ANALOGUE OF OPTICAL BISTABILITY IN DRIVEN JOSEPHSON JUNCTIONS

S.R. Shenoy
School of Physics, University of Hyderabad
Hyderabad - 500134, India

1. Introduction

When open systems are pushed away from equilibrium by an external drive, they settle down to steady states, with the external energy flux balanced by dissipations. Two competing types of dissipations can combine to produce nonlinearities and multiple stable roots in the steady state equations. Transitions between these states are called nonequilibrium or dissipative phase transitions [1,2]. The stationary solution of the Fokker-Planck equation serves to define a free energy-like potential. One can then talk about first-order and second-order transitions, just as in the equilibrium case.

The first example of a second-order dissipative phase transition was the single-mode laser [3], with the incoherent pump intensity being the drive, the coherent photon number being the order parameter, and the competing dissipations being the atomic decay rate and the cavity photon leakage rate. A much-studied recent example of a first-order dissipative phase transition is optical bistability [4]. Here the drive and order parameter are the incident and transmitted coherent radiation intensity, respectively. The competing atomic decay processes $\gamma_\perp$, $\gamma_\parallel$ and photon leakage K, produce an effective nonlinearity of the refractive index that yields two possible transparencies, for a given value of the incident intensity. The mathematical formalism involves two-level atoms or spins linearly coupled to photons.

In superconductivity, it is well known that certain combinations of fermion operators, acting on occupied/unoccupied pair states, can be regarded as pseudo-angular momentum operators acting on spin up/spin down states [5]. This pseudo-spin formalism has been applied to Josephson junctions, where the (total) raising or lowering operator $S^{\pm}$ transfers a pair from one superconductor to the other. The total z component operator S^z counts the number of excess pairs on one side.

Given the formal description of optical bistability in terms of two-level spins, it is natural to ask if an analogous radiation-induced dissipative transition can occur in a Josephson junction, with external photons coupling to the tunneling pairs.

It turns out that, within a simple model, the answer is 'yes', but with important differences that simplify the problem. These differences arise because the underlying algebra is that of fermi operators, and not actual angular momenta—certain commutators are manifestly negligible if physically appropriate normalizations are chosen. The problem, thus simplified, could be of pedagogic usefulness as an introduction to bistability and hysteresis [6,7].

The system is a Josephson tunnel junction, of capacitance C and with an external resistance R across it. The oxide layer forms an electromagnetic resonance cavity of lowest resonant frequency ω_c. The order parameter is a self-consistently developed d.c. voltage across the junction. (By the Josephson relation, this is also the ac current frequency, $\bar{\omega} = 2e\bar{V}/\hbar$). An external microwave source $\Omega \simeq \bar{\omega}$ is a drive, but it turns out that there could be other drives, that act additively.

The results have been published [6,7] and we will only outline them below, emphasizing some aspects not treated elsewhere.

In Section 2 we present the coupled pseudo spin and photon operator equations, and reduce the problem to one dimension by adiabatic elimination of fast modes. Various possible drives are considered. In Section 3 we examine the conditions for experimental observability of bistability and consistency of the assumptions made. Section 4 considers noise and dynamic correlation effects at the limit of metastability. Finally in Section 5, we place the effect in perspective with other d.c. jump and dissipation effects in Josephson junctions.

2. The Langevin Equations

The hamiltonian for the system is [6,8]

$$H = H_C + H_J + H_{cav} + H_{ext} , \tag{2.1}$$

where the capacitance or charging energy of the junction is

$$H_C = \frac{1}{2}\frac{(2eS^z)^2}{C} ; \tag{2.2}$$

the pair coupling hamiltonian that gives rise to the Josephson tunneling current is

$$H_J = -\frac{\hbar I_J}{4e}(S^+ + S^-) ; \tag{2.3}$$

the hamiltonian describing cavity photons and their interaction with the tunneling current is

$$H_{cav} = i\hbar T(\omega_c)(S^- - S^+)(a + a^+) + \hbar\omega_c a^+ a ; \tag{2.4}$$

and the coupling of the current to external radiation is

$$H_{ext} = i\hbar\int d\Omega' T(\Omega')(S^- - S^+)\, W(\Omega\Omega')\, N_{ext}^{1/2} \cos\Omega' t. \tag{2.5}$$

