

QUANTIZED STRINGS AND QCD

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1. Introduction

It is now generally believed that the $N = \infty$ QCD is equivalent to some string model (see e.g. [1,2]). This equivalence is understood in the sense of the string ansatz for the Wilson loop average:

$$W[C] \equiv \frac{1}{N} \langle \text{tr} P \exp - \oint_C A_\mu dx_\mu \rangle = \sum_{\text{surfaces, } \partial S=C} \exp(-I_{\text{string}}) \quad (1)$$

However the main question about the type of this string is still open. But suppose we have solved the Makeenko-Migdal equation and thus know the string action I_{string} (which can be an effective one obtained after integrating out some internal string variables). Then the natural problem is how to calculate mesons Green's functions (and thus their spectrum and scattering amplitudes). One may naively think that the old "dual string" approach (see e.g. [3]) may be useful: start with the known string action and define the amplitudes trying to mimic the dual string definitions. However, we will show that the knowledge of I_{string} is not sufficient - one must first calculate $W[C]$ as a functional and then sum over closed paths with appropriate measure. The point is that the QCD string is not a free one but has quarks at the ends (let us stress that only open strings naturally appear from the QCD field formalism if the string ansatz (1) is assumed). In sect. 4 we work out the definition of the mesons scattering amplitudes starting with the QCD path integral and assuming (1). We illustrate our definition on the example of the 2-dimensional QCD.

Lacking the final form for the string ansatz in (1) one can try to study the properties of various possible string models in attempt to establish their common features, or, ^{we show} to find some distinguished one. This is the topic of sect. 2 where, that the Brink-Di Vecchia-Howe-Polyakov (BDHP) model [5,6] seems to be the most simple and tractable one among other known bose string ansatze. Sect.3 is devoted to the formal BDHP string theory: we temporarily forget about QCD applications and explicitly calculate the free BDHP string scattering amplitudes, defining them in analogy with the dual string case. The results seem not to support the conjecture that substituting the

Nambu action by the BDHP one we get tachyons free string theory in $d=4$. A realistic QCD motivated string theory should be the result of the proper account of the quark end point terms as stressed in Sect. 4.

2. String ansatze

Let us list the following three bose string models which may be considered in connection with QCD according to (1):

Nambu model [7]: $\mathcal{D} \subset \mathbb{R}^2$, $h_{ab} = \partial_a x_\mu \partial_b x_\nu$, $h = \det h_{ab}$
 $I_N = M^2 \int_{\mathcal{D}} \sqrt{|h|} d^2 z$, $M^{-2} = 2\pi\alpha'$, $W_N[c] = \int_{\mathcal{D}} [\mathcal{D}x] e^{-I_N}$ (2)

Eguchi model [8,9]: $I_E = M^{4\nu} \int_{\mathcal{D}} h^{\nu\nu} d^2 z$, $x|_{\partial\mathcal{D}} = c$, $\kappa(\nu) = \left(\frac{2\nu-1}{\nu}\right) \cdot \frac{1}{2\nu}$,
 $W_E[c] = \int_0^\infty da e^{-a/2} \int_{\mathcal{D}} [\mathcal{D}x] \exp(-\kappa(\nu) I_E)$, $a = \int_{\mathcal{D}} d^2 z$.

BDHP or "gravitational" model [5,6]:

$$I_G = M^2 \int_{\mathcal{D}} \frac{1}{2} g^{ab} \sqrt{g} \partial_a x_\mu \partial_b x_\nu d^2 z$$
, $W_G = \int_{\mathcal{D}} [\mathcal{D}g \mathcal{D}x] e^{-I_G}$ (3)
 $x|_{\partial\mathcal{D}} = c$

The study of these models gave the following results [10]: (1) They are equivalent not only at the classical level (minimal surfaces

