PREGEOMETRY

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All the existing experimental evidences, though not so many, clearly support the general relativity of Einstein as a theory of gravitation. So far, extended investigations have been made, based on the premise of general relativity. Even the theories of induced gravity,[1] or pregeometry, where the Einstein action is derived from a more fundamental stage, are not free of this premise. However, if the principle of general relativity is true, it should be a manifestation of some underlying dynamics, just like the Kepler's law for the Newtonian gravity, or like the law of definite proportion in chemical reactions for atoms, etc. So we would like to ask here why the physical laws are generally relative, instead we premise it. The purpose of this talk is to propose a model to give a possible answer to the question. By general relativity, we mean the general covariance of physical laws in the curved spacetime. Our solution, in short, is that it is because our four-spacetime is a four-dimensional vortex-like object in a higher-dimensional flat spacetime, where only the special relativity is assumed. To be specific, we adopt the dynamics of the Nielsen-Olesen vortex[2] in a six-dimensional flat spacetime, and show that general relativity actually holds in the four-spacetime. Furthermore we will show that the Einstein equation in the four-spacetime is effectively induced through vacuum fluctuations, just as in Sakharov's pregeometry.[1]

We start with the Higgs Lagrangian in a six-dimensional flat spacetime

$$\mathcal{L} = -\frac{1}{4} F_{MN}F^{MN} + D_M\phi^\dagger D^M\phi + a|\phi|^2 - b|\phi|^4 + c \tag{1}$$

where $F_{MN} = \partial_M A_N - \partial_N A_M$ and $D_M\phi = \partial_M + ieA_M$.

This has the 'vortex' solution[2]

$$A_M = \epsilon_{0123MN}\, A(r)\, X^N/r\,, \quad \phi = \varphi(r)\, e^{in\theta}\,, \tag{2}$$

$(r^2 = (X^5)^2 + (X^6)^2)$, where $A(r)$ and $\varphi(r)$ are the solutions of the differential equations,

$$\begin{aligned} &-\frac{1}{r}\frac{d}{dr}\left(r\frac{d}{dr}\varphi\right) + \left[\left(\frac{n}{r} + eA\right)^2 - a + 2b\varphi^2\right]\varphi = 0 \\ &-\frac{d}{dr}\left(\frac{1}{r}\frac{d}{dr}rA\right) + \varphi^2\left(e^2A + \frac{en}{r}\right) = 0 \end{aligned} \tag{3}$$

The 'vortex' is localized within the region of $O(\epsilon)$ $(\epsilon = 1/\sqrt{a})$ in two of the space dimensions (X^5, X^6) , leaving a four-dimensional subspacetime $(X^0 - X^3)$ inside it. For large a , the curved 'vortices' with curvature $R \ll a$

become approximate solutions,[3] which we denote by A_M^o and ϕ^o. Let the center of the 'vortex' be $X^M = Y^M(\xi^\mu)$ $(\mu = 0-3)$, and take the curvilinear coordinate x^M such that, near the 'vortex',

$$X^M = Y^M(x^\mu) + n_m^M\, x^m, \quad (M = 0-3, 5, 6,\ \mu = 0-3,\ m = 5, 6) \tag{4}$$

where X^M is the Cartesian coordinate, and n_m^M are the normal vectors of the 'vortex'. (Hereafter Greek suffices stand for 0-3, small Latin, 5, 6, and capital, 0-3, 5, 6). Then, the solution is

$$A_M^o = \epsilon_{0123MN}\, A(r)\, x^N/r, \quad \phi^o = \varphi(r)e^{in\theta}. \ (r^2 = x^m x^m). \tag{5}$$

The S-matrix element between the states Ψ_i and Ψ_f is given by

$$S_{fi} = \int \prod_{X^M} dA_M\, d\phi\, d\phi^\dagger\, \exp\left[i\int \mathcal{L}\, d^6X\right] \Psi_f^* \Psi_i \prod_{X^M} \delta(\partial_M A^M) \tag{6}$$

We assume that the path-integration is dominated by the field configurations of the approximate solutions (5) and small quantum fluctuations around it. To estimate it, we first extract the collective coordinate by inserting

$$1 = \int \prod_{X_{/\!/}} dY^M(\xi^\mu)\, \delta(Y^M(\xi^\mu) - C^M(\xi^\mu)) \tag{7}$$

where $C^M(\xi^\mu)$ is the center of the distribution of $|\tilde\phi|^2$ $(\tilde\phi = \phi - \sqrt{a/2b})$ in the normal plane $N(\xi^\mu)$ of the 'vortex' at $x^\mu = \xi^\mu$,

$$C^M(\xi^\mu) = \int_{N(\xi^\mu)} X^M |\tilde\phi|^2 d^2X_\perp \Big/ \int_{N(\xi^\mu)} |\tilde\phi|^2 d^2X_\perp \tag{8}$$