In the above equation, $T(\omega_c)$ is a coupling constant between the current and the electromagnetic field coming from the familiar $\int d^3x\, \vec{J}.\vec{A}$ interaction, In terms of junction parameters

$$T(\omega_c) = \frac{1}{4}\left(\frac{\pi}{\hbar\omega_c C}\right)^{1/2} I_J \tag{2.6}$$

where I_J is the maximal Josephson current in zero dc magnetic field. The operators a, a^+ are cavity photon descruction and creation operators and S^-, S^+ are fermion operator combinations that obey [6,8]

$$[S^z, S^{\pm}] = \pm S^{\pm} \tag{2.7a}$$

$$[S^+, S^-] \approx \frac{2}{m^2} S^z \tag{2.7b}$$

In this physically preferred normalization

$$\langle S^{\pm} \rangle = e^{i\,\theta(t)} , \tag{2.8}$$

where the angular brackets are steady state averages and $|\langle S^{\pm} \rangle| = 1$, with the current from (2.3) being $I_J \sin\theta$. In (2.7b) $m \gg 1$ is of order the system size. N_{ext} is the number of externally supplied photons in the cavity and is not treated as an operator. $W(\Omega, \Omega')$ is the external photon spectral distribution, centred on Ω, with width covering the hysteresis region. A dc magnetic field of wavenumber $\propto \Omega$ is applied, to provide current-photon coupling.

The nonequilibrium average $\langle S^z \rangle$ is related to the quantum phase difference $\theta(t)$ and voltage $V(t)$ across the junction, by the Josephson relation

$$\frac{2e}{\hbar} \frac{2e \langle S^z \rangle (t)}{C} = \frac{2e}{\hbar} V(t) = \dot{\theta}(t) \tag{2.9}$$

The other Langevin equations are

$$\dot{a} = -(i\omega_c + K)a - T(\omega_c)(S^+ - S^-) + f_a(t) \tag{2.10}$$

$$\dot{S}^z = -\frac{S^z}{RC} - \frac{I_J}{4ei}(S^+ - S^-) - T(\omega_c)(a + a^+)(S^+ + S^-)$$

$$- 2N_{ext}^{\frac{1}{2}} \int d\Omega' \, T(\Omega') \, W(\Omega, \Omega')(S^+ + S^-) \cos\Omega' t + f_z(t) \tag{2.11}$$

Dissipative terms representing the photon leakage $-Ka$, and capacitor discharge through the external resistance, $-S^z/RC$ have been added to the Heisenberg equations of motion. (In optical bistability language, $\gamma_\perp = 0$, $\gamma_\parallel = (RC)^{-1}$). The quality factor Q of the cavity is $Q = \omega_c / 2K$. Noise operator terms f_a, f_z represent photon shot noise and generalized Johnson noise in the resistor that includes zero point effects [9]. They obey

$$\langle f_a(t) \, f_a^+(o) \rangle = 2K \, \delta(t) \tag{2.12}$$

with others $\langle f_a f_a \rangle = 0 = \langle f_a^+ f_a^+ \rangle$ and

$$\langle f_z(t) \, f_z(o) \rangle = \frac{\hbar\omega_c}{4e^2 R} \, \mathrm{Coth} \, \frac{\hbar\omega_c}{2k_B T} \, \delta(t) \tag{2.13}$$

Now the following approximations are made: (i) replace operators by nonequilibrium averages; (ii) put $a(t) = \tilde{a}(t) \, e^{-i\theta(t)}$, where $\tilde{a}$ is slowly varying on a scale Ω^{-1}, and time average on this scale; (iii) assume

$$K \gg RC \tag{2.14}$$

so the photons are fast modes, and $\dot{\tilde{a}} \approx 0$.

Then, in terms of dimensionless scaled variables,

$$f = \omega/\omega_c = 2\,eV/\hbar\omega_c = (2e/\hbar\omega_c)(2e\langle S^z\rangle/C) \tag{2.15}$$

and, on a slow time scale $\gg \Omega^{-1} \approx \omega^{-1}$,

$$\dot{f}(t) = -\frac{1}{RC}\left\{ f + \frac{2Q\alpha}{[1+4Q^2(f-1)^2]} - \mu \right\} + F_f(t) \tag{2.16}$$

Here the I_J Sin θ term has averaged to zero. The random force obeys

$$\langle F_f(t)\, F_f(o)\rangle = \tau^{-1}\,\delta(t) \tag{2.17}$$

where, approximating f-dependence of by its f = 1 value,

$$\tau^{-1} = \frac{8e^2 Q\alpha}{\hbar\omega_c\, RC^2}\left(1+\frac{1}{2Q\alpha}\right) \tag{2.18}$$

