DYON BLACK HOLE IN THE TOMIMATSU-SATO-YAMAZAKI SPACE-TIME

Masahiro Kasuya
Institute for Nuclear Study, University of Tokyo,
Midori-cho, Tanashi, Tokyo 188, Japan

The symmetry between electric and magnetic charge which is inherent in Maxwell's equations does not seem to be realized in nature. Dirac pointed out, however, that quantum mechanics does not preclude the existence of magnetic monopoles [1], and Schwinger proposed the dyon (a pole possessing both electric and magnetic charges) [2]. This dyon lies in the Abelian theory U(1).

On the other hand, in non-Abelian theory, 't Hooft and Polyakov obtained spherically symmetric classical monopole solution of the SO(3) Yang-Mills theory coupled with a triplet Higgs field [3]. Shortly afterwards, Julia and Zee showed that the same theory also admitted dyon [4]. The magnetic monopoles and the dyons may play an important role in grand unified theories.

A solution of the Einstein-Maxwell equation in the Kerr space-time was obtained by Newman et al. [5]. This solution represents a rotating mass and electric charge. Tomimatsu-Sato [6] discovered the series of solutions for the gravitational field of a rotating mass, following Ernst's formulation of axisymmetric stationary fields [7]. Furthermore Yamazaki obtained the charged Kerr-Tomimatsu-Sato family of solutions with arbitrary integer distortion parameter for gravitational fields of rotating masses [8].

We present an exact stationary rotating dyon solution in the Tomimatsu-Sato-Yamazaki space-time [9]. Our solution is characterized by five parameters (mass M, angular momentum J, electric charge Q, magnetic charge Φ, and distortion parameter δ). Let us start with the following Lagrangian density, which describes the electromagnetic field induced by an Abelian dyon in curved space-time ($\hbar$=c=1)

$$L = \sqrt{-g}\ (-\frac{1}{16\pi G}R - \frac{1}{4}g^{\mu\rho}g^{\nu\sigma}F_{\mu\nu}F_{\rho\sigma})\ , \qquad (1)$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + {}^*G_{\mu\nu}$ and ${}^*G_{\mu\nu}$ is the Dirac string term. We can express the line element of stationary and axisymmetric space-time in the form

$$ds^2 = f^{-1}[e^{2\gamma}(d\rho^2+dz^2)+\rho^2 d\varphi^2]-f(dt-\omega d\varphi)^2\ , \qquad (2)$$

where f, ω and γ are functions of ρ and z only.

Our exact dyon solution for (1) and (2) can be expressed as follows The metric functions are

$$f = \mathcal{A}_\delta / \mathcal{B}_\delta \ ,$$
$$\omega = 2GMq(1-y^2)\mathcal{C}_\delta / \mathcal{A}_\delta \ , \qquad (3)$$
$$e^{2\gamma} = \mathcal{A}_\delta / [p^{2\delta}(x^2-y^2)^{\delta^2}] \ .$$

We shall follow the notation of Ref. [9].
It is worth while noting that γ has the same form as in vacuum field. Here the relations among the Weyl coordinates (t,ρ,z,φ), the Boyer-Lindquist coordinates (t,r,θ,φ), and the prolate spheroidal coordinates (t,x,y,φ) are

$$\rho = K(x^2-1)^{1/2}(1-y^2)^{1/2} \ , \quad z = Kxy \ ,$$
$$r = Kx + GM \ , \quad \cos\theta = y \ ,$$
$$(GMp\sigma)^2 = (K\delta)^2 = (GM)^2 - a^2 - \frac{G}{4\pi}(Q^2+\Phi^2) \ , \qquad (4)$$
$$a = J/M = GMq\sigma \ ,$$
$$p^2+q^2 = 1 \ , \quad \sigma^2+|\tau|^2 = 1 \ , \quad \tau = (4\pi G)^{1/2}\tau_0 \ .$$

Next the electromagnetic potentials are

$$4\pi GM\sigma^2\mathcal{B}_\delta A_0 = Q(\sigma\eta+\zeta)+\Phi\sigma\kappa \ , \quad A_1=A_2=0 \ ,$$
$$A_3 = \omega A_0 + \frac{K}{8\pi GM\sigma^2}[Q(\sigma\hat{U}+\hat{V}-\frac{\delta}{pq})-\Phi\sigma(\hat{w}-\frac{2\delta}{p})] \ , \qquad (5)$$

with the dyon solution ξ of the Ernst equation given by

$$\sigma^{-1}\xi = \frac{\eta + i\kappa}{\zeta} = \frac{\mu+\nu}{\mu-\nu} \ , \qquad (6)$$

ζ, η and κ being real functions, μ and ν being complex functions of x, y, p and q, and taking $4\pi GM\tau_0 = Q+i\Phi$. The presence of a Dirac string in our solution may be seen from A_3 in (5). In fact the monopole term in A_3 does not vanish on the negative semi-infinite line of the symmetry axis. Our dyon solution reduces to the magnetic monopole solution in the case Q = 0.