$W \sim \exp(-M^2 A_{\min}[c])$) but also in the semiclassical approximation near a minimal surface and thus predict the same long-range $\bar{g}g$ potential $V_{\text{semiclass.}} = M^2 R - \gamma/R + \text{const}$, $\gamma = \frac{\pi}{12} \cdot \frac{d-2}{2}$

for $C = (R \rightarrow \infty) \times (T \rightarrow \infty)$; (2) They are equivalent in the leading $1/d$ approximation for the static potential (cf. [9]): $V(d \rightarrow \infty) =$

$= M^2 R (1 - R_c^2/R^2)^{1/2}$, $R_c^2 = \pi d / 12 M^2$; (3) these models are

not equivalent as exact functional integrals (e.g. beyond the semiclassical approximation) being different quantum analogs of the same

classical theory; (4) the BDHP model is the most simple and tractable at the quantum level. The main fact is that in order to preserve the $O(d)$ (Lorentz) symmetry of $W[c]$ one is to use $O(d)$ -

invariant and thus incomplete gauges on x_μ (or g_{ab}). Then

additional Weyl symmetry of the BDHP action and its polynomiality in x_μ are essential simplifications. For example, if we consider

(instead of W) the formal BDHP string partition function

(with $\partial_n x|_{\partial\mathcal{D}} = 0$ boundary conditions) then it is easy to obtain the effective action, integrating the conformal anomaly [6];

(5) However, one cannot establish the analogous local representation

for $W[c]$ due to the nontrivial boundary condition $x|_{\partial\mathcal{D}} = c$:
 in the gauge $g_{ab} = e^{2\sigma} \delta_{ab}$ we get $W_G = \int d\sigma \exp(-\tilde{I}_G[\sigma, c])$
 $\tilde{I}_G = \frac{d}{2} \text{tr} \log \Delta_{0c} - \frac{1}{2} \text{tr} \log \Delta_1$ = non-local functional of
 σ and c_μ :

$$\left\{ \begin{array}{l} \Delta_0 = -\nabla_a \nabla^a \\ x_\mu|_{\partial\mathcal{D}} = c_\mu \end{array} \right. , \quad \Delta_{1ab} \gamma^b = -(\nabla_c \nabla^c + R/2) \gamma_a .$$

Thus no (26-d) -coefficient arise. As for the formal partition function, it can be evaluated through the anomaly also for locally supersymmetric fermi string models, as was shown for the spinor string in [11] and for the string with spin and charge in [12] . In the latter case one has

$$\begin{aligned} Z_{CF} &= \int [d\psi_\mu d\bar{\psi}_\mu dA_a d\chi_a d\bar{\chi}_a d\bar{g}_{ab}] e^{-I_{CF}} = \int d\sigma d\lambda d\mathcal{Z} e^{-\tilde{I}_{CF}} , \\ I_{CF} &= M^2 \int_{\mathcal{D}} d^2z \sqrt{g} \left\{ \frac{1}{2} g^{ab} \partial_a \psi \cdot \partial_b \psi^* + \frac{1}{2} \bar{\psi} \gamma^a \partial_a \psi + A_a \bar{\psi} \gamma^a \psi + \right. \\ &\quad \left. + (\partial_a \psi^* + \bar{\psi} \gamma_a) \bar{\chi}_b \gamma^a \gamma^b \psi + \text{c.c.} \right\} , \\ \tilde{I}_{CF} &= \frac{2-d}{4\pi} \int d^2z \left[\frac{1}{2} (\partial\sigma)^2 - \frac{1}{2} (\partial\mathcal{X})^2 + i \bar{\lambda} \hat{\partial} \lambda \right] , \end{aligned}$$

where $(\psi_\mu = \chi_\mu + i\gamma_\mu, \bar{\psi}_\mu)$ form d complex matter multiplets, interacting with $N = 2$ two-dimensional supergravity fields (g_{ab}, χ_a, A_a) and we used the gauges $g_{ab} = e^{2\sigma} \delta_{ab}, \chi_a = \frac{1}{2} \partial_a \lambda, A_a = \frac{1}{2} \epsilon_{ab} \partial_b \mathcal{X}$. Three terms in (4) are due to the conformal, axial and superconformal anomalies respectively. As was stressed in [12] and [10] the Liouville non-linearities ($\sim e^\sigma$) do not arise in supersymmetrical cases due to the absence of quadratic (L^2) divergences. (Even for the bose BDHP case one can formally neglect L^2 -terms or put the renormalized value of the cosmological constant Λ to zero). However, it is instructive to show how the Λ -term can be introduced in the fermi string theory on the example of quantization of the $N=1$ 2-dimensional supergravity. This is a non-trivial theory with the compact spaces partition function given