By $\prod_{X_{/\!/}}$, we mean the product over the four parameters ξ^μ with the invariant measure. Then, we transform them into the representation in the curvilinear coordinate x^M, and we change the path-integration variables A_M and ϕ to their quantum fluctuations $B_{\bar N} = A_{\bar N} - A_{\bar N}^o$ and $\sigma = \phi - \phi^o$, retaining the terms up to quadratic in them.

$$\begin{aligned} S_{fi} = \int \prod_{X_{/\!/}} dY^M \prod_{x^M} dB_{\bar N}\, d\sigma\, d\sigma^\dagger\, \delta(\sqrt{-g}\, \nabla_{\bar N} B^{\bar N}) \prod_{X_{/\!/}} \delta(\tilde C^M) \\ \exp\left[i\int (\mathcal{L}_0 + \mathcal{L}_2)\sqrt{-g}\, d^6x\right] \Psi_f^* \Psi_i \ , \end{aligned} \tag{9}$$

with

$$\mathcal{L}_0 = \mathcal{L}\ (\phi = \phi^o,\ A_M = A_M{}^o) \tag{10}$$

$$\begin{aligned} \mathcal{L}_2 = & -\tfrac{1}{2} g^{LM} \nabla_L B_{\bar N} \nabla_M B^{\bar N} + B_{\bar N} B^{\bar N} e^2 |\phi^o|^2 \\ & + g^{LM} (D_L^o \sigma)^\dagger (D_M^o \sigma) - 4ie V^{\bar N M} B_{\bar N}\, \mathcal{I}m\, (\sigma^\dagger D_M^o \phi^o) \\ & + a|\sigma|^2 - b\,[\, 4|\phi^o\sigma|^2 + 2\, \mathcal{R}e\, (\sigma^\dagger \phi^o)^2]\ , \end{aligned} \tag{11}$$

$$\tilde{C}^m = \int x^m |\tilde{\Phi}|^2 dx^5 dx^6 \Big/ \int |\tilde{\Phi}|^2 dx^5 dx^6 \tag{12}$$

$$= \frac{1}{J_0}\int x^m [|\sigma|^2 + 2\,\mathrm{Re}(\tilde{\Phi}^o\sigma^\dagger)\{1 - \frac{2}{J_0}\int \mathrm{Re}(\tilde{\Phi}^o\sigma^\dagger)\,dx^5dx^6\}]\,dx^5dx^6, \tag{13}$$

and $\tilde{C}^\mu = 0$, where the barred suffices stand for the local Lorentz frame indices, $V^{\bar{N}M}$, the vielbein, g^{LM}, the metric tensor, ∇_M, the covariant differentiation, $D^o_M = \nabla_M + ieA^o_M$, and $J_0 = \int |\Phi^o|^2 dx^5 dx^6$. The Lagrangian $\mathcal{L}_2$ indicates that, outside the 'vortex', any low energy fields are suppressed because of the high barrier of $|\phi^o|^2$. Inside the 'vortex', $g_{m\mu} = O(R/a) \ll 1$, $g_{mn} = -\delta_{mn} + O(R/a)$ and $B_{\bar{M}}$ reduces to the four-vector $B_{\bar{\mu}}$ and the two scalers $B_{\bar{m}}$. Thus, the spacetime looks like four-dimensional and curved to observers with large scale. It is easily checked that the action is invariant under the general coordinate transformation of the curved four-spacetime, i.e. the physical laws are generally relative!

