GLUON CONDENSATION AND CONFINEMENT OF QUARKS

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The purpose of my talk is to argue that if gluon condenses in the form of color singlet, then the dual variable becomes a good coordinate in the low energy region and acquires a mass gap.

The vacuum of Quantum Chromodynamics (QCD, quarks are neglected for the moment) will be filled with nontrivial configuration of gluonic fields $A_\mu^a(x)$: they condense in the vacuum in the form of color singlet and of Lorentz scalar. We have to take into account the effect of the gluon condensation when we study the vacuum state and the excitation spectra.

Any gluonic operator can be used for the discussion of the gluon condensation as long as it has correct quantum numbers: color singlet and $J^{PC}=0^{++}$. We can use $G_{\mu\nu}^2(x)$, $A_\mu^a(x)A^{a\mu}(x)$ etc. Here $G_{\mu\nu}^a$ is the covariant field strength. The gauge has to be fixed of course in the case $A_\mu^a(x)A^{a\mu}(x)$. The most convenient operator should be picked up according to the following criteria, 1) The condensation of the selected operator is easy to discuss. 2) In case it condenses the effect on the excitation spectra is clearly seen. The operator $G_{\mu\nu}^2$ has been studied but it lacks the second property. The operator chosen here is the zero momentum mode $A_\mu^{a(0)}$ of $A_\mu^a(x)$ in the axial gauge $A_3^a(x)=0$,

$$A_\mu^a(x) = A_\mu^{a(0)} + A_\mu^{a'}(x) , \qquad \int A_\mu^{a'}(x)d^4x = 0 . \tag{1}$$

Although $\langle A_\mu^{a(0)}\rangle=0$ we can assume $\langle A_\mu^{a(0)}A^{a(0)\mu}\rangle\neq 0$. The residual gauge symmetry in $A_3^a(x)=0$ gauge is fixed by specifying the prescription to avoid the singularity of the gluon propagator $\langle A_\mu^a A_\nu^b\rangle_p$ at $p_3=0$ in momentum space. The conventional one is the principal part prescription. We assume for the moment the condensation of $A_\mu^{a(0)}$ in the above sense. At the end of the talk the condensation is discussed.

In case $A_\mu^{a(0)}$ condenses, the excitation spectra are determined by substituting (1) in the Lagrangian,

$$\int \mathcal{L}(A_\mu^a)\, d^4x = -\frac{1}{4}\, G_{\mu\nu}^{(0)2}\Omega$$

$$- \frac{1}{2} \iint A^{\mu a'}(x) M^{ab}_{\mu\nu}(x-y) A^{\nu b'}(y) d^4x d^4y$$

$$+ (A')^3 \text{ term} + (A')^4 \text{ term} , \qquad (2)$$

where $\Omega=\int d^4x$. The matrix $M^{ab}_{\mu\nu}(p)$ is not a positive definite matrix so that the squared mass matrix $M^{ab}_{\mu\nu}(p=0)$ may have negative (i.e. tachyonic) eigenvalues. We have seen that this is indeed the case for SU(2) color group. For SU(3) the same phenomenon occurs since SU(2) is a subgroup of SU(3). Therefore $A^{a'}_{\mu}$ is not a stable coordinate: we have to condense above tachyonic unstable modes to reach the really stable vacuum. The situation is similar to that discovered by Nielsen and Oleson[1]. Instead of condensing unstable modes we make a dual transformation and find that the unstable modes are absent if the theory is written in terms of the dual potential. Moreover the condensation of $A^{a(0)}_{\mu}$ yields a positive definite squared mass matrix to the dual potential.

