Consistency of Coupling in Supergravity with Propagating Lorentz Connexion

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§1. Introduction

Gauge principle is nowadays the most elegant and successful scheme in grand unifications of three kinds of interactions other than the gravitational interaction. The recent hierarchy problem of grand unification theories motivates us to consider their supersymmetric extensions (superunifications). Once supersymmetry is introduced into the description of nature, there is no theory other than supergravity [1], which is consistent with the gravitational interaction.

On the other hand, there has been another approach to the gravitational interaction, i.e. Poincaré gauge theory [2,3]. This theory is a kind of gauge theory of Poincaré group (P_m, J_{rs}), where the generators P_m and J_{rs} are treated as equally as possible. In particular, the Lorentz connexion gauge field $\omega_\mu{}^{rs}$ is treated as an independent propagating field.

With these developments in mind, we have tried in our previous papers [4∿6] to unify these two approaches, i.e. supergravity theory and Poincaré gauge theory. In other words, we attempted to present "a super Poincaré gauge theory", or "a supergravity theory with propagating Lorentz connexion" [4∿6].

This kind of theory is relevant to the problem of hidden symmetry in extended supergravity theories. Consider for example the recent result of N=8 extended supergravity showing the appearance of the hidden local SU(8) symmetry from the original local Lorentz group SO(1,10) in the 11-dimensional space-time before the dimensional reduction [7]. The study of the propagation mechanism of SO(1,10) Lorentz connexion may be hence as important as that of SU(8) gauge fields.

Our supergravity theory with propagating Lorentz connexion is based on the nonminimal and reducible multiplet of Breitenlohner (126 Breitenlohner potentials) [8]. It consists of two multiplets: the supergravity multiplet (SG) and the Lorentz connexion multiplet (LC). In our previous papers [4∿6], we have presented the Lagrangians for these multiplets: $\mathcal{L}_{SGa}$ and $\mathcal{L}_{LCa}$. The former is a supersymmetric extension of the "massless case" of Poincaré gauge theory [3] and the latter contains the kinetic terms of LC.

The simplest example of $\mathcal{L}_{LCa}$ was already proposed by Breitenlohner , which contains the YM-type kinetic term of $\omega_\mu{}^{rs}$, i.e. $-(1/4)eR_{\mu\nu}{}^{rs}(\omega)\times R^{\mu\nu}{}_{rs}(\omega)$ [8]. We named this Lagrangian $\mathcal{L}_{LCa1}$ in our papers [4∿6].

As was noticed in our paper [4], theories with the Lagrangian of the "massless case" in Poincaré gauge theory may generally have the problem of inconsistent couplings of $\omega_\mu{}^{rs}$ to spinor fields. In our theory, the coupling of $\omega_\mu{}^{rs}$ to its spinor partner $\lambda_\alpha{}^{tu}$ is shown to be consistent to all orders in the total Lagrangian $\mathcal{L} = \mathcal{L}_{SGa} + \mathcal{L}_{LCa1}$ in spite of the "masslessness" of the Lagrangian $\mathcal{L}_{SGa}$. We regard this as an important result of supersymmetry of our theory.

The Lagrangian $\mathcal{L}_{LCa1}$, however, has the problem of negative energy ghosts reflecting noncompactness of the Lorentz group. To circumvent this difficulty we have presented two new Lagrangians named $\mathcal{L}_{LCa2}$ and $\mathcal{L}_{LCa3}$, which are written in local forms in superspace as [4]