The parameters that set the scale are $(\Omega \approx \omega_c)$

$$\alpha = \frac{8e^2}{\hbar}\left(\frac{T(\omega_c)}{\omega_c}\right)^2 R\ ; \quad N_c^{-\frac{1}{2}} \simeq \frac{8e^2}{\hbar}\,\frac{T(\Omega)}{\omega_c}\, W(\omega_c, \Omega) \tag{2.19}$$

The first term in (2.16) is the Joule loss term, the second, resonant term, comes from fast mode elimination. It says that photon escape loss is largest when the ac Josephson current is on resonance with the cavity frequency, $f = \omega/\omega_c = 1$. The last term, with opposite sign, is the total drive term

$$\mu = (N_{ext}/N_c)^{\frac{1}{2}} + 2eV_b/\hbar\omega_c \tag{2.20}$$

Motivated by the analogy with optical bistability we have considered [6] an external photon drive. Another possibility is as follows. From (2.9), the first term $\sim -V/RC$ says that, in the absence of all other terms, the voltage across the capacitance decays to zero. With a battery V_b in series with the resistance and (junction) capacitance, the charging up of the capacitance, in the absence of other terms would lead to $V \rightarrow V_b$ at long times. The decay term for the R.C. circuit is $-(V-V_b)/RC$. Hence (2.20) is obtained: the two drives are additive. From (2.5), an oscillatory voltage of maximum amplitude $\propto N_{ext}^{\frac{1}{2}}$, across the junction with a d.c. blocking capacitor, is another possible drive.

The main point to be kept in mind is that for bistable effects discussed below, the d.c. voltage across the junction is the order parameter, and must not be pinned down.

3. The conditions for Bistability

From (2.16) a stochastically equivalent Fokker-Planck equation can be written down. The stationary solution $P_o(f,\mu) = e^{-(2\tau/RC)U(f,\mu)}$ defines a potential $U(f,\mu)$ with extrema at $f = \bar{f}$ satisfying the deterministic version of (2.16),

$$\mu - \bar{f} = \frac{2Q\alpha}{1+4Q^2(\bar{f}-1)^2} . \quad (3.1)$$

Within a range of parameters defined by

$$\mu_{c2} < \mu < \mu_{c1} \quad (3.2a)$$

$$Q^2 \alpha \geqslant 2/3\sqrt{3} \quad (3.2b)$$

there are three solutions of (3.1), $\bar{f}_1 < \bar{f}_3 < \bar{f}_2$ with $\bar{f}_{1,2}$ corresponding to local minima of $U(f,\mu)$ and $\bar{f}_3$ being a maximum. This simple bistability condition should be compared with that of optical bistability [4] where the full angular momentum commutation relations enter. $\bar{f}(\mu)$ is an S-shaped curve with size and shape determined by the external resistance, for fixed junction parameters.

From (3.2b) it is at first sight easy to satisfy the bistability conditions: for given R, C, I_J, the oxide layer should be well-formed making a good cavity, with large Q. However, for $\alpha = \alpha_o + \delta$, $(Q^2\alpha_o = 2/3\sqrt{3})$ and $\delta \ll 1$ the spinodal points $(\mu_{c1,2}\ \bar{f}(\mu_{c1,2}))$ can be evaluated to leading order in δ. The result is $\mu_{c1} - \mu_{c2} \sim \delta^{3/2}/Q$ and $\bar{f}(\mu_{c2}) - \bar{f}(\mu_{c1}) \sim \delta^{\frac{1}{2}}/Q$. Thus 'good' cavities, with $Q \gg 1$ have a small bistability region. On the other hand, very poor cavities, $Q \sim 1$ have diffuse, overlapping cavity modes, washing out the hysteresis. The junction parameters have to be carefully chosen, for the phenomena to be seen.

For (2.16) to be valid, the fast-mode assumption of (2.14) should hold and that is most easily satisfied for small R. For the hysteresis curve to be well described by the extrema $\bar{f}(\mu)$ alone, and not be washed out by fluctuations the stationary probability must be sharply peaked,

$$2\,\mathcal{T}/RC = \frac{\hbar\omega_c C}{2e^2}\,(1+2Q\alpha) \gg 1 \quad (3.3)$$

This is most easily satisfied for large R. (Note that this condition arises from a Fokker-Planck description and would not arise from a study of steady states alone).

