ONE-LOOP DIVERGENCES AND ß-FUNCTIONS IN SUPERGRAVITY THEORIES

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1. Introduction

Supergravity (SG) was invented with a hope to solve the problem of infinities in the Einstein theory. Now came the time of explicite calculations (of counter-terms, ß-functions, off shell and asymptotic behaviour) which are to reveal the structure and the status of quantum SG. Several new results on this way are the topic of this report.

Let us first remind a number of known facts about the infinities in ungauged O(N) SG's (for refs. see [1]): (1) N = 1,...,8 SG's are on-shell finite in L=1,2-order (L is the number of loops): L=1 - diagram calculations of infinities of the S-matrix elements; L=1,2 - general argument of the absence of an on-shell non-vanishing superinvariant; (2) $L \geqslant 3$: there exist superinvariants - candidates for on-shell divergences; (3) N-extended SG's are (off shell) finite (in d-dimensions) for $L < 2(N-1)/(d-2)$, e.g. the N=8, d=4 theory is infinite for $L \geqslant 7$ (some plausable argument based on supergraph power counting rules [2]); (4) N = 8 SG is divergent for $L \geqslant 3$ (implicite argument treating N=8 SG as a $\alpha' \to 0$, $d=10 \to 4$ limit of the superstring theory [3]). Thus different approaches seem to leave the only possibility for finiteness if the actual coefficient of an admissable superinvariant in the (e.g. L=3) infinities is zero. Two examples of such kind of "zeroes" were already found in SG at L=1 order: the absence of topological and gauge field action counter-terms for $N \geqslant 3$ and $N \geqslant 5$ respectively (cf. [1]). A new one - the absence of the off shell L=1 Weyl tensor squared type infinities in the N=8 and N=1, $d=10 \to 4$ theories - will be reported in sect.2, where we discuss the L=1 off shell infinities in gauged O(N) supergravities [4] . Here we make a conjecture that the N=8 SG may be L=1 off-shell finite (cf. [2]) which, if true, may imply an improvement of higher loop behaviour.

Suppose, however, that N=8 SG fails to be finite at 3-loop order. At least two possible modifications of the approach can then be suggested: (i) consider the N=8 SG to be only a low energy manifestation of some fundamental ultraviolet finite superstring theory in d=10 space-time (with six compact dimensions) [3] ; (ii) change the SG lagrangian by adding super-extensions of the curvature squared invariants in order to get a power counting renormalizable theory (just like it can be done already the Einstein theory, see e.g. [5]). It is the second secnnd possibility that we propose here (see sect.4). Sn.3 is devoted to the discussion of the one-loop ß-function [6] (Sect. 3) in conformal supergravities, i.e. the superextensions of the Weyl tensor squared invariant.

2. Off shell one-loop divergences in gauged O(N) supergravities [4]

In order to get a realistic theory one should consider the gauged version of the O(N)-Poincare supergravity (and also try to invent some viable mechanism for a spontaneous supersymmetry breaking). For example, the simplest gauged SG-theory-the O(2)-one-has the following lagrangian [7] $(\Lambda = -3m^2,\ m = 2g/\kappa)$

$$\mathcal{L}_2 = -\frac{1}{\kappa^2}(R-2\Lambda) + \frac{1}{4g^2}F^2_{\mu\nu} + \frac{1}{2}\epsilon^{\mu\nu\lambda\rho}\bar{\psi}^i_\mu\gamma_5\gamma_\nu\mathcal{D}_\lambda\psi^i_\rho + m\bar{\psi}^i_\mu\sigma_{\mu\nu}\psi^i_\nu + \frac{1}{2}m^{-1}\bar{\psi}^i_\mu F^{+ij}_{\mu\nu}\psi^j_\nu + \dots, \tag{1}$$

where $F^{ij}_{\mu\nu} = \varepsilon^{ij}F_{\mu\nu}$, $F^{+}_{\mu\nu} = F_{\mu\nu} + \gamma_5 F^{*}_{\mu\nu}$, i,j = 1,2 and g is dimensionless gauge coupling). This theory is one-loop on-shell renormalizable and one can ask about the value of the ß-function for g. In the first calculation [8] done in the background gravitational sector $\beta(g)$ was implicitly obtained by establishing the Λ-term renormalization and then using the relation $\Lambda\kappa^2 = -12g^2$. The results

$$\begin{array}{cccccccc} N: & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ \beta: & -\frac{26}{3} & -5 & -2 & 0 & 0 & 0 & 0 \end{array} \tag{2}$$

were then rederived by a "heuristic" calculation in the background gauge field sector[9]. The reasoning of [9] contained a number of unjustified assumptions like the validity of the formula $\beta_0(s) = -4\left(\frac{1}{12} - s^2\right)(-1)^{2s}C_2$ (for the cnntribution of spin s field in the gauge field ß-function) for the gravitino ($s = 3/2$) and the possibility to obtain the total result by simple summation of contributions of all spins of the SG multiplet.