$$\mathcal{L}_{LCa3}(z) = \frac{1}{16}V(z)(R(z))^2, \tag{1.1}$$

$$\mathcal{L}_{LCa3}(z) = -V(z)(\bar{R}(z))^2 , \tag{1.2}$$

with $\bar{R}(z)\equiv(1/24)\varepsilon_{mnrs}V^{m\mu}(z)V^{n\nu}(z)R_{\mu\nu}{}^{rs}(z)$ in the notation of Ref. [9]. By examining the free Lagrangians $\mathcal{L}^{(0)}_{LCa2}(x)$ and $\mathcal{L}^{(0)}_{LCa3}(x)$ in component fields, we have shown the absence of negative energy ghosts. The physical helicity states of these Lagrangians are $(\partial_\mu v^\mu(0^+), \gamma_\rho\partial_\sigma\lambda^{\rho\sigma}(\pm 1/2), \tilde{D}^{\rho\sigma}(0^-))$ and $(\partial_\mu a^\mu(0^-), \gamma_\rho\partial_\sigma\lambda^{\rho\sigma\prime}(\pm 1/2), \tilde{D}^{\rho\sigma\prime}(0^+))$, respectively, where $v^\mu(a^\mu)$ is the vector (axial vector) component of $\omega_\mu{}^{\rho\sigma}$, and $\lambda^{\rho\sigma\prime}(\tilde{D}^{\rho\sigma\prime})$ is the dual component of $\lambda^{\rho\sigma}(\tilde{D}^{\rho\sigma})$ [4,6]. The consistency check of $\omega_\mu{}^{rs}$-$\lambda_\alpha{}^{tu}$ couplings, however, is very difficult, since interaction terms in $\mathcal{L}_{LCa2}$ and $\mathcal{L}_{LCa3}$ are far complicated than those in $\mathcal{L}_{LCa1}$ [4,6].

In order to obtain at least trilinear couplings in $\mathcal{L}_{LCa2}$, we have set up a local tensor calculus for the Breitenlohner potentials. It consists of a multiplication rule of two scalar multiplets, a D-type Lagrangian and a correspondence rule between a scalar curvature superfield R(z) and a scalar multiplet. These rules are established up to the first correction terms (required by local supersymmetry) to the global rules

[5]. By using these results, we derived the interaction terms necessary for consistency check of $\omega_\mu{}^{rs}$-$\lambda_\alpha{}^{tu}$ trilinear couplings in $\mathcal{L}_{LCa2}$ [5].

The consistency check of trilinear couplings of $\omega_\mu{}^{rs}$ to its spinor partner $\lambda_\alpha{}^{tu}$ is to show that there is a nontrivial solution satisfying the simultaneous equations: $T^{[\mu\nu]}(\lambda) = 0$ and the free field equation of $\lambda_\alpha{}^{rs}$ [4,6]. Intuitively the former originates from the absence of the antisymmetric part of the Einstein tensor in the "massless case" in Poincaré gauge theory. In the next section we perform this consistency check by using the explicit form of $T^{[\mu\nu]}(\lambda)$ from trilinear terms in $\mathcal{L}_{LCa2}$.

§2. Consistency of Couplings

Our check of coupling consistency of $\omega_\mu{}^{rs}$ to $\lambda_\alpha{}^{tu}$ is to show the existence of nontrivial and propagating solutions of the simultaneous equations: $T^{[\mu\nu]}(\lambda) = 0$ and the free field equation $\not\partial\gamma_\rho\partial_\sigma\lambda^{\rho\sigma} = 0$ [6]. The form of $T^{[\mu\nu]}(\lambda)$ is obtained from trilinear $e_\mu{}^m$-$\lambda_\alpha{}^{rs}$-$\lambda_\beta{}^{tu}$ terms in $\mathcal{L}_{LCa2}$

In order to simplify the caluculation, we decompose $\lambda_\alpha{}^{\rho\sigma}$ into the following three components:

$$\lambda_\alpha{}^{\rho\sigma} = \phi_\alpha{}^{\rho\sigma} + \frac{1}{2}(\gamma^\rho\phi^\sigma - \gamma^\sigma\phi^\rho)_\alpha + \frac{i}{12}(\sigma^{\rho\sigma}\phi)_\alpha, \tag{2.1}$$

$$\phi^\rho = \gamma_\sigma\lambda^{\sigma\rho} - \frac{1}{4}\gamma^\rho\phi, \qquad \phi \equiv -i\sigma_{\rho\sigma}\lambda^{\rho\sigma}, \qquad (\gamma_\rho\phi^\rho \equiv 0, \quad \gamma_\rho\phi^{\rho\sigma} \equiv 0). \tag{2.2}$$

The field equation of $\lambda_\alpha{}^{\rho\sigma}$ then takes the form

$$\not\partial(\gamma_\rho\partial_\sigma\lambda^{\rho\sigma}) = \not\partial(\partial_\rho\phi^\rho + \frac{1}{4}\not\partial\phi) = 0, \tag{2.3}$$

which is invariant under the following gauge transformation with the paramater $\varepsilon^{\rho\sigma}$:

$$\delta\lambda^{\rho\sigma} = \delta\phi^{\rho\sigma} = \varepsilon^{\rho\sigma} \quad (\gamma_\rho\varepsilon^{\rho\sigma} \equiv 0). \tag{2.4}$$

