1}{113}$$

choosing $\alpha_{e.m} \sim \frac{1}{137}$ forces $\Lambda_{QCD} \sim 10$ keV

The problem is caused by the light $M\sim 10$ TeV color octet superfield $Y_{8,1}$ which is essential to the inverse hierarchy mechanism. We have not been able to construct a model without this field.

II. Gravitational Problems

1) Gravitino mass density of the Universe

Pagels, Primack[3], Weinberg[4] bounds on the SUSY breaking scale

$$M < 2\times 10^6 \text{ GeV} \quad \text{or} \quad M > 10^{11} \text{ GeV}$$

(actually the lower bound on M may be higher depending on the details of R symmetry breaking. We make the <u>optimistic</u> assumption that our

model's R symmetry can be arranged so that this mimimal bound is satisfied).

In an inverse hierarchy model

$$\alpha \frac{M^2}{X} \sim 100 \text{ GeV and } 10^{14} \text{ GeV} \leqslant X \leqslant 10^{18} \text{ GeV}$$

The first estimate is due to the fact that $SU_2 \times U_1$ breaking in these models is a 2 loop radiative correction[2]. So

$$10^9 \text{ GeV} \leqslant M \leqslant 10^{11} \text{ GeV}$$

The model is barely compatible with the minimal bound. Factors of π^2 could be important. Note compatibility $\Rightarrow$ $X \gtrsim 10^{18}$ GeV.

2) Gravitational corrections to the effective potential classical coupling to supergravity implies[5]

$$V = \left| \frac{\partial W}{\partial \phi} \right|^2 \rightarrow \left[\left| \frac{\partial W}{\partial \phi} + \frac{\phi}{M_p^2} W \right|^2 - \frac{1}{M_p^2} \left| W \right|^2 \right] e^{\frac{\phi^+ \phi}{M_p^2}}$$

In Witten's original scenario $M_p = 0(M)$ in the fundamental gravitational Lagrangian and the physical Planck mass arises from a non-minimal gravitational coupling $\sqrt{g}\, X\, R$ to the X field. But in this case the above gravitational correction completely swamps the radiative corrections which create the inverse hierarchy effect. So we must give up Witten's attractive explanation of the ratio between the supersymmetry breaking scale M and the Planck scale (this problem was pointed out to me by Ed Witten).

III. The Revenge of Hierarchy

$SU_2 \times U_1$ breaking must be a radiative effect since tree level scales M, X are both $>>$ 100 GeV. To obtain a nontrivial effective potential for H, $\bar{H}$ we must couple them to SUSY breaking. There are two (?) possibilities

$$\delta W = \beta \bar{H}_i Y^i_j H^j \Rightarrow \text{wrong } SU_5 \text{ breaking (Witten)}^1$$

$$\delta W = \beta \bar{H}_i A^i_j H^j$$

$$\langle A \rangle \propto \begin{matrix} 2 & & \\ & 2 & \\ & & -3 \\ & & & -3 \end{matrix}$$

gives mass of 0(M) to all components of H, $\bar{H}$. Solution by fine tuning

$$\delta' W = \gamma \bar{H} H$$

tune γ (accuract $10^{-10} \sim 10^{-11}$) to cancel doublet mass. Brilliant solution of fine

tuning problem - the sliding scalar:

$$\delta' W = \gamma Z \bar{H} H$$

(proposed independently by Witten, T.B. and Kaplunovsky, Dine + Fischler, Dimopoulos + Raby)[6,2]

If the radiative effective potential favors parallel expectation values of the doublet components of H, $\bar{H}$ (and it does for some values of parameters)[2,6] z will adjust itself to exactly cancel the doublet mass.

Unfortunately there are radiative corrections to the effective potential for Z which are much bigger than this term, so <Z> will probably not sit at the right point. We still have a fine tuning problem of $O(10^{-10})$. This problem is more serious than the others but it may be curable by a change in the treatment of H, $\bar{H}$. The other problems seem to be quite general features of inverse hierarchy models at least for an SU_5 gauge group. We have not been able to find models based on SO_{10}, Thus we conclude that Witten's inverted hierarchy does not work.

References

1. E. Witten, Phys. Lett. 105B, 271 (1981).
2. T. Banks and V. Kaplunovsky, Tel Aviv preprints TAUP 1028-82 TAUP 1011-82 and manuscript in preparation.
3. H. Pagels and J. Primack, Phys. Rev. Lett. 48, 223 (1982).
4. S. Weinberg, Phys. Rev. Lett. 48, 1303 (1982).
5. J. Bagger and E. Witten, Princeton preprint (1982).
6. M. Dine and W. Fischler, IAS preprint (1982).
   S. Dimopoulos and S. Raby, Los Alamos preprint (1982).
   E. Witten, (private communication).