To provide understanding of the agreement of these two calculations of $\beta(g)$ one should study the off shell divergences in the combined gravitational-gauge field background sector of the effective action ($g_{\mu\nu} \neq \delta_{\mu\nu}$, $A_\mu \neq 0$, $\psi_\mu = 0$). The one-loop divergences for various fields can be evaluated using the formula

$$(\log \det \Delta)_\infty = \frac{1}{(4\pi)^2} \int d^4x \sqrt{g} \left(\bar{b}_0 L^4 - \frac{1}{2} \bar{b}_2 L^2 - \bar{b}_4 \log \frac{L^2}{\mu^2} \right), \quad (3)$$
$$L \to \infty ,$$

where $\Delta = -\mathcal{D}^2 + X$ and

$$\bar{b}_0 = \nu , \quad \bar{b}_2 = \rho_1 R + \rho_2 \Lambda + \rho_0 \varkappa F_{\mu\nu}^2 , \quad \varkappa \equiv \frac{\kappa^2}{g^2} ,$$
$$\bar{b}_4 = \beta_1 R^*R^* + \beta_2 W + \frac{1}{3}\beta_3 R^2 + \beta_4 R\Lambda + \beta_5 \Lambda^2 + \quad (4)$$
$$+ \beta_6 \mathcal{D}^2 R + \gamma_1 \varkappa R_{\mu\nu} T_{\mu\nu} + \gamma_2 \varkappa^2 T_{\mu\nu}^2 + \gamma_3 \varkappa (\mathcal{D}_\mu F_{\mu\nu})^2 +$$
$$+ \beta_0 F_{\mu\nu}^2 , \qquad W = R_{\mu\nu}^2 - \frac{1}{3} R^2 , \; T_{\mu\nu} = F_{\mu\lambda} F_{\nu\lambda} - \frac{1}{4} g_{\mu\nu} F^2 .$$

The central point is establishing the gravitino contribution (in the standart background gauge $\gamma_\mu \psi_\mu^i = \xi_\mu^i(x)$)

$$Z_\psi = (\det \Delta_{3/2})^{1/2} (\det \Delta_{gh})^{-3/2} , \quad \Delta_{gh}^{ij} = \hat{\mathcal{D}}^{ij} + 2m\delta^{ij} ,$$
$$\Delta_{3/2\,\mu\nu}^{\;ij} = \Sigma^\lambda_{\mu\rho} (\mathcal{D}_\lambda)^{ij}_{\rho\nu} - m\delta^{ij} g_{\mu\nu} + m^{-1} F^{+\,ij}_{\mu\nu} , \quad \Sigma^\lambda_{\mu\nu} = -\frac{1}{2}\gamma_\nu \gamma^\lambda \gamma_\mu .$$

"Squaring" the $\Delta_{3/2}$-operator we get <u>additional</u> F^2-infinities due to mixing of the "mass" and the non-minimal coupling terms. We found that it is this mixing that is essential for the correctness of the $\beta_0(s)$ -formula for $s = 3/2$ used in [9] . The results for different spins contributions in the gravitational and gauge infinities are the following (we also utilize the old off-shell results for the Einstein-Maxwell system [10]):

s	ν	β_0	β_1	β_2	β_3	β_4	β_5	β_6	ρ_1	ρ_2
2	2	$-$	$\frac{53}{45}$	$\frac{7}{10}$	$\frac{3}{4}$	$-\frac{26}{3}$	20	$-\frac{19}{15}$	$-\frac{23}{3}$	20
$\frac{3}{2}$	-2	$-\frac{13}{3}$	$-\frac{233}{720}$	$-\frac{77}{60}$	$-\frac{1}{3}$	$-\frac{4}{9}$	$\frac{44}{9}$	$\frac{1}{60}$	$\frac{1}{6}$	$\frac{16}{3}$
1	2	$\frac{11}{3}$	$-\frac{13}{180}$	$\frac{1}{5}$	0	0	0	$-\frac{1}{10}$	$-\frac{2}{3}$	0
$\frac{1}{2}$	-2	$-\frac{2}{3}$	$\frac{7}{720}$	$\frac{1}{20}$	0	0	0	$\frac{1}{60}$	$\frac{1}{6}$	0
0	1	$-\frac{1}{6}$	$\frac{1}{180}$	$\frac{1}{60}$	$\frac{1}{24}$	$-\frac{1}{9}$	$\frac{2}{9}$	$\frac{1}{30}$	$\frac{1}{6}$	$-\frac{2}{3}$