by

$$\begin{aligned} Z &= \sum_{(\text{topologies})} \int_{(\text{fixed topology})} [d\bar{g}_{ab} d\chi_a dA] e^{-I(\alpha, \mu)} , \quad (5) \\ I &= \alpha \chi + \alpha \int d^2z \sqrt{g} \left\{ -i \bar{\chi}_a \gamma^a \gamma_5 \epsilon^{mn} \partial_m \chi_n + \right. \\ &\quad \left. + \frac{i}{4} \bar{\chi}_a \gamma^b \gamma^a \chi_b - \frac{1}{2} A^2 \right\} + \mu \int d^2z \left(\frac{i}{2} \bar{\chi}_a \gamma_5 \epsilon^{ab} \chi_b - A \right) \sqrt{g} , \end{aligned}$$

where A is the auxiliary scalar field (cf. [13]) and $\chi = \frac{1}{\sqrt{\pi}} \int R \sqrt{g} d^2z$ is the Euler number, while the μ -term is the analog of the Λ -term (cf. [14]). Only taking in account the gauge measure (in the gauge $g_{ab} = e^{2\sigma} \delta_{ab}$, $\chi_a = \frac{1}{2} \partial_a \lambda$) we get the correct counting of degrees of freedom and

$$Z = \sum_{\chi} e^{-\alpha \chi} \int_{(\text{fixed } \chi)} \partial \sigma \partial \lambda \partial A e^{-\tilde{I}},$$

$$\tilde{I} = \frac{10}{12\pi} \int d^2z \left\{ \frac{1}{2} (\partial \sigma)^2 + i \bar{\lambda} \partial \lambda - \frac{1}{2} A^2 e^{2\sigma} \right\} + \mu \int d^2z \left\{ \bar{\lambda} \lambda e^{\sigma} - A e^{2\sigma} \right\}. \quad (6)$$

Integration over A gives the super-Liouville theory with $\Lambda = \mu^2$ (which probably leads to some space-time foam picture as the corresponding 2-dim gravity model [15]).

3. Free BDHP string amplitudes

Here we forget about the connection with QCD and study the formal BDHP string theory assuming the Neumann boundary condition

$\partial_n X_\mu|_{\partial\mathcal{D}} = 0$ in the path integral. This theory has recently attracted much attention due to a hope that the proper account of the anomalous degree of freedom σ (with the action $\tilde{I} = \frac{26-d}{12\pi} \int d^2z [\frac{1}{2} (\partial \sigma)^2 + \mu^2 e^{2\sigma}]$ + boundary terms, $\partial_n \sigma|_{\partial\mathcal{D}} \sim \mu \exp(\sigma)$) may help to avoid tachyons and $d=26$ restriction in loops. As a simplest test of this hope one can put $\mu = 0$ and study the corresponding scattering amplitudes. Still the main problem is the absence of a natural definition of the BDHP strings amplitudes. Thus we are to follow the old dual string definition or try to invent some new one. The realization of this program gave the following results [10]:

I Open strings ($\mathcal{D} = \mathbb{C}^+$, $\partial\mathcal{D} = \mathbb{R}$), on-shell definition a la ref. [4]:

$$V(p_1, \dots, p_n) = \int d\mu_{KN} \prod_{i=1}^n |z_{i+1} - z_i|^{-\alpha' m_0^2} Z[J_0], \quad (7)$$

$$p_i^2 = -m_0^2, \quad z_i \in \partial\mathcal{D}, \quad J_0^\mu = \sum_{i=1}^n p_i^\mu \delta^{(\mu)}(z - z_i)$$

$$Z[J] = \langle \exp(i \int d^2z J_\mu X^\mu) \rangle_{\partial_n X|_{\partial\mathcal{D}} = 0}, \quad d\mu_{KN} = \prod_{i=1}^n dz_i$$