This implies that the $\phi^{\rho\sigma}$ component of $\lambda^{\rho\sigma}$ is unphysical and can be always gauged away. We then obtain

$$T^{[\mu\nu]}(\lambda) = [\ \frac{1}{16}\bar{\zeta}\gamma_5\sigma^{\mu\nu}\not\partial(\partial_\rho\phi^\rho - \frac{1}{4}\not\partial\phi)$$
$$+(\text{terms containing } \phi^{\rho\sigma} \text{ or vanishing by the use of the free field equation } \not\partial\gamma_\rho\partial_\sigma\lambda^{\rho\sigma} = 0)] - (\mu\leftrightarrow\nu). \tag{2.5}$$

The simultaneous equations for our consistency check are now

$$\bar{\zeta}\gamma_5\sigma^{\mu\nu}\not{\partial}(\partial_\rho\phi^\rho - \frac{1}{4}\not{\partial}\phi) = 0 \tag{2.6}$$

$$\not{\partial}(\partial_\rho\phi^\rho + \frac{1}{4}\not{\partial}\phi) = 0 \tag{2.7}$$

We can find nontrivial solutions of (2.6) and (2.7) satisfying

$$\not{\partial}\partial_\rho\phi^\rho = 0 \tag{2.8}$$

$$\not{\partial}\phi = 0 \tag{2.9}$$

Since these equations give nontrivial propagating solutions, we conclude that the trilinear $\omega_\mu{}^{rs}$-$\lambda_\alpha{}^{tu}$ couplings in our theory are indeed consistent couplings [6].

§3. Physical Helicity States and Global Supersymmetry

We show here that the $\partial_\mu\phi^\mu$ component (but not ϕ) of $\lambda_\alpha{}^{\rho\sigma}$ describes the physical helicity states $J=\pm 1/2$ by studying global supersymmetric covariances.

Under global supersymmetry, the left hand side of $\not{\partial}\partial_\mu\phi^\mu = 0$ (2.8) is transformed into

$$\delta(\not{\partial}\partial_\rho\phi^\rho) = \frac{1}{2}\varepsilon\Box\partial_\rho v^\rho + \frac{1}{2}(\gamma_5\sigma_{\rho\sigma}\varepsilon)\partial^{[\rho}\partial_\tau\tilde{D}^{\sigma]\tau}$$
$$-\frac{i}{2}(\sigma^{\rho\sigma}\varepsilon)\Box\partial_\rho v_\sigma - \frac{i}{4}\varepsilon^{\rho\tau\upsilon\omega}(\sigma_{\rho\sigma}\varepsilon)\partial^\sigma\partial_\tau\tilde{D}_{\upsilon\omega} \tag{3.1}$$

The first line contains the left sides of the free field equations of v^μ and $\tilde{D}^{\rho\sigma}$, while the second line contains the terms removed by appropriate gauge conditions [6]. In a similar way, the left sides of the free field equations of v^μ and $\tilde{D}^{\rho\sigma}$ are transformed into that of $\lambda_\alpha{}^{\rho\sigma}$ [6]. These facts imply that $\partial_\mu\phi^\mu$ describes the physical helicity states $J=\pm 1/2$, that are transformed into $\partial_\mu v^\mu(0^+)$ and $\tilde{D}^{\rho\sigma}(0^-)$ under global supersymmetry.

§4. Concluding Remarks

We summarize here our main conclusions:

(1) There is no negative energy ghost in $\mathcal{L}_{LCa2}$ and $\mathcal{L}_{LCa3}$.

(2) Although $\mathcal{L}_{SGa}$ is a supersymmetric extension of the "massless case" Lagrangian of Poincaré gauge theory, the $\omega_\mu{}^{rs}$-$\lambda_\alpha{}^{tu}$ couplings are consistent up to the trilinear interaction order.

As far as we know, no other theory of supergravity with propagating Lorentz connexion possesses both of these two properties (1) and (2).

Future studies include

(1) Checking coupling consistency of other interactions and to higher orders.

(2) Quantizing fields and discussing renormalizability.

(3) Making our theory more realistic.

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