SCALE INVARIANT SCALAR-TENSOR THEORY AND THE ORIGIN OF GRAVITATIONAL CONSTANT AND PARTICLE MASSES

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Scalar-tensor theory seems to offer a natural alternative to the standard theory of gravitation, especially when one tries to unify the theory of gravitation and the theory of elementary particles. It also seems inevitable that any scalar-tensor theory results in a variable-G theory. This is, however, not always true. We present a simple viable scalar-tensor theory the result of which is not a variable-G theory in the usual sense. We suggest that the next simplest model will give a true variable-G theory.

We consider the fundamental Lagrangian[1)]

$$\mathcal{L} = \sqrt{-g}\left(\frac{1}{2} f^{-2}\phi^2 R - \frac{1}{2}\epsilon g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + L_M + L_I\right), \tag{1}$$

where ϕ is the scalar gravitational field; $\epsilon=\pm 1$ depending on whether ϕ is a normal or ghost field, L_M and L_I being the matter and interaction Lagrangians, respectively. The coupling constant f^2 is dimensionless, expected to be of the order of unity. (We use the unit with $c=\hbar=1$.) For L_M we assume the Lagrangian of massless matter fields with dimensionless coupling constants, as in any of gauge theories. We also choose

$$L_I = -c_0\phi^4 - g\bar{\psi}\psi\phi + \cdots , \tag{2}$$

where ψ is a typical spinorial matter field. The Lagrangian (1) is characterized by the absence of dimensional constants, and is invariant under global scale transformation.

We assume a decomposition

$$\phi^2(x) = u(t) + \sigma(x), \tag{3}$$

where $\sigma(x)$ is a usual spacetime-dependent field, while u(t), called the cosmological background value (BGV), may depend only on the cosmic time t. The BGV u(t) may change so slowly that it may be viewed as a constant in most physical phenomena except those that take place on a cosmological time scale. We then find from the first term of (1) that the effective time-dependent gravitational constant G(t) is given by

$$G(t) = (f^2/8\pi)\, u^{-1}(t). \tag{4}$$

The second term of (2) gives the effective mass

$$m(t) = g\, u^{1/2}(t). \tag{5}$$

From (4) and (5) we obtain the relation

$$G(t)\, m^2(t) = \text{const.} \tag{6}$$

This is a crucial consequence of this simplest model in which there is only one scalar field that plays a dual role ("single-scalar model").

We derive the field equations as given by

$$G_{\mu\nu} = f^2 \mathcal{T}_{\mu\nu} = f^2 \phi^{-2} \left[T_{\mu\nu} - f^2 (g_{\mu\nu}\Box - \nabla_\mu \nabla_\nu) \phi^2 \right], \tag{7a}$$

$$f^{-2}\phi R + \epsilon \Box \phi + \partial L_I/\partial\phi = 0, \tag{7b}$$

together with other matter field equations, where $T_{\mu\nu}$ is the symmetric energy-momentum tensor of the matter as well as ϕ. We must impose

$$\nabla_\mu \mathcal{T}^{\mu\nu} = 0, \tag{8}$$

in order to be consistent with LHS of (7a). The scalar field equation (7b) can be put into a simpler form

$$Z^{-1} \Box \phi^2 = 0, \tag{9}$$

with $Z^{-1} = \epsilon + 6f^{-2}$, where we have used the trace of (7a) and the matter field equations. On RHS of (9) we have the trace of the matter energy-momentum tensor which vanishes due to the scale invariance. The scalar field has now no direct matter source. No scalar long-range force occurs in the limit of weak gravitational field, thus leaving the experimental tests of general relativity unaffected. In this sense the theory is completely viable for any value of f^2.[2)] This is in sharp contrast with Brans-Dicke theory;[3)] in order to meet observational constraints, their coupling constant $\omega(=\epsilon f^2/4)$ is severely bounded almost to the extent that the theory does not make much sense. ($|\omega|\gg 60$.)

The factor Z^{-1} would vanish if one chooses a conformal coupling, $f^2=6$, $\epsilon=-1$, as in conformally invariant theories. Then, however, we have no field equation of ϕ^2. The scalar field no longer has a dynamical degree of freedom, and thus making the theory viable again. This is the way Dirac and other authors formulated their variable-G theories.[4)] This approach is not satisfactory, however, because the theory has no built-in principle to determine the scalar field, and eventually G(t).[5)] One has to appeal to some outside principle, like Dirac's Large Numbers Hypothesis. We insist that any physical quantity which develops with time must be dynamical. From this point of view we avoid the conformal coupling and assume $Z^{-1}\neq 0$. We hence obtain

$$\Box\phi^2=0. \tag{10}$$

We reiterate that requiring scale invariance and conformal noninvariance is an almost unique choice if we want a viable scalar-tensor theory maintaining a dynamical degree of freedom of the scalar field.[2)]