The dual formalism in the axial gauge has been given by Halpern[2] with the result

$$\int [dA_\mu] \exp\{- \frac{i}{4} \int (\partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu)^2 d^4x\} \qquad (3)$$

$$= \int [dG_{\mu\nu}] \delta(D_\mu \tilde{G}^{\mu\alpha}) \exp\{- \frac{i}{4} \int G^2_{\mu\nu} d^4x\} \qquad (4)$$

$$= \int [dG_{\mu\nu}][dB^\nu] \delta(n_\mu B^\mu) \exp i\int (B^a_\nu D_\mu \tilde{G}^{\mu\nu} - \frac{1}{4} G^2_{\mu\nu}) d^4x \qquad (5)$$

$$= \int [dB^\alpha] \exp i\int \mathcal{L}(B) d^4x . \qquad (6)$$

In (4) $D^{ab}_\mu = \delta^{ab}\partial_\mu + g f^{acb} A^c_\mu$ with $A^c_\mu(x) = \frac{1}{\partial_3} G^c_{3\mu}$ and $n_\mu=(0,0,0,1)$. In the following indices $\alpha,\beta,\gamma\cdots$ take 0,1,2, while $\mu,\nu,\rho,\sigma\cdots$ take 0,1,2,3. The Lagrangian for the dual potential B^α is written symbolically as[2]

$$\mathcal{L}(B) = - \frac{1}{4} \widetilde{(\partial_\mu B^a_\nu - \partial_\nu B^a_\mu)} N^{-1\ ab}_{\mu\nu,\rho\sigma} \widetilde{(\partial_\rho B^b_\sigma - \partial_\sigma B^b_\rho)} \qquad (7)$$

where the tilde indicates the dual tensor as $\tilde{F}_{\mu\nu} \equiv \frac{1}{2}\varepsilon_{\mu\nu\rho\sigma} F^{\rho\sigma}$, $B^a_3=0$ and $N^{ab}_{\mu\nu,\rho\sigma} = [1+g\int^{x_3} dx'_3 B]^{ab}_{\mu\nu,\rho\sigma}$.

In the representation (5) we see that the theory is invariant under $G^a_{\mu\nu} \to G^a_{\mu\nu}$ and

$$B^a_\mu \to B^a_\mu + D^{ab}_\mu \Lambda^b + g f^{abc} \int^{x_3} G^b_{3\mu} \Lambda^c \, dx'_3$$

with $\Lambda^a(x)$ arbitrary function. This is due to the following identity

$$G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + gf^{abc}A^b_\mu A^c_\nu + \varepsilon_{3\gamma\mu\nu}J^{a\gamma} ,$$

where $A^a_\mu = \frac{1}{\partial_3} G^a_{3\mu}$ and $J^{a\gamma} = \int^{x_3} D^{ab}_\mu \tilde{G}^{b\mu\gamma} dx'_3$. The Ward identities are derived from the above transformation and we see, by the similar procedure as in the conventional case, that the dual potential is massless to any finite order of perturbation: writing the propagator as

$$i\langle B^a_\mu B^b_\nu \rangle_p = \frac{\delta^{ab} g_{\mu\nu}}{p^2_\mu + \Pi(p^2_\mu, (n_\mu p^\mu)^2)} + \text{gauge term} ,$$

we can show that Π behaves near $p^2_\mu = 0$ as Ap^2_μ with some constant A. We have also checked explicitly in the form of (6) that the one loop correction produces the correct amount of logarithmic divergence: $A \sim \frac{11}{3} g^2 C_2(G) \ln\Lambda$ which corresponds to the β-function behaving as $\beta(g) \underset{g\sim 0}{\sim} -\frac{11}{3} g^3 C_2(G)$ where $f^{abc} f^{dbc} = \delta^{ad} C_2(G)$.

Now in the presense of the condensation $A^{a(0)}_\mu$, we separate it as in (1) and from the variable $A^{a'}_\mu$ we switch to the dual variable. Since $G^a_{3\mu} = \partial_3 A^a_\mu = \partial_3 A^{a'}_\mu$, $A^{a'}_\mu(p) = iP \frac{1}{p_3} G^a_{3\mu}(p)$ where P denotes the principal part. Writing $[dA_\mu] = [dA^{(0)}_\mu][dA'_\mu]$ and following the same procedure as above we arrive at

$$(3) = \int [dA^{(0)}][dB^\alpha] \exp i \int \mathcal{L}(A^{(0)}_\mu, B_\alpha)$$

where

$$\mathcal{L}(A^{(0)}_\mu, B_\alpha) = -\frac{1}{4}\widetilde{(D^{(0)}_\mu B_\nu - D^{(0)}_\nu B_\mu)}^a N^{-1\ ab}_{\mu\nu,\rho\sigma} \widetilde{(D^{(0)}_\rho B_\sigma - D^{(0)}_\sigma B_\rho)}^b , \tag{8}$$