The important fact is the <u>negative</u> "sign" of the gravitino contribution (β_2) in the "Weyl" infinities, which can be contrasted to the positive ones for S = 0,1/2, 1 fields. It should be stressed that the statement (cf. [10]) about " $\beta_2 > 0$ for any spin" is not actually applicable for S = 2 and 3/2 (in the background gauges). One should take into account that here β_2 is gauge dependent (cf. [11] for S = 2 case and note that the "one-loop" gravitino lagrangian is superinvariant only if the background space is the Einstein space).

The final expression for the one-loop off shell infinities in the gauged O(N) supergravity can be written in the form

$$\bar{b}_0 = 0\,, \quad \bar{b}_2 = \rho_1' \bar{R} + \rho_2' \Lambda + 4\kappa^2 \rho_0 \mathcal{L}_2\,,$$
$$\bar{b}_4 = \beta_1 R^* R^* + \beta_2 \left(\bar{R}_{\mu\nu}^2 - \tfrac{1}{3} \bar{R}^2 \right) + \tfrac{1}{3}\beta_3 \bar{R}^2 + \beta_6 \partial^2 R + \qquad (5)$$
$$+ \gamma_3 \varkappa (\partial_\mu F_{\mu\nu})^2 + \alpha_1 \varkappa \bar{R} + \alpha_2 \varkappa T_{\mu\nu} \bar{R}_{\mu\nu} + 4g^2 \beta_0 \mathcal{L}_2\,,$$
$$\mathcal{L}_2 = -\tfrac{1}{\kappa^2}(R - 2\Lambda) + \tfrac{1}{4g^2} F_{\mu\nu}^2\,, \quad \bar{R}_{\mu\nu} = R_{\mu\nu} - \Lambda g_{\mu\nu} - \tfrac{\kappa^2}{2g^2} T_{\mu\nu}\,, \quad \bar{R} = R - 4\Lambda\,,$$

, where the coefficients are given in the table

N	0	1	2	3	4	5	6	8	$N=1$, $d=10$
β_0	–	–	$-\frac{26}{3}$	-5	-2	0	0	0	–
β_1	$\frac{53}{45}$	$41/48$	$11/24$	0	0	0	0	0	0
β_2	$\frac{7}{10}$	$-7/12$	$-5/3$	$-5/2$	-3	-3	-2	0	0
β_3	$\frac{3}{4}$	$5/12$	$1/12$	$-1/4$	$-1/2$	$-1/2$	0	1	$13/12$
β_6	$-19/15$	$-5/4$	$-4/3$	$-3/2$	$-21/15$	$-1/3$	$11/3$	$26/3$	$22/5$
γ_3	–	–	$1/3$	$1/2$	$2/3$	$11/12$	$4/3$	$13/6$	–
α_1	–	40	64	88	114	146	190	178	–
α_2	–	–	-2	$-5/2$	$-17/6$	$-7/2$	-6	-5	-3
ρ_1'	$-23/3$	$-15/2$	-8	-9	-10	-10	-8	-4	$-31/3$
ρ_2'	$-32/3$	$-14/3$	$-4/3$	0	0	0	0	0	0
ρ_0	–	–	0	0	0	0	0	0	–

Several conclusions follow from these results:

(1) we explicitly demonstrate that <u>gauged</u> SG's are on-shell (L=1) renormalizable up to topological infinity (i.e. after the use of

$$R_{\mu\nu} - \Lambda g_{\mu\nu} - \frac{\kappa^2}{2g^2} T_{\mu\nu} = 0\,, \quad R - 4\Lambda = 0, \quad \partial_\mu F_{\mu\nu} = 0);$$

(2) the on shell renormalizations of Λ and g^2 are given by the <u>same</u> coefficient β_0 , explaining the agreement of the results of refs.[8] and [9] ;
(3) there is no on-shell quadratic divergences in all theories with $N \geqslant 3$;
(4) N = 8 SG and the theory, obtained by a reduction of the N=1, d=10 SG (the last column of the table) are off shell finite in the gravitational 2^+-sector (have $\beta_2 = 0$). Thus the N=8 SG is distinguished by having a maximal degree of the off shell finiteness: $\beta_0 = \beta_1 = \beta_2 = 0$. One may even conjecture that it is completely off shell finite (for L=1) when treated in a suitable background supergauge where $\alpha_1 = \alpha_2 = 0$ (it may turn out that also $\beta_3 = \gamma_3 = 0$ in this gauge if the N=8 superextension of R^2 does not exist).

