GENERALIZED STRING AMPLITUDE AND WAVE EQUATION FOR HADRONS

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A gauge invariant hamiltonian formulation of hadron dynamics involving a generalized string amplitude is proposed. Resulting partial integro-differential wave equations reflect the shielding of the long range potential and determine a universal logarithmic derivative of the hadronic wave functions. Because of this boundary condition, the new wave equation allows confined solutions without an explicit confinements potential in it.

Instead of the conventional string amplitude $\exp\{ig\int \vec{\underset{\sim}{A}}\cdot d\vec{x}\}$ used in gauge invariant formulations of hadronic systems, I propose to study a modified string operator on a t plane

$$U(2,1) = \exp\{g\int_1^2 [i\vec{\underset{\sim}{A}}(x) + \lambda\vec{\underset{\sim}{E}}(x)]\cdot d\vec{x}\}\,, \tag{1}$$

($\underset{\sim}{A} = A^a\lambda^a/2$). In spite of its explicitly non-covariant definition, the operator is useful in taking into account an infinite number of soft gluons created on the string as well as the shielding of the long range force due to splittings of the string. Using (1), I construct a gauge invariant quark operator

$$q_{\alpha\beta}(1,2) = \mathrm{Tr}^c\,[\psi_\alpha(1)\,U^T(2,1)\,\psi_\beta{}^+(2)], \tag{2}$$

where α and β are Dirac indices, and flavor indices have been suppressed. Tr^c is the trace over color spin. Time development of the operator (2) can be derived in the same way as in my previous paper,[1] which was for the case $\lambda = 0$. Two new features arise because of the $\underset{\sim}{E}$ term inserted in (1). Time derivative of A(x) is equivalent to a c-number operator $\frac{1}{i}\frac{\partial}{\partial\lambda}$, since $\dot{A} = -E$. Time derivative of E is equal to insertion of the quark current $-j$, which, after use of a Fiertz identity, gives a product of two-string operators. In the following I consider q(1,2) at a large distance and neglect all the derivatives of the field operators (B, $\nabla \times E$ and $\nabla \times B$). Strictly speaking, transverse (to the string) momentum of gluons should be kept under the large distance approximation. However, it would give rise to the transverse oscillation of the string, which will be studied elsewhere. Thus

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$$i\,\dot{q}\,(1,2) = H_D\, q(1,2) + \frac{\partial}{\partial\lambda} q\,(1,2)$$

$$- \frac{i}{2} g^2\lambda \int_1^2 d\vec{x} \cdot \sum q^{(a)}(1,x)\vec{\alpha} \quad q^{(a)}(x,2)$$

$$+ \frac{i}{6} g^2\lambda \int_1^2 d\vec{x} \cdot \vec{j}_s(x) \quad q(1,2) \cdots \tag{3}$$

where

$$H_D\, q(1,2) = [-i\alpha \cdot \nabla_1 + \beta m]\, q - q\, [i\alpha \cdot \overleftarrow{\nabla}_2 + m] \,. \tag{4}$$

$\sum_a$ in the third term means a summation over all possible flavors. $\vec{j}_s$ is the color- and flavor-singlet quark current, and represents an emission of a singlet vector meson from the string. For vacuum-vacuum matrix element of (3), I may neglect this term, and also keep only the vacuum intermediate state neglecting all the hadronic ones, since the latter tend to give short range forces. Defining

$$S(1,2) = \langle 0 \mid q(1,2) | 0 \rangle , \tag{5}$$

I obtain an equation

$$H_D\, S(1,2) + \frac{\partial}{\partial\lambda} S(1,2) - \frac{i}{2} g^2\lambda \int_1^2 d\vec{x} \cdot S(1x)\vec{\alpha}\, S(x,2) = 0 \tag{6}$$

S(1,2) can be expanded in Dirac matrices like

$$S(1,2) = -i\ \alpha\cdot\hat{r}\ S_1(r) + \beta\ S_2(r) + i\beta\ \alpha\cdot\hat{r}\ S_3(r) \quad . \tag{7}$$

If S(1,2) were the equal-time limit of a quark propagator for which a spectral representation holds, then we would have $S_3=0$. Since no spectral representation holds for S(1,2) which has no color-singlet hadronic intermediate states we may include S_3 term. In fact as shown below S_3 term is necessary in order to obtain the spontaneous breaking of the chiral symmetry which says

$$< \bar{\psi}(0)\ \psi\ (0) >_0 = -S_2(0) \neq 0. \tag{8}$$

Introducting (7) into (6) I obtain

$$\frac{\partial S_1(r)}{\partial\lambda} - \frac{1}{2}\ g^2\lambda \int_0^r dz\ [S_1(r-z)S_1(z) + S_2(r-z)S_2(z) + S_3(r-z)S_3(z)]=0,$$

$$-2\ \left(\frac{\partial S_3(r)}{\partial r} + \frac{2}{r} S_3(r)\right) + \frac{\partial S_2(r)}{\partial\lambda} - g^2\lambda \int_0^r dz\ S_1(r-z)\ S_2(z)=0,$$

$$2\ \frac{\partial S_2(r)}{\partial r} + \frac{\partial S_3}{\partial\lambda} - g^2\lambda \int_0^r dz\ S_1(r-z)\ S_3(z) = 0\ . \tag{9}$$

If $S_3 = 0$, then the third equation tells that S_2 = const. and hence must vanish. The solution $S_2=S_3=0$ represents a chirally symmetric solution. The time reversal requires that the amplitudes S_1, S_2 and S_3 in (7) are all real, if the parameter λ is taken to be real. The solution of eq. (9) is not well defined without a precise knowledge of the boundary conditions in variable λ . Nevertheless we may obtain a physically reasonable solution in the following way. Neglecting quark kinetic energy terms in (9) I obtain an asymptotic solution for large r,

$$S_1(r) = A(\lambda)\ e^{-D(\lambda)r};\ A = -\frac{dD}{d\lambda}/g^2\lambda$$

$$S_2(r) = b_2 S_1\ ,\quad S_3(r) = b_3 S_1\ , \tag{10}$$

where b_2 and b_3 are constants independent of satisfying $b_2^2 + b_3^2 = 1$. Thus, I have

$$S_1'/S_1 = S_2'/S_2 = S_3'/S_3 = -D \tag{11}$$

To obtain a picture of the overall solution, I consider three regions in r. In the inner region I, the system is free (short range forces are being neglected), so that $S_2 = 1$ and $S_3 = 0$. In the outer region III, (10) gives the leading asymptotic terms. In the intermediate region II, I may extrapolate from outside and set

$$-\frac{\partial S_{2,3}}{\partial\lambda} = (W-Kr)\ S_{2,3} \tag{12}$$

where

$$W = -A^{-1}\frac{dA}{d\lambda}\ ,\qquad K = -\frac{dD}{d\lambda}\ .$$

The Kr term should cancel the integral terms in (9) in region III. However, in region II, the integral involves $S_{2,3}$ in region I, so that cancellation will not be complete. Thus, in region II, neglecting the integral terms, the second and third equations give the Breit-type equation with an effective eigenvalue W and a linear potential Kr. The equation must be solved with a boundary condition (11) imposed at a certain radius r = R inside region II. In this way the Klein paradox associated with a linear potential[2] is completely avoided. Details of the solution, as well as its relation to the pion wave function will be discussed elsewhere.

References

1. H. Suura, Phys. Rev. D20, 1412 (1979).
2. D. A. Geffen and H. Suura, Phys. Rev. D16, 3305 (1977).