COSMOLOGICAL PHASE TRANSITION IN MICROCANONICAL GRAVITY

G. Horwitz
Racah Institute of Physics
The Hebrew University
Jerusalem, Israel

Phase transitions in cosmology whether first or second order, spontaneous or dynamic were first studied by methods which ignore or exclude gravitational effects (Linde, 1974). The basis for the exclusion was the assumption that the effects are local, and that since the relevant times are much later than Planck times quantum gravity is unimportant and thus gravity enters only as a background metric which can be removed by studying results in a local inertial frame. Gradually various gravitational effects have been added: the effect of curvature in addition or instead of the Higgs mass for symmetry breaking (Grib, Mostepanenko and Frolov, 1977), topological effects (Avis and Isham, 1978), gravitational effects in growth rate of bubbles (Coleman and De Luccia, 1980).

In the present work we introduce a completely new element: reasonably realistic models in which there exists a nonquantum gravitational contribution to the entropy based on a microcanonical (MCE) rather than grandcanonical treatment of the statistical mechanics. The Hawking thermal states of de Sitter cosmology (Gibbons and Hawking, 1977) is an example of such gravitational entropy, but the latter is not a relevant model for phase transitions. Horwitz and Weil (1982) have developed a self-consistent generalization of such thermal states for a closed FRW universe with a positive cosmological constant Λ which has both scalar "radiation" and gravitational entropy. The dynamics is that of the usual classical models except that Λ and the invariant temperature are not independent parameters. In the present work we apply this method to a hyperbolic universe with $\Lambda < 0$ and a conformally coupled scalar field with a ϕ^4 self interaction. Such systems have been found to have a ground state with broken symmetry due to negative curvature even without mass.

The basis of our approach is the use of a Gibbsian definition for a thermodynamic equilibrium (TDE) MCE, generalized to apply to appropriate cosmological mocels and extended to include gravity. The standard Gibbs entropy is based on equal weight sums over states in a shell of (nearly) fixed constants of motion (COM) including energy, angular momentum, etc. For cosmological models, the energy which is not conserved can be replaced by the so-called dilatation operator D. Projecting the stress energy tensor $T_{\mu\nu}$ on the conformal time-like Killing vector ξ^{μ}_{c} characterizing FRW universes, a conserved quantity is obtained giving the COM D. This also requires a vanishing trace, hence invariance to Weyl transformations of the action. Thus instead of the standard definition for static systems, we have (units: $c = \hbar = G = 1$)

$$\exp S/k = \mathrm{Tr}\,\delta(D - D_0) \tag{1}$$

where

$$D = \int_{\Sigma_0} d\,\Sigma_{\mu} \sqrt{-g}\, T^{\mu}_{\nu}\, \xi^{\nu}_{c} \tag{2}$$

with Σ_0 a space-like hypersurface, $T_{\mu\nu}$ the functional derivative of the action with respect to the metric, the conformal time-like Killing vector satisfying $\xi_{c\mu;\nu} + \xi_{c\nu;\mu} = f\, g_{\mu\nu}$ where in appropriate coordinates f depends only on time. D is then a COM provided the action $I(\phi,\psi,g_{\mu\nu})$ is invariant to Weyl transformations by an arbitrary local scale function $\Omega(x^\mu):\phi \to \phi/\Omega$, $\psi \to \psi/\Omega$, $g_{\mu\nu} \to \Omega^2\, g_{\mu\nu}$. For the purposes of our model, we have included two scalar bosons, in addition to the metric.

For a FRW universe, with gravity appearing only as a background metric, and considering two alternative coordinate systems: one involving cosmic time $ds^2 = dt^2 - a^2\, d\sigma^2$ and the other conformal time: $ds^2 = a^2(d\eta^2 - d\sigma^2)$, where $d\sigma^2$ is a given time-independent element depending only on a parameter k which is +1 for closed, -1 for open and 0 for spatially flat spaces. Applying our entropy definition (1) we readily find the Tolman-Ehrenfest result for entropy:

$$S = 8\pi/3\, a_s\, T^3(\eta) = 8\pi/3\, a_s\, T^3(t) a^3(t) \tag{3}$$

their condition for TDE being

$$T(t)a(t) = \text{const.} = T(\eta)\ . \tag{4}$$

We wish to stress the interpretation of this situation as a state of global and not only local (in time) equilibrium. This is a maximum entropy, collisions preserving the state which is homogeneous and isotropic, but temperatures and various densities as functions of proper (cosmic) time vary with the expansion; the fixed entropy is stretched over growing proper volume elements in a self similar fashion. The truly new element of our analysis which goes beyond the above rederivation and reinterpretation of known results involves the inclusion of gravitational contributions with the use of a MCE and not based on quantum gravity.