3. One-loop ß-function in pure conformal supergravities [6]

Conformal supergravities are $U(N)$ ($N = 1, \ldots, 4$) superconformal extensions of the Weyl invariant $W = R_{\mu\nu}^2 - \frac{1}{3} R^2$. The lagrangian has the following structure [12]

$$\mathcal{L}_N = \alpha^{-2} (W)_{ss} = \alpha^{-2} \Big[W - \frac{4-N}{4N} F_{\mu\nu}^2 (A) - F_{\mu\nu}^{ij\,2} (V) + \\ + 4 \epsilon^{\mu\nu\rho\sigma} \bar{\phi}_\rho^i \gamma_5 \gamma_\sigma \mathcal{D}_\nu^+ \phi_\mu^i + \ldots \Big] , \tag{6}$$

where $\phi_\mu = \frac{1}{3} \gamma^\nu (\mathcal{D}_\nu \psi_\mu - \mathcal{D}_\mu \psi_\nu + \frac{1}{2} \gamma_5 \epsilon_{\mu\nu\rho\sigma} \mathcal{D}_\sigma \psi_\rho)$, $\mathcal{D}^+ = \partial + A + V + \ldots$, while A_μ and V_μ are $U(1)$ and $SU(N)$ gauge fields and ψ_μ^i is "conformal gravitino". Why these theories are interesting: (1) they are gauge theories with the maximal known group -superconformal group, including the ordinary and conformal supersymmetries, scale and chiral $U(N)$ transformations; 2) they are power counting renormalizable due to higher derivatives in the kinetic terms, $L_{linear} = h \Box^2 h + \bar{\psi} \partial^3 \psi + A \Box A + \ldots$ (Note that for the correct counting of degrees of freedom one should properly account for the "averaging over gauges" operators, e.g. for the N = 1 theory we have $\nu_{tot} = (6)_h + (-8)_\psi + (2)_A = 0$); 3) N = 1,2,3-theories are asymptotically free, while the N=4 theory is finite (in one-loop). More explicitly, one can obtain the following results for the ß-function for the dimensionless coupling α :

$$N = 1 \quad : \quad \beta_I = \beta_g + \beta_\psi + \beta_A = \frac{17}{2} \quad , \tag{7}$$

where $\beta_g = \frac{199}{15}$ is the value for the pure Weyl theory [5], $\beta_\psi = -149/30$ is the conformal gravitino W-infinities in the gravitational sector established in [6] with the use of the algorithms for the divergences of the 4-th and 3-d order differential operators and $\beta_A = 1/5$ is the axial vector field contribution.

N=2: $\beta_{\underline{II}} = \beta_g + 2\beta_\psi + \beta_A + \beta_V($ SU_2-gauge field V_μ^{ij} $) + \beta_\chi($2 spinors χ^i $) + \beta_T($1 antisymmetric tensor field $T_{\mu\nu}^{ij}$, $\mathcal{L}_T \sim T \Box T$ $) = \frac{13}{3}$;

N=3: $\beta_{\underline{III}} = \beta_g + 3\beta_\psi + \beta_A + \beta_V(SU_3) + \beta_\chi(9\chi^{ij}) + \beta_E($3 complex scalars E_i $) + \beta_T(3T) + \beta_\Lambda($1 spinor Λ, $\mathcal{L}_\Lambda \sim \bar{\Lambda}\hat{\partial}^3\Lambda$ $) = 1$;

N = 4:

$$\beta_{\underline{IV}} = \beta_g + 4\beta_\psi + \beta_V(SU_4) + \beta_\chi(20\chi^{ij}_k) + \beta_E(10E_{(ij)}) + \beta_T(6T) + \beta_\Lambda(4\Lambda_i) + \beta_C(1 \text{ complex scalar } C,\ \mathcal{L}_C \sim C^*\Box^2 C) = 0 .$$