Now we see that the Einstein action is induced through vacuum polarizations. The effective action S^{eff} for it is given by

$$S^{eff} = -i\ln\int \prod_{x^M} dB_{\bar{N}}\, d\sigma\, d\sigma^\dagger\, \delta(\sqrt{-g}\,\nabla_{\bar{N}}B^{\bar{N}}) \prod_{x_\mu} \delta(\tilde{C}^m)\exp\left[i\int\sqrt{-g}\,\mathcal{L}_2\, d^6x\right]. \tag{14}$$

Exponentiating the argument of the δ -functions by $\delta(x) = \int dk\, e^{ikx}$, we get

$$S^{eff} = -i\ln\int \prod_{\xi^\mu} dw_m \prod_{x^M} dB_{\bar{M}}\, d\sigma\, d\sigma^\dagger\, dv\, \exp\left(i\int(\Xi\Phi + \Phi^\dagger\Delta\Phi)\,d^6x\right) \tag{15}$$

with $\Phi^\dagger = (B^{\bar{M}}, \sigma^\dagger, \sigma)$ (16)

$$\Xi = \sqrt{-g}\left(\nabla_{\bar{M}}v\,,\ \ w_m x^m\tilde{\Phi}^{o\dagger}/J_0\,,\ \ w_m x^m \tilde{\Phi}^o/J_0\right) \tag{17}$$

$$\Delta = \sqrt{-g}\begin{bmatrix} \eta_{\bar{M}\bar{N}}(\frac{1}{2}\nabla_L\nabla^L + e^2|\phi^o|^2) & ieD^o_{\bar{M}}\phi^{o\dagger} & -ieD^o_{\bar{M}}\phi^o \\ -ieD^o_{\bar{N}}\phi^o & \frac{1}{2}D^o_L D^{oL} + \frac{a}{2} - 2b|\phi^o|^2 + \delta^m_{11}w_m & -b(\phi^o)^2 + \delta^m_{12}w_m \\ ieD^o_{\bar{N}}\phi^{o\dagger} & -b(\phi^{o\dagger})^2 + \delta^m_{21}w_m & \frac{1}{2}D^o_L D^{oL} + \frac{a}{2} - 2b|\phi^o|^2 + \delta^m_{22}w_m \end{bmatrix} \tag{18}$$

where δ^m is the nonlocal operator in 5-6 plane,

$$\delta^m(x,x') = \frac{1}{2J_0}x^m\delta(x-x') + \frac{1}{2J_0^2}(x^m + x'^m)\begin{bmatrix}\tilde{\Phi}^o(x) \\ \tilde{\Phi}^o(x)^\dagger\end{bmatrix}\left[\tilde{\Phi}^o(x')^\dagger\ \ \tilde{\Phi}^o(x')\right] \tag{19}$$

Performing the path-integration in $B_{\bar{N}}$, σ, $\sigma^\dagger$, and v , we get (with $\Xi_0 = \Xi|_{v=0}$)

$$S^{eff} = \frac{1}{2}i\,\mathrm{Tr}\ln\Delta + \frac{1}{2}i\,\mathrm{Tr}\ln\left[\partial_M\sqrt{-g}\,(\Delta^{-1})^{MN}\sqrt{-g}\,\partial_N\right] - \frac{1}{4}\int \Xi_0^\dagger\Delta^{-1}\Xi_0\, d^6x \tag{20}$$

S^{eff} in (20) is estimated perturbatively in $h^{MN} = g^{MN} - \eta^{MN}$ ($\eta^{MN} = diag(1,-1,-1,-1,-1,-1)$) and w . The propagator is given by the inverse of $\Delta|_{h^{MN}=0,\,w=0} \equiv \Delta_0$. Δ_0 can be separated into two parts Δ_0^{sp} and Δ_0^{ex} which operates on four-space variables x^μ , and the extra space variables x^m, respectively. Furthermore, these Δ_0's are block-diagonalized into two parts Δ_0^V and Δ_0^S, which operate on the four-vector B^μ and the coupled scalars $(S^{(1)}, S^{(2)}, S^{(3)}, S^{(4)}) = (B^5, B^6, \sigma, \sigma^\dagger)$, respectively. They are given by

$$\Delta_0^{V,sp} = \frac{1}{2}\Box \ , \quad \Delta_0^{S,sp} = \frac{1}{2}\Box \ , \quad \Delta_0^{V,ex} = -\frac{1}{2}\partial_\ell\partial_\ell + e^2|\phi^0|^2$$

$$\Delta_0^{S,ex} = \begin{pmatrix} (-\frac{1}{2}\partial_\ell\partial_\ell + e|\phi^0|^2)\eta_{mn} & ieD_n^0\phi^{0\dagger} & -ieD_n^0\phi^0 \\ -ieD_m^0\phi^0 & -\frac{1}{2}D_\ell^0D_\ell^0 + \frac{a}{2} - 2b|\phi^0|^2 & -b(\phi^0)^2 \\ ieD_m^0\phi^{0\dagger} & -b(\phi^{0\dagger})^2 & -\frac{1}{2}D_\ell^0D_\ell^0 + \frac{a}{2} - 2b|\phi^0|^2 \end{pmatrix} \tag{21}$$