, where $d\mu_{KN}$ is the Koba-Nielsen

measure and the averaging is made with the help of the BDHP string path integral. We get: $d=26$; $Z_G = \prod |z_i - z_j|^{2\alpha' p_i \cdot p_j}$ i.e. the Veneziano amplitude; $d < 26$; $Z_G \sim \prod |z_i - z_j|^{2\alpha' p_i \cdot p_j}$, $\alpha_{ij} = 2\alpha' p_i \cdot p_j - \alpha'$ and the poles are given by $\alpha(0) = -\alpha' m_0^2 + \alpha/2 = 1 \rightarrow \alpha' m_0^2 \simeq 15.6$ or -0.94 . Thus there is the non-tachyonic ground state solution; the amplitude is dual only for $d = 26$. Analogous results follow [10] if we start with the heuristic off-shell definition [6, 16]

$$\Gamma(p_1, \dots, p_n) = \left\langle \prod_{j=1}^n \int d^2 z_j \sqrt{g(z_j)} e^{i p_j \cdot X(z_j)} \right\rangle, z_j \in \mathcal{D} \quad (8)$$

Here for $d < 26$ we get the physical trajectory $\alpha' m_0^2 \simeq 1.34 - 0.7$ (along with the tachyonic one). However, (8) does not lead to the Veneziano amplitude for $d=26$ and thus is probably consistent only for closed string case.

II Closed strings ($\mathcal{D} = \mathcal{C}$)

Starting with (8) we get for $d=26$: Γ has poles at $\frac{\alpha' p_i^2}{2} = 2$, it describes only the ground state tachyon scattering in the Shapiro-Virasoro model [17]. For $d < 26$: $\Gamma \sim \int \prod d^2 z_j \prod |z_i - z_j|^{2\alpha' p_i \cdot p_j - \alpha_{ij}}$, $\alpha_{ij} = \frac{24}{26-d} (1 - \frac{\alpha'}{4} p_i^2) (1 - \frac{\alpha'}{4} p_j^2)$ and thus the spectrum contains the old tachyon $1 - \frac{\alpha'}{4} p_i^2 = 0$ along with a new physical state $1 - \frac{\alpha'}{4} p_i^2 = \frac{-(26-d)}{24}$. It remains to be seen if the inclusion of the Liouville term may eliminate tachyons and give unitary and factorizable loop diagrams. However it should be understood that a priori they will have no relation to QCD, where the expression for scattering amplitudes turns to be different from (7) or (8) even if the BDHP ansatz is assumed in (1).

4. $N \rightarrow \infty$ QCD string scattering amplitudes

Trying to derive the string amplitudes from QCD we are to start with the field theoretic definition for meson Green's functions

$$\begin{aligned} G(x_1, \dots, x_n) &= \langle (\bar{q}q)_{x_1} \dots (\bar{q}q)_{x_n} \rangle_{\text{connected}} = \\ &= N^{-n/2} \left(\frac{\delta^n \log Z[m]}{\delta m(x_1) \dots \delta m(x_n)} \right)_{m(x) = m}, \end{aligned} \quad (9)$$

$$Z[m] \sim \int \mathcal{D}A \mathcal{D}\bar{q} \mathcal{D}q \exp(-I[m]),$$

where $I[m] = \int d^4x \left(\frac{1}{4g} F_{\mu\nu}^2 + i\bar{q}(\not{\partial} + m(x))q \right)$.

