DIMENSIONAL REDUCTION

Peter G. O. Freund
The Enrico Fermi Institute and the Department of Physics
The University of Chicago, Chicago, Illinois 60637

Higher dimensional unified theories (i.e., generalized Kaluza-Klein theories)[1] have been considered for quite some time now. What is a simple gravity (or supergravity[2]) theory in a N+4-dimensional space-time becomes a theory involving gravity, and lower spin Bose (and Fermi) fields in four dimensions, upon the compactification of N space-like dimensions. Besides the number N of these extra dimensions, 4-dimensional Physics depends on the nature of the compact (and very small) N-dimensional manifold M_N. Depending on M_N we can envisage the low energy gauge group $G_{LE} = SU(3)_{color} X (SU(2) X U(1)_{electroweak}$ as the isometry group G_{KK} of M_N. Then the simplest possibility[3] is N=7 with $M_N = CP_2 X S^2 X S^1$. In this case there is no grand unification of the usual kind[4] whatsoever, $G_{KK} \approx G_{LE}$. Grand unification can be included by choosing a manifold M_N with isometry group G_{KK} larger than G_{LE}, e.g., $G_{KK} = SU(5)$, or by acquiring in the process of dimensional reduction gauge symmetries beyond those corresponding to G_{KK} (this happens when reducing 11-dimensional supergravity[5] to 4-dimensions when an additional Cremmer-Julia gauged SU(8) symmetry is nonlinearly realized by the scalar fields).[6] If the 4-dimensional theory possesses more gauged symmetry than G_{LE}, the question is at what scale this symmetry is restored. Standard renormalization group reasoning claims[7] this to occur at a length scale of $\ell_{GUT} \sim 10^{-15}\ GeV^{-1}$ or $\ell_{SUSY} \sim 10^{-16}\ GeV^{-1}$ in the minimal supersymmetric case. These scales being very far from present-day "physical" scales, the question can be asked as to whether at such scales 4-dimensional Physics is still applicable.

To answer this question we have to estimate the size of the small dimensions.[1] Requiring the Yang-Mills piece of the reduced 4-dimensional lagrangian to have the correct normalization relative to the Einstein piece and to have the proper minimal coupling say to charged scalar fields determines the size of the small dimension

$$\ell = 4\pi\, G^{1/2} (g^2/4\pi)^{-1/2} \tag{1}$$

where G is Newton's gravitational constant and g the Yang-Mills coupling constant. Equation (1) yields $\ell \sim 10^{-17} GeV^{-1}$, very close to ℓ_{SUSY}. It thus appears that in generalized Kaluza-Klein theories by the time the grand unification scale is reached (4+N)-rather than 4-dimensional Physics applies.[8] In particular, dimensionality is increased even before quantum gravity effects become large. This has prompted me[9] and also Ramond[10] to consider cosmologies in which the "effective" space-dimensionality is time dependent. Earlier work on such cosmologies is due to Chodos and Detweiler.[11]

Specifically we start from 11-dimensional supergravity[5] in which case supersymmetry requires an antisymmetric tensor Bose-matter field $A_{\mu\nu\rho}$ which is known[12] to produce preferential compactification of either 7 or 4 space-like dimensions. The first case is of course interesting and we generalize the Ansatz of reference 12) to include a cosmological time-dependence. The splitting of space-time into a 4-dimensional physical space-time and a 7-dimensional manifold M_7 is then automatic. A solution in which the cosmological scale of ordinary (3-dimensional) space increases linearly with cosmological time t, whereas the scale of M_7 increases only as $t^{1/7}$ is found. The 4-dimensional gravitational constant turns out to decrease at 1/t as proposed by Dirac.[13]

An alternative solution has M^7 a sphere of small time-independent radius, and 4-dimensional space time, an anti-de-Sitter universe with cosmological factor $R(t)=R(0)\cos\alpha t$ where α is determined by the 11-dimensional gravitational constant and by the, here time-independent, scale of the antisymmetric tensor field.

These solutions do not involve the Fermi-matter fields and as such should not be valid in the matter dominated era. They also break down at too early times where quantum gravity matters, but they can reasonably be expected to be relevant in some time interval around the dimensional transition where the 11-dimensional manifold splits into $M_4 \times M_7$. Other cosmological solutions along similar lines have been discussed in references 9)-11). New scales are introduced in these solutions: the time t_s at which all d-dimensions have comparable sizes, the actual size $\ell(t_s)$ of the dimensions at time t_s and the strength of gravity at that time. Depending on the details of the evolution, these scales can considerably exceed the present day Planck (10^{-19} GeV^{-1}) and Kaluza-Klein (10^{-17} GeV^{-1}) scales. The cosmology of the very early universe is thus seriously affected.

The preferential compactification of 7 space dimensions has been achieved here at the classical level as in reference 12). The alternative could be entertained that the choice of 4 large dimensions occurs at the quantum level, by somehow summing over all possible dimensionalities and for dynamical reasons ending up with four large physical dimensions in some approximation. Such a possibility was considered by others as well.[14]

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References

1) Th. Kaluza, Sitzungsber. Preuss. Akad. Wiss. Phys. Math. Kl. 966 (1921);
O. Klein, Z. Phys. 37, 895 (1926);
B. de Witt, Dynamical Theories of Groups and Fields, (Gordon and Breach, New York, 1965) p. 139;
R. Kerner, Ann. Inst. H. Poincaré 9, 143 (1968);
A. Trautman, Rep. math. Phys. 1, 29 (1970);
Y.-M. Cho and P.G.O. Freund, Phys. Rev. D12, 1711 (1975).

2) D.Z. Freedman, P. van Nieuwenhuizen and S. Ferrara, Phys. Rev. D13, 1314 (1976)
S. Deser and B. Zumino, Phys. Lett. 62B, 335 (1976).

3) E. Witten, Nucl. Phys. B186, 412 (1981).

4) J.C. Pati and A. Salam, Phys. Rev. D8 (1973) 1240;
H. Georgi and S.L. Glashow, Phys. Rev. Lett. 32 (1974) 438.

5) E. Cremmer, B. Julia and J. Scherk, Phys. Lett. 76B, 409 (1978).

6) E. Cremmer and B. Julia, Nucl. Phys. B159 (1979) 141;

7) H. Georgi, H. Quinn and S. Weinberg, Phys. Rev. Lett. 33 (1974) 451.
S. Dimopoulos, S. Raby and F. Wilczek, Phys. Rev. D24 (1981) 1681;
S. Weinberg, Harvard preprint HUTP-81/A047 (1981);
N. Sakai and T. Yanagida, Nucl. Phys. B197 (1982) 533;
M.B. Einhorn and D.R.T. Jones, Nucl. Phys. B196 (1982) 475;
J. Ellis, D.V. Nanopoulos and S. Rudaz, CERN preprint TH 3199 (1981).

8) P.G.O. Freund, preprint EFI 82/23.

9) P.G.O. Freund, preprint EFI 82/24.

10) P. Ramond, Univ. of Florida preprint UFTP 82-21.

11) A. Chodos and S. Detweiler, Phys. Rev. D21, 2167 (1980).

12) P.G.O. Freund and M.A. Rubin, Phys. Lett. 97B, 233 (1980).

13) P.A.M. Dirac, Proc. Roy. Soc. (London) A165, 199 (1935).

14) R. Geroch, private communication.
S. Shenker, private communication.