On the other hand, the non-Abelian dyon solution can be obtained from the Abelian dyon solution and vice versa using the extended Arafune-Freund-Goebel singular gauge transformation [9],[10],[11].

Let us discuss the space-time structure of our dyon solution. The metric functions $\mathcal{A}_\delta$ and $\mathcal{B}_\delta$ in (3) are written in the form, using (6), [12]

$$\mathcal{A}_\delta = (\mu\nu^* + \mu^*\nu)/2 , \qquad (7)$$
$$\mathcal{B}_\delta = [(\sigma+1)^2\mu\mu^* + (\sigma^2-1)(\mu\nu^*+\mu^*\nu) + (\sigma-1)^2\nu\nu^*]/(2\sigma)^2 .$$

The ergosurfaces are obtained by taking $\mathcal{A}_\delta = 0$. On the equatorial plane (y=0) the complex functions μ and ν in (6) become real $\mathcal{M}$ and $\mathcal{N}$. The metric functions $\mathcal{A}_\delta$ and $\mathcal{B}_\delta$ are then of the form

$$\mathcal{A}_\delta(y=0) = \mathcal{M}(x,p)\mathcal{N}(x,p) ,$$
$$\mathcal{B}_\delta(y=0) = [\{(\sigma+1)\mathcal{M}(x,p) + (\sigma-1)\mathcal{N}(x,p)\}/(2\sigma)]^2 ,$$
$$\mathcal{N}(x,p) = (-1)^\delta \mathcal{M}(-x,p) .$$

The position of ergosurfaces on the equatorial plane is determined by $\mathcal{A}_\delta(y=0) = 0$, and there exist ring singularities determined by $\mathcal{B}_\delta(y=0) = 0$, i.e., $(\sigma+1)\mathcal{M} + (\sigma-1)\mathcal{N} = 0$, on the equatorial plane. The metric g_{11} becomes infinity at $x = \pm 1$. We find that the number of ergosurfaces is δ for $x>1$ and also δ for $x<-1$, and that the number of ring singularities is $[\delta/2]$ for $x>1$ and $\delta-[\delta/2]$ for $x<-1$.

We obtain the proper area A of the surface x=1 in the Tomimatsu-Sato-Yamazaki space-time for our dyon

$$A = \frac{4\pi}{\delta}(GM)^2(1+2p\sigma+\sigma^2)\left(\frac{q}{p}\right)^{\delta-1}\sqrt{(-1)^{\delta^2-1}} . \qquad (8)$$

Therefore our dyon solution has an event horizon for arbitrary odd integer δ; there exists an event horizon at x=1, i.e.,

$$r = GM + \frac{1}{\delta}\left[(GM)^2 - a^2 - \frac{G}{4\pi}(Q^2+\Phi^2)\right]^{1/2} .$$

Thus, for arbitrary odd integer δ, our dyon solution represents a black hole with four hairs provided that

$$(GM)^2 \geq a^2 + \frac{G}{4\pi}(Q^2+\Phi^2) .$$

The special case $\delta=1$ is the Kerr-Newman case of our dyon. Furthermore it can be easily seen that the series of Weyl solutions (a=q=0) has no event horizons except for the case $\delta=1$, i.e. the Schwarzschild and Reissner-Nordström solutions.

We have presented an exact dyon solution for which the space-time metric takes the Tomimatsu-Sato-Yamazaki form. Our solution has five parameters, i.e., the mass M of the dyon, the angular momentum J, the electric charge Q, the magnetic charge Φ, and the distortion parameter δ, and reduces to the rotating monopole solution in the case Q=0. Using our extended Arafune-Freund-Goebel singular gauge transformation, the non-Abelian dyon solution may be obtained from the Abelian dyon solution, and vice versa. Therefore an observer at large distances cannot distinguish between the Abelian dyon and the non-Abelian dyon. In the Abelian dyon we can treat Q and Φ as independent. However, this is not the case for the non-Abelian dyon, and the finite energy Prasad-Sommerfield solution [13] in flat space-time is known. Finally there is the problem of naked ring singularities of the fields with $\delta = 3,5,7,\cdots$. This remains an open problem since Einstein's general relativity is not applicable to the ring singularities.

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