With the help of the proper time representation for the quark determinant and taking the large N limit (cf. [2]) we finally get

$$G(x_1, \dots, x_n) = N^{1-n/2} \int_0^\infty \frac{dT}{T} \oint_{C(0)=C(T)} \mathcal{D}C_\mu(z) J_{1/2}[C] W[C] \times \prod_{j=1}^n \int_0^T d\tau_j \delta^{(4)}[x_j - c(\tau_j)] \quad (10)$$

where W is given by (1) and $J_{1/2}[C] = \int \mathcal{D}\pi \, t_z \rho \exp \left\{ i \int_0^T d\tau [\pi_\mu (\dot{x}_\mu + \dot{x}_\mu) - m] \right\}$ is the quark end point factor.

Using the momentum representation and "covariantizing" the closed path integral we have

$$G(p_1, \dots, p_n) = \int \mathcal{D}e(t) \mathcal{D}c(t) \prod_{j=1}^n \int_0^1 dt_j e(t_j) e^{i \sum_j p_j \cdot c(t_j)} \times J_{1/2}[c, e] W[c, e] \quad (11)$$

where e^2 is a one-dimensional metric on C (in the proper time gauge $\dot{e} = 0$, $\int_0^T e dt = \int_0^T d\tau$). Suppose now that the string ansatz (1) is valid for W , which can be conveniently written as

$$W[c, e] = \int \mathcal{D}g_{ab} \int \mathcal{D}x(z) \exp(-I[g, x]) \quad (12)$$

$(g_{tt}|_{\partial\Omega} = e^2(t)) \quad (x|_{\partial\Omega} = c(t))$

where $g_{tt} = g_{ab} \frac{dz^a}{dt} \frac{dz^b}{dt}$ and I is some effective action (e.g. the BDHP one). [It is interesting to note that the final result (11), (12) prompts the open-string analog of the off-shell-closed string amplitude definition (8) for the free BDHP string:

$\Gamma(p_1, \dots, p_n) = \langle \prod_{j=1}^n \int_{\partial\Omega} dz_j e(z_j) e^{i p_j \cdot x(z_j)} \rangle$, where $\langle \dots \rangle = \int \mathcal{D}e \int \mathcal{D}g \delta[g_{tt}|_{\partial\Omega} - e^2] \int \mathcal{D}x \delta[x|_{\partial\Omega} - c]$. It is this (z_j -reparametrization invariant) definition that should probably be used in the future studies of the BDHP string]. As for $J_{1/2}$, it can be expressed as follows (cf. [18])

$$J_{1/2}[c, e] = \int \mathcal{D}\pi_\mu(t) \int \mathcal{D}\psi(t) \mathcal{D}\bar{\psi}(t) \exp \left[i \int_0^1 dt \cdot \left\{ \pi_\mu \dot{c}_\mu + i \bar{\psi} \dot{\psi} - e(t) \bar{\psi} (\partial_\mu \pi_\mu - m) \psi \right\} \right], \quad (13)$$

or $J_0[c, e] = \exp \left\{ - \int_0^1 dt \left[\frac{1}{2} e^{-1} \dot{x}^2 + m^2 e \right] \right\}$ if we neglect the spins of quarks. The important consequence of (11) and (12) is that

we cannot explicitly integrate over $X_\mu(z)$ because of the non-trivial end point factor $J_{1/2}$ (or J_0). Really, if this factor was absent, we could rewrite two integrals $\int \mathcal{D}C \int \mathcal{D}x$ over the "boundary and interior" as one integral over the whole domain and then assume the Neumann boundary conditions on X_μ , providing the possibility to obtain the explicit expression for the amplitude analogous to those of sect.3 (with ξ_j playing the role of Koba-Nielsen variables). The conclusion is that contrary to the free string case (sect.3) here we first need to find $W[C]$ as a functional then integrate over C_μ and e . However, the expression for W is difficult to obtain even for the simplest BDHP string case (see sect. 2). In this situation some approximations are needed, for example, the semiclassical one for W ($W \sim \exp(-M^2 A_{min}[C])$) or the semiclassical approximation for the total (string + "ends") action. The second approach was already initiated in a number of papers [19, 20] where it was shown that "ends" are essential to obtain a reasonable spectrum of hadrons. However, these attempts are to be improved (if trying to go beyond the semiclassical approximation) by changing the Nambu action, e.g., by the BDHP (or some fermi string) one and also by using the proper quark end point term (13) instead of the "phenomenological" Bars-Hanson's one (used also in [20]), which actually not follows from QCD