Thus in the above mentioned work of Horwitz and Weil (1982) for a k = +1 and $\Lambda > 0$ FRW universe with conformally coupled massless bosons or in the presently discussed k = -1, $\Lambda < 0$ FRW universe with massless conformally coupled bosons having a ϕ^4 self interaction, the microcanonical TDE state found includes gravitational contributions. In both cases, these are self-consistent analogues of the TDE states of a BH in a radiation cavity (Hawking, 1976; Gibbons and Perry, 1978). Thermal fluctuations of the quantum radiation field couple to the classical fields of gravity through the microcanonical constraint, leaving an overall gravitational contribution to the entropy unrelated to quantum gravity or Planck times. In order to define such a TDE state which includes gravity one requires a Weyl invariant gravitational theory defined with an extra scalar field ψ in which the gravitational action

$$I'_G = (8\pi)^{-1}\int d^4 x \sqrt{-g}\, [\tfrac{1}{2}R - \Lambda] + \text{surf. terms} \tag{5}$$

is replaced by

$$I_G = -3(8\pi)^{-1}\int d^4 x (-g)^{\frac{1}{2}} [g^{\mu\nu}\psi_{,\mu}\psi_{,\nu} - \tfrac{1}{6} R\psi^2 + \tfrac{1}{3}\Lambda\psi^4]\ . \tag{6}$$

This leads to an expression for an action including the scalar, massive bosons conformally coupled with self interaction. The action is Weyl invariant even with the mass, but we shall treat here only $m^2 = 0$.

$$I = \tfrac{1}{2}\int d^4x(-g)^{\frac{1}{2}}\Big\{[g^{\mu\nu}\phi_{,\mu}\phi_{,\nu} - \tfrac{1}{6}R\,\phi^2 - m^2\,\phi^2\,\psi^2 - \lambda\,\phi^4] - 3(4\pi)^{-1}[g^{\mu\nu}\psi_{,\mu}\psi_{,\nu} - \tfrac{1}{6}R\,\psi^2 + \tfrac{\Lambda}{3}\psi^4]\Big\} \quad (7)$$

where the line element for k = -1 FRW is

$$ds^2 = a^2(\eta)[d\eta^2 - dx^2 - \sinh^2 x(d\theta^2 + \sin^2\theta\, d\Phi^2)]\,. \quad (8)$$

The standard choice of conformal gauge $\Omega = \psi^{-1}$ reduces I to the standard form (5) with the mass term $-m^2\phi^2$. Instead, after limiting our discussion to the case where $g_{\mu\nu}$ retains the form (8) with arbitrary $a(\eta)$, we now choose our conformal gauge with $\Omega = a(\eta)$, which then reduces the background metric to a static one - that in (8) with the $a(\eta)$ deleted. In the quantized version of this theory, this means we shall neglect spin-2 excitations (gravitons), although the method can readily be extended to include them. (It is a peculiar situation that presently to carry out classical SM for gravity one must introduce quantum gravity.) We subsequently interpret the transformed ψ to be the dynamic factor of the cosmic scale function, in our quasi-classical theory. Notice, in such a static background metric, there are no conformal trace anomalies although mass generation can be retained with a $m^2\phi^2\psi^2$ term. This choice also leads to a static energy operator, and the result that the Hamiltonian defined by (2) is equivalent to the canonical definition.

We now outline our calculation using (1), (2) and (7), choosing D = 0 corresponding to Einstein's equations when ψ is identified as the scale function, and taking a Fourier-Laplace transform of the delta function (1) becomes

$$\exp S/k = \mathrm{Tr}\int d\beta/2\pi i \exp - \beta D \quad (9)$$

$$= \int[d\phi]\int[d\psi]\int d\beta/2\pi i \exp - I_E \quad (10)$$

where the Euclideanized action I_E

$$I_E = \tfrac{1}{2}V\int_0^\beta du[\dot\phi^2 + \phi_{,i}\phi^{,i} - \phi^2 + \lambda\,\phi^4] - 3/8\pi\, V\int_0^\beta du[\dot\psi^2 - \psi^2 + \psi^4/R_0^2] \quad (11)$$

with the boundary conditions $\phi(0) = \phi(\beta) = \phi_0$; $\psi(0) = \psi(\beta) = \psi_0$ and V is the (infinite) spatial 3-surface of the hyperbolic universe, and we have assumed that ψ depends only on the (imaginary) time. Since we seek only extremal values of ψ, which remain homogeneous and isotropic of course, significant fluctuations depend on x^i, but we ignore them for the present. We have written our cosmological constant $\Lambda = -3/R_0$ and $R/6 = -1$ for the static hyperbolic universe of scale length unity. This kind of model is known to lead to a ground state with broken symmetry even without a Higgs mass. The extremal solution of the ϕ and ψ satisfy the equations

$$\ddot\phi_{cl} + \phi_{cl} - 2\lambda\phi_{cl}^3 = 0 \quad (12)$$

$$\ddot\psi_{cl} + \psi_{cl} - 2/R_0^2\,\psi^3 = 0 \quad (13)$$

respectively. The functional integrals for fixed boundary values of ϕ_0 and ψ_0 are expanded about these classical solutions. The quadratic quantum fluctuations of the ψ's are dropped as we seek only classical gravity contributions. The quadratic expansion of the ϕ's is the one-loop term for the scalar conformal bosons, and we will take here the crudest high temperature approximation, that its contribution to the entropy is A/β^3 where $A = \pi^2/45$. Properly one should calculate it self-consistently, but we are here only looking for the qualitative modification of standard treatment by our gravitational inclusive MCE formalism. There remain the integrals over ψ_0, ϕ_0 and $\delta\beta$: where we also require a quadratic expansion around the extremum of β. The first integrals of (12) and (13) yield

$$\tfrac{1}{2}[\dot{\phi}^2 + \phi^2 - \lambda\,\phi^4] = \tfrac{1}{2}[\dot{\phi}^2 + V_1] = \omega \tag{14}$$

$$\tfrac{1}{2}[\dot{\psi}^2 + \psi^2 - \psi^4/R^2] = \tfrac{1}{2}[\dot{\psi}^2 + V_2] = \varepsilon\ . \tag{15}$$