$$D^{(0)ab}_\mu \equiv \delta^{ab}\partial_\mu + gf^{acb} A^{c(0)}_\mu .$$

Expanding $\int \mathcal{L}(A^{(0)}_\mu, B^\alpha) d^4x$ in the form

$$\frac{1}{2}\int B^{a\alpha}(p) \mathcal{M}^{ab}_{\alpha\beta}(A^{(0)}, p) B^{b\beta}(-p) \frac{d^4p}{(2\pi)^4} + B^3 \text{ term} + \cdots , \tag{9}$$

we define the mass of B by the term

$$\frac{1}{2} B^{a\alpha}(0) \mathcal{M}^{ab}_{\alpha\beta}(A^{(0)}, 0) B^{b\beta}(0) = \frac{1}{2}(\varepsilon^{\alpha\beta\gamma} gf^{abc} A^{c(0)}_\beta B^b_\gamma(0))^2 .$$

Since it is written as a square, we have a positive semi-definite squared mass matrix. The dual potential is a stable coordinate: unstable modes which are present in the spectrum of $A^{a'}_\mu$ field are eliminated by the dual transformation.

Once the dual potential aquires mass gap, the system is in the magnetic Higgs phase — dual to the usual Higgs phase. The dual loop introduced by 't Hooft[3] shows the perimeter law and we expect the area law for the Wilson loop and the color electric flux tube will be formed as a dual Meissner effect. We do not discuss this scenario here.

The solution to the U(1) problem is also provided by the above mechanism. From (5) we see that B^{ai} and $\varepsilon_{ii'}G^a_{3i'}$ are canonically conjugate pairs (Here i,i'=1 or 2, $\varepsilon_{12}=-\varepsilon_{21}=1$, $\varepsilon_{11}=\varepsilon_{22}=0$) and by (9), propagators take the form in the low energy region

$$i\langle B^a_\alpha B^b_\beta\rangle_p = i(1/m^2)\langle G^a_{3\alpha}G^b_{3\beta}\rangle_p = \delta^{ab}g_{\alpha\beta}(1/p^2_\mu - m^2) \ . \qquad (10)$$

Consider $K^\mu=(g^2/16\pi^2)\varepsilon^{\mu\nu\rho\sigma}A^a_\nu(\partial_\rho A^a_\sigma+\frac{g}{3}\ f^{abc}A^b_\rho A^c_\sigma)$. We have to explain why $\partial_\mu K^\mu$ does not vanish at zero momentum. If we substitute (1) in $K_3(x)$ there appears a term proportional to $A^{a'}_\alpha(x)A^{b(0)}_\beta A^{c(0)}_\gamma$ which gives a non-zero contribution to

$$\int d^4x\langle\partial_\mu K^\mu(x)\partial_\nu K^\nu(0)\rangle = \int d^4x\langle\partial_3 K_3(x),\partial_3 K_3(0)\rangle$$

because of (10). Recall here $\partial_3 A^{a'}_\mu=G^a_{3\mu}$. The above quantity gives a non-zero mass to η' meson as is well known.

Finally we discuss the condensation of $A^{a(0)}_\alpha$. The effective potential $V(A^{a(0)}_\alpha)$ of $A^{a(0)}_\alpha$ can be calculated in loop expansion. We know that V has a non-trivial minimum at one loop level. Our conclusion is that we can develop the series expansion of V in such a way that the position of the minimum found at one loop level is shifted slightly for small coupling if the effect of the higher order terms of the series are taken into account. These shifts are in accordance with the renormalization group equations.

Details are found in Ref.4 where we also discuss the whole subjects in this talk.

References

1 N.K. Nielsen and P. Olesen, Nucl. Phys. B144, 376 (1978).

2 M.B. Halpern, Phys. Rev. D19, (1979); Phys. Letters 81B, 245 (1979).

3 G. 't Hooft, Nucl. Phys. B138, 1 (1978).

4 R. Fukuda, "Gluon Condensation and the Field Strength Formulation in Quantum Chromodynamics" preprint RIFP-469 (to appear in Prog. Theor. Phys. August 1982).