$$I_{BH} = \int_0^T d\tau \left(\frac{i}{2} \bar{\Psi} \frac{\dot{c}^\mu}{\sqrt{\dot{c}^2}} \delta_\mu \vec{\partial}_\tau \Psi - m \bar{\Psi} \Psi \sqrt{\dot{c}^2} \right) \quad (14)$$

(note that (13) and (14) may again be considered as providing different quantum extensions of the same classical theory).

Finally let us illustrate our result for the amplitude (11) on the example of the 2-dimensional QCD, where the expression for W is explicitly known for the simple curve C [1,21]: $(g=g'N)$
 $W[C] = \exp(-g/2 \int_0^T d\tau \epsilon_{\mu\nu} c_\mu \dot{c}_\nu)$. Using the proper time gauge in (11) and assuming that quarks are spinless and have equal masses, we get the following expression for the $\bar{q}q$ - mesons off shell scattering amplitude

$$G(p_1, \dots, p_n) = N^{1-n/2} \int_0^T \frac{dT}{T} e^{-\frac{m^2 T}{2}} \prod_{j=1}^n d\tau_j \phi dC(\tau) \cdot \exp \left[- \int_0^T d\tau \left(\frac{1}{2} \dot{c}_\mu^2 + \frac{g}{2} \epsilon_{\mu\nu} c_\mu \dot{c}_\nu \right) + i \sum_{j=1}^n p_\mu^j c_\mu(\tau_j) \right] \quad (15)$$

Calculating the path integral we are left with

$$G(p_1, \dots, p_n) = N^{1-n/2} \delta^{(n)} \left(\sum_{j=1}^n p_j \right) \int_0^\infty \frac{ds}{s} e^{-\frac{m^2 s}{2}} \left[\frac{\delta/2}{\sin s} \right]^n$$

$$\prod_{j=1}^n \int_0^1 dt_j \exp \left\{ \frac{1}{4\alpha' \sin s} \sum_{\kappa, e} (p_\kappa^\mu p_e^\mu A_{\kappa e}(t) - \right. \\ \left. - \epsilon_{\mu\nu} p_\kappa^\mu p_e^\nu B_{\kappa e}(t) \right\}, \quad (16)$$

$$s = T\delta/2, \quad t_\kappa = \tau_\kappa/T, \quad \omega_{\kappa e} = s - \delta(t_\kappa - t_e),$$

$$A_{\kappa e} = \cos \omega_{\kappa e} \cdot \theta(t_\kappa - t_e) + \cos \omega_{e\kappa} \cdot \theta(t_e - t_\kappa),$$

$$B_{\kappa e} = \sin \omega_{\kappa e} \cdot \theta(t_e - t_\kappa) - \sin \omega_{e\kappa} \cdot \theta(t_e - t_\kappa).$$

This amplitude resembles the dual-like ones with "cos" and "sin" instead of "log's". The spectrum of mesons is given by the poles of the propagator (c.f. with the approach of ref. [22])

$$G(p_1, p_2) = \delta(p_1 + p_2) G(p_1),$$

$$G(p) = \int_0^\infty \frac{ds}{s} e^{-m^2 s/\alpha'} \left(\frac{\delta/2}{\sin s} \right) \int_0^1 d\xi \left(\int_0^{1-\xi} d\eta + \int_0^\xi d\eta \right) \times \\ \times \exp \left\{ \frac{p^2}{4\alpha'} [\operatorname{ctg} s (1 - \cos \delta\eta) - \sin \delta\eta] \right\} \equiv \sum_{n=0}^\infty \frac{a_n^2}{p^2 + m_n^2}.$$

It can probably be connected with the longitudinal spectrum of the string with masses at the ends [23] or the t'Hooft spectrum [1].

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