The self-consistent solution if obtained by solving

$$3A/\beta^4 = \omega - 3/4\pi\,\varepsilon = \bar{\omega}/\lambda - 3R_0^2/8\pi\,\bar{\varepsilon} \tag{16}$$

$$\beta = 4\int^{y_-} dy[\bar{\omega} - y^2 + y^4]^{-\frac{1}{2}} = 4K(t^2)/y_+ \tag{17}$$

$$\beta = 2\int_0^{x_-} dx[\bar{\varepsilon} - x^2 + x^4]^{-\frac{1}{2}} = 2K(s^2)/x_+ \tag{18}$$

where the K are complete elliptic integrals of the first kind and $\bar{\omega} = \dot{y}^2 + y^2 - y^4$ with $y = \lambda^{\frac{1}{2}}\phi$; and $\bar{\varepsilon} = \dot{x}^2 + x^2 - x^4$ with $x = \psi/R_0$. Furthermore, $x_\pm^2 = \tfrac{1}{2}[1 \pm (1 - 4\bar{\varepsilon})^{\frac{1}{2}}]$ and $y_\pm^2 = \tfrac{1}{2}[1 \pm (1 - 4\bar{\omega})^{\frac{1}{2}}]$.

We then find two kinds of solutions which are shown to be local entropy maxima for appropriate values of ,R . One preserves the symmetry for the interacting bosons (Fig. 1a and Fig. 1b).

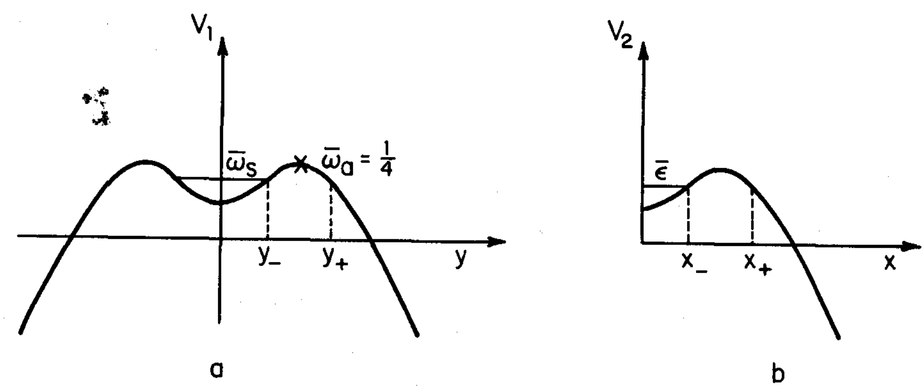

Figure 1

$\bar{\omega}_S$ represents a solution which preserves symmetry and $\bar{\omega}_a = 1/4$ the broken symmetry solution. The $\bar{\varepsilon}$ solution is similar for both cases.

ψ is positive since it represents the scale function. The other is the broken symmetry solution shown in Fig. 1a. The entropy for the two solutions are respectively, (E, elliptic integral of second kind)

$$S_{sym}/V = -3R_0^2/8\pi[\tfrac{1}{3}\,x_+][K(s^2)] + \tfrac{4}{3}\,y_+/3[K(t^2) - E(t^2)] \tag{19}$$

$$S_{asym}/V = \beta/8\lambda - [R_0^2/8\pi]\ \ x_+[K(t^2) - E(t^2)]\ . \tag{20}$$

There is no second-order phase transition possible since the temperature β^{-1} is a constant, for given (λ, R_0). Both symmetric and broken symmetry solutions can be locally stable in certain cases for equal values of λ and R_0. The symmetric solution is generally of lower entropy than the broken symmetry solution, so that it would appear that a first-order phase transition is possible, although the details of the thermodynamic analysis have yet to be carried out.

The cosmological solution is found by analytic continuation to real time

$$a(t) = 2^{-\frac{1}{2}} R_0 (1 - \gamma \cos^2 2t/R_0)^{\frac{1}{2}} \tag{21}$$

with $$\gamma \equiv \sqrt{1 - 4\varepsilon} \tag{22}$$

range $$x_- \leq a/R_0 \leq x_+$$

period $$0 \leq t \leq \pi/2\, R_0 \quad .$$

Thus we have evaluated the MCE entropy of this model and found its dynamical solution, finding a possible first-order phase transition. This method is capable of wide generalization and the study of its consequences is being pursued.

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