(where we assumed an appropriate gravitational coupling of the scalar C). Interpreting $-\alpha^2$ as the $U(N)$ gauge coupling, we get the following sequence (cf. with the O(N) Poincare SG sequence (2))

$$\begin{array}{lcccccc} N & : & 1 & 2 & 3 & 4 \\ \beta(-\alpha^2) & : & -\frac{17}{2} & -\frac{13}{3} & -1 & 0 \end{array} \qquad (8)$$

However, the conformal supergravities lack a low energy correspondence with the Einstein theory. That is why in order to get a viable theory one should add also the ordinary (linear in curvature) supergravity term in the lagrangian.

4. Renormalizable supergravity models

Let us consider the following lagrangian

$$\mathcal{L} = -\frac{1}{k^2}(R)_{ss} + \frac{1}{\alpha^2}(W)_{ss} - \frac{1}{3\sigma^2}(R^2)_{ss} , \qquad (9)$$

where the brackets denote the corresponding superextensions. This theory is renormalizable, possesses the correct Einstein limit but lacks perturbative unitarity due to the presence of ghosts. However, ghosts here fill a supermultiplet and thus may decouple in some non-perturbative way. The physical spectrum contains gauge fields of Poincare SG, while $U(N)$ gauge fields of conformal SG are in fact auxiliaries for Poincare SG. Some generalization of (9) probably exists for N=4, ..., 8 with higher spin fields being auxiliary (propagating) in P_0incare (conformal) SG-parts.

One can prove that the inclusion of the $(R+...)$-term in the conformal SG lagrangian increases the value of the Weyl coupling $\beta(\alpha)$-function. Thus we get the asymptotically free behaviour for α also for N=4 theory and may hope for $\beta(\alpha)=0$ for some N > 4. In turn, the addition of conformal SG term to the P_0incare one (1) changes the renormalization of the physical gauge field coupling g: all negative gravitino and matter fields conributions in $\beta(g)$ are suppressed due to higher derivative terms in the conformal SG part. Therefore the value of $\beta(g)$ is the same as for the free O(N) gauge field in the flat space-time, i.e. corresponds to the asymptotic freedom in contrast to the non-asymptotically free results in pure Poincare SG case (2). Now let us mention a possibility that a superextension of the R^2-term may not exist for some N > 2. Then the superconformal theory

$$\mathcal{L}=\left(-\frac{1}{12}R\phi^2-\frac{1}{2}\partial_\mu\phi\partial_\mu\phi+\lambda\phi^4+\frac{1}{\alpha^2}W\right)_{ss} \qquad (10)$$

is an attractive candidate for a fundamental theory if it has $\beta(\alpha)=0$ (no superconformal anomalies and possible solution of the problem of ghosts). The presence of conformal supergravity term may also help to provide a spontaneous supersymmetry breaking. The important fact is that one can add some matter multiplets to (9) or (10) without destroing renormalizability. Obtaining in this way a sufficient spectrum of particles (or taking in account that additional particles may appear as monopoles after a spontaneous supersymmetry breaking) we get a power counting renormalizable asymptotically free (or finite) unified theory.

In conclusion we want to point out that the renormalizable supergravity (9) can be considered as an "induced supergravity" theory. Suppose we start with the lagrangian

$$\mathcal{L}=\left(\frac{1}{4g^2}F_{\mu\nu}^2+i\bar{\psi}\hat{\mathcal{D}}\psi\right)_{sc}+\frac{1}{\alpha^2}\left(W\right)_{ss}, \qquad (11)$$

containing massless gauge and spinor matter fields interacting with the external conformal supergravity fields and also the pure conformal SG part. Assuming that the regularization breaks the conformal symmetry but preserves the general covariance and local supersymmetry, we get (according to the ideas of "induced gravity" approach

[13]) the following effective lagrangian

$$\mathcal{L} = -\frac{1}{k^2_{ind}}\left(R - 2\Lambda_{ind}\right)_{ss} - \frac{1}{36^2_{ind}}\left(R^2\right)_{ss} + \\ + \frac{1}{\alpha^2_{ren}}\left(W\right)_{ss} + \dots \quad , \tag{12}$$

where k_{ind} , Λ_{ind} and 6_{ind} are finite calculable constants. One can probably induce the P_0incare SG term in (12) even without matter terms in (11) (i.e. starting only with conformal SG term). Thus the conformal supergravity itself may be a true theory on some fundamental level.

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