The compatibility of the sharp-peaking and fast-moding conditions depend on the physics of the problem. Using (2.19), (3.3) can be written as

$$1 > 2\ (e^2/C\ \hbar\omega_c)\ \left[(2Q(T(\omega_c)/\omega_c)^2 (e^2R/\hbar) + 1\right] \quad (3.4)$$

each bracket of which is less than or around unity, for a range of accessible physical parameters. The first bracket is the ratio of the charging energy to the photon energy; the second is the ratio of the $\vec{j}.\vec{A}$ field-current coupling to the photon energy, and the third is the ratio of the resistance to $\hbar/e^2$ that sets the scale for localization or insulator behaviour. Thus although (3.3), (2.14) seem to impose opposing constraints, there can still be enough freedom to satisfy both, with reasonable parameters [6].

An estimate of the relaxation and metastable decay rates for the system has also been made [6]. This is useful in gauging whether hysteresis will be easily achieved or not [10].

4. Dynamic Correlations

The deterministic relaxation rate within a given well, from linearizing (2.16) is

$$\frac{1}{T_1} = 1 - \frac{16Q^3 \alpha (\bar{f}-1)}{[1+4Q^2(\bar{f}-1)^2]} \tag{4.1}$$

$\frac{1}{T_1}$ vanishes like $\sim |\mu - \mu_{c1,2}|^{\frac{1}{2}}$, analogous to 'critical slowing down' but here at spinodal values $\mu_{c1,2}$ where the metastable well flattens and disappears. The relaxation rate T_1^{-1} appears in noise and dynamic correlations that can be experimentally measured.

Voltage fluctuations across the junction, from (2.16), (2.17) are given by

$$\langle (f(t)-\bar{f}) \; (f(o)-\bar{f}) \rangle = (T_1/2\,\mathcal{T}) \; e^{-t/T_1} \tag{4.2}$$

in a gaussian approximation. This yields a Lorentzian noise spectrum, $[(\omega T_1)^2+1]^{-1}$ that narrows and rises at the transitions at $\mu_{c1,2}$: the time scale of the fluctuations in voltage increases.

The Josephson radiation line shape is related to the current-current correlation and hence to

$$\langle S^+(t)S^-(o)\rangle = \langle e^{i(\theta(t)-\theta(o))}\rangle \approx e^{i\omega_c \bar{f} t} \; e^{-\frac{1}{2}\langle(\theta(t)-\theta(o))^2\rangle} \tag{4.3}$$

This yields a gaussian spectrum $\sim \exp[-(\omega - \omega_c\bar{f})^2/(\omega_c^2 T_1/\mathcal{T})]$ that broadens and falls at transition : the extent of the fluctuations in voltage, increases. From (4.3) the dispersion in ω is $\langle(\Delta\omega)^2\rangle \sim (T_1/T)$. As $\langle(\Delta\omega)^2\rangle \sim \langle(f-\bar{f})^2\rangle$ through the Josephson relation, this is consistent with the static limit of (4.2).

A numerical study of the dynamic correlations through eigenfunction expansions and continued fraction methods [11] has been done [12]. It confirms the validity of the approximations made.

Besides the (mean-field) exponent occurring in the relaxation time, other critical and spinodal exponents can be defined [7].

5. Comparison with other dc effects

The Josephson junction exhibits a family of dc effects [13] when suitably driven and it is useful to place our results, with a radiation drive, in perspective.

Three 'good cavity' effects rely on frequency matching and not on photon leakage, i.e. they exist in the $Q \rightarrow \infty$ limit. The Shapiro steps in the dc characteristic [14] occur when $\bar{\omega} = n\Omega$, n = 1,2,3. Fiske steps occur when $\bar{\omega} = n\,\omega_c$ a cavity resonance [15]. In both cases, a d.c. current source is applied to the junction. A d.c. voltage effect observed by Chen et al [16] involves irradiating an unbiassed junction but this is also a good cavity effect, best seen for $Q \longrightarrow \infty$. Moreover, it is small in our parameter regime [6] .

In another effect [17] a peak occurs in the tunneling current as a function of magnetic field for very poor cavities $Q \sim 1$ with no standing waves. The dc voltage across the junction is pinned down.

In contrast the effect we have considered [6] involves both cavity modes and dissipation, in an intermediate Q regime, with both $Q \longrightarrow 0$ and $Q \longrightarrow \infty$ killing it. The effect is radiation intensity driven, assuming a broad $\Omega \sim \omega$ spectrum and is not just a frequency matching effect. It crucially depends on a resonant dissipation, with the dc voltage free to change. It differs from other effects,has connections with optical bistability, and seems worth investigating, experimentally.

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