We now assume the spatially flat Robertson-Walker metric with the pressureless matter. In accordance with the decomposition in (3), the BGV parts of (7a), (8) and (10) are calculated to be

$$3H^2=f^2 J_{00}=f^2u^{-1}\left(\frac{1}{2}\epsilon\dot{v}^2+c_0v^4-3f^{-2}H\dot{u}+\rho\right), \tag{11a}$$

$$\dot{J}_{00}-3Hu^{-1}\left(\epsilon\dot{v}^2+f^{-2}\ddot{u}+\frac{1}{2}f^{-2}H\dot{u}+\rho\right)=0, \tag{11b}$$

$$\frac{d}{dt}(a^3\dot{u})=0, \tag{11c}$$

respectively, where $H=\dot{a}/a$ with $a(t)$ the scale factor of the universe, $u=v^2$, and ρ the density of the matter and $\sigma(x)$. We solve (11c) to obtain

$$a(t)=K\dot{u}^{-1/3}(t). \tag{12}$$

Substituting this into (11a) and (11b), and eliminating ρ, we obtain

$$\frac{\dddot{u}u}{\dot{u}}-\frac{3}{2}\frac{\ddot{u}^2u}{\dot{u}^2}-\frac{1}{2}\ddot{u}-\frac{3}{16}\epsilon f^2\frac{\dot{u}^2}{u}=-\frac{3}{2}c_0f^2u^2. \tag{13}$$

A solution u(t) of this third-order nonlinear differential equation will determine simultaneously G(t), m(t) and a(t) through (4), (5) and (12), respectively, being in conformity with Mach's principle.

In solving (13) we must give integration constants in the form of initial or boundary conditions. In this way we derive dimensional quantities in nature starting from the Lagrangian that has no dimensional constants. The same situation is typical in any spontaneous symmetry breaking.

Ignoring RHS of (13), for the moment, we solve (13) in a systematic way. Among the solutions, we find "asymptotically standard solutions" in which u(t) approaches a finite constant as $t\to\infty$. This implies that G(t) and m(t) also approach constant values and a(t) tends to the standard Einstein-de Sitter solution $t^{2/3}$. We may propose an interesting conjecture that the standard theory gives an accurate description of the present or recent universe just because we are already in the asymptotic region of t.

On the other hand, we may not be in the asymptotic epoch, or one of other solutions may be a true solution. Corresponding to Dirac's atomic gauge, we then apply a conformal transformation which brings $\phi(x)$ into a constant v_*, so that the particle mass $m_*=gv_*$ is also a constant. In this "microscopic unit system" in which the time is measured by using atomic clocks, the gravitational constant G_* is also a true constant due to (6). For this reason our theory is not a variable-G theory in the usual sense.

In spite of a truly constant G_* our theory is certainly different from the standard theory because the scalar field is still present, showing itself through a time-dependent cosmological term. We find that the Lagrangian after the conformal transformation is given by

$$\mathcal{L} = \sqrt{-g_*}\left(\frac{1}{16\pi G_*}R_* - \frac{1}{8\pi G_*}\Lambda + L_{*M} + L_{*I} - m_*\bar{\psi}_*\psi_*\right), \tag{14a}$$

where the stars indicate the quantities in the microscopic unit system, and the effective cosmological term Λ is found to be

$$\begin{aligned}\Lambda(t) &\sim |Z|^{-1/2} G_* g_*^{\mu\nu}\partial_\mu(v_*\ln\phi)\partial_\nu(v_*\ln\phi)\\ &\sim (\dot{v}/v)^2 \sim t^{-2}.\end{aligned} \tag{14b}$$

The relation (6) from which $G_*=$ const follows may be avoided to give a true variable-G theory, if we include another scalar field ("two-scalar model"). As still another consequence of this next simplest model of a viable scalar-tensor theory, one of the scalar fields may acquire a non-zero mass μ, giving a finite-range Yukawa potential which is added to the Newtonian potential:[2)]

$$V(r) = \frac{GM}{r}\left(1 + \alpha e^{-\mu r}\right). \tag{15}$$

A plausibility argument gives $\mu^2 \sim (Gm^2)m^2$, where m is a typical particle mass. For a choice $m \sim 1$ GeV, we find the force-range $\mu^{-1} \sim 10^5$ cm = 1 km. Experimental searches for any deviation from the purely Newtonian behaviors in this distance range are now under way.[6)]

References

1) For details, see Y. Fujii, UT-Komaba 82-8.
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5) For a similar view, see also J. D. Bekenstein and A. Meisels, Phys Rev. D22(1980), 1313.
6) See the papers cited in the second of refs.2.