where $\Box = \eta^{\mu\nu}\partial_\mu\partial_\nu$. Then, the propagators for each class are given by

$$[\Delta_0^{V\,-1}]^{\mu\nu} = \eta^{\mu\nu}\sum_k (\Box + m_k^2)^{-1} V_k(x^m) V_k(x'^m) \ ,$$
$$[\Delta_0^{S\,-1}]^{(a)(b)} = \sum_k (\Box + m_k'^2)^{-1} S_k^{(a)}(x^m) S_k^{(b)}(x'^m) \ , \tag{22}$$

where V_k, $S_k^{(a)}$, m_k^2 and $m_k'^2$ are the solutions and the eigenvalues of the differential equations in the extraspace.

$$\Delta_0^{V\,ex} V_k = m_k^2 V_k \tag{23}$$
$$\Delta_0^{S,ex\,(a)(b)} S_k^{(b)} = m_k'^2 S_k^{(a)}$$

The argument of the logarithms in (20) is expanded as follows

$$\Delta = \Delta_0 (1 + \Delta_0^{-1}\Delta_{int}) \ , \tag{24}$$

$$\partial_M\sqrt{-g}\,[\Delta^{-1}]^{MN}\sqrt{-g}\,\partial_N = 1 + \Delta_0'^{\,-1} + \partial_m(\Delta_0^{-1})^{mn}\partial_n + \Delta'_{int} \ , \tag{25}$$

where Δ_{int} and Δ'_{int} are the interaction parts including $h^{\mu\nu}$ and w, and

$$\Delta_0'^{\,-1} = \sum_k m_k^2 (\Box + m_k^2)^{-1} V_k(x^m) V_k(x'^m) . \tag{26}$$

We expand the logarithms in (20), and get series of one-loop diagrams with external $h^{\mu\nu}$ and w lines attached. These diagrams diverge quartically in the ultraviolet region. We introduce the momentum cutoff Λ much larger than $\sqrt{a}$, and calculate the divergent contributions. The diagrams with vertices which involve extra-space operators are less divergent.

After this, the same argument as in the pregeometry[1] leads to the Einstein action in the four-dimensional curved space. Namely, the divergent contributions are

$$S^{eff} = \int \sqrt{-g} \left[(N_0 \alpha_0 + N_1 \alpha_1 + \alpha_c) \Lambda^4 + (N_0 \beta_0 + N_1 \beta_1 + \beta_c) \Lambda^2 R \right] d^4 x \tag{27}$$

plus less divergent terms, where N_0 and N_1 are the numbers of the scalar and the vector bound-states in (23), respectively, and $\alpha_0, \alpha_1, \beta_0, \beta_1$ are calculable constants of $O(1)$. The values are found in literatures,[1,4] though we should be careful, since they depend on the cutoff-method and even on the gauge.

α_c and β_c are the contributions from the continuum states in (23). Now, together with the contributions from $\mathcal{L}_0$, we finally get the Einstein action

$$S = \int \sqrt{-g} \, (\lambda + \frac{1}{16\pi G} R) \, d^4 x \tag{28}$$

where

$$\lambda = \int \mathcal{L}_0 \, dx^5 dx^6 + (N_0 \alpha_0 + N_1 \alpha_1 + \alpha_c) \Lambda^4 , \tag{29}$$

$$\frac{1}{16\pi G} = (N_0 \beta_0 + N_1 \beta_1 + \beta_c) \Lambda^2 .$$

In conclusion, in this model:

1) The principle of general relativity is induced, instead it is premised.
2) The Einstein equation is induced just as in Sakharov's pregeometry.
3) Two kinds of internal symmetries are induced, those of the transformation and the excitation in the extra-space. The former is somewhat like isospin, while the latter, generation. This suggests a new mechanism for unification of the interactions.
4) When the gravitational field is quantized, the ultraviolet divergences should be cut off at the inverse of the size of the 'vortex', which may be much smaller than the Planck mass. If this is the case, we can by-pass the problems of renormalizability of gravity.
5) Particles with sufficiently high energy can penetrate into the extra dimensions.
6) At very high temperatures,[5] or high densities, the 'vortex' is spread out over the extra-space revealing the higher dimensional spacetime.

References

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