RELAXATIONAL DYNAMICS OF SPIN-GLASSES NEAR TRANSITION TEMPERATURE

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The dynamic behaviour of spin-glasses is of interest from several points of view. (i) At low temperatures, the relaxation of magnetisation and other associated properties show very slow, non-exponential behaviour. (ii) In order to explain the low temperature thermodynamic properties, viz., magnetic specific heat, one has to study the spectrum of low energy excitations. (iii) On higher temperature side, one is interested in understanding how the spin-motion freezes to give the spin-glass phase as the temperature is lowered. It is with the last aspect that we shall be concerned with in this lecture.

The seminal paper which has given us first understanding of how spin-motion freezes to cause a cooperative phase transition at a well defined temperature is due to Edwards and Anderson [1]. Since this work, a vast amount of papers have been written on the subject, and it is now well recognised that the question of phase transition in spin-glasses is a very complex one, involving very delicate competition between randomness and critical fluctuations. Due to glassy nature of freezing in spin-glasses, it is believed that the key to understanding the spin-glass state is via the dynamics. For the discussion of the current theoretical situation, we refer to some recent review articles [2-5]. In this lecture, we shall describe only the work of Edwards and Anderson (EA) in detail.

The starting point of the discussion is the Hamiltonian given by

$$H = \sum_{ij} J_{ij} \vec{S}_i \cdot \vec{S}_j \tag{1}$$

where J_{ij}'s are the exchange constants, the sum is over the pairs of sites and S_i are taken to be two dimentional unit vectors. J_{ij}'s are taken to be random quantities with zero average and a non-zero mean square average:

$$\langle J_{ij}^2 \rangle_R = \bar{J}_{ij}^2 \tag{2}$$

where $\langle \; \rangle_R$ denotes the averaging over the distribution of J_{ij}'s. Since the idea here is to merely illustrate concepts, we shall take J_{ij}'s to be non-zero only for nearest neighbours. Due to zero mean value of J_{ij}'s, unlike ordered magnetic phases, the quantity $\langle\langle \vec{S}_i \rangle_T \rangle_R$ (where $\langle \; \rangle_T$ denotes the averaging with respect to the thermal ensemble) remains zero at all temperatures. So EA introduced a rather novel way to describe the frozen state [6]. They considered the average $\langle\langle \vec{S}_i(t) \cdot \vec{S}_i(0) \rangle_T \rangle_R$, and argued that even though there is no magnetic long range order of any kind in the frozen state, the frozen state is different from, say the paramagnetic state, in the sense that the spins retain infinitely a memory of their directions. Mathematically

$$\lim_{t\to\infty} \langle\langle \vec{S}_i(t) . \vec{S}_i(0) \rangle_T \rangle_R = q_i \neq o \quad \text{for} \quad T \leqslant T_g \tag{3}$$

In the mean field description, q_i is independent of the site index i and plays the role of order parameter for the second order phase transition. Thus the EA picture of the phase transition is : as the temperature decreases the spin-motion slows down and at a well defined temperature T_g, the various spins freeze in different directions depending upon their local environment in a somewhat collective manner.

Since $\vec{S}_i$'s are fixed length vectors, we rewrite (1) in terms of angles in the plane, and add a kinetic energy term for planar rotators

$$H = \sum_{ij} J_{ij} \operatorname{Cos}(\theta_i - \theta_j) + \frac{I}{2} \sum_i \dot{\theta}_i^2 \tag{4}$$

where I is the moment of inertia of the rotators. The equations of motion are

$$I \ddot{\theta}_i + \nu \dot{\theta}_i = - \sum_j J_{ij} \operatorname{Sin}(\theta_i - \theta_j) + f_i(t) \tag{5}$$

In writing down (5), we have allowed the spins to interact with a heat bath in the usual way. Here ν is the coefficient of viscosity which causes a drag on the motion of the rotators. The force $f_i(t)$ is a random one which helps maintain the system in equilibrium with the heat bath at temperature T. As is customary, $f_i(t)$ is taken to be a Gaussian, delta correlated random force satisfying the relation

$$\langle f_i(t) f_j(t') \rangle = \nu kT \, \delta_{ij} \, \delta(t-t') \tag{6}$$

In the long time limit, i.e., $t \gg \nu^{-1}$, the inertial effects on the velocity are washed out, and it is sufficient to consider the equations

$$\nu \dot{\theta}_i = - \sum_j J_{ij} \operatorname{Sin}(\theta_i - \theta_j) + f_i(t) \tag{7}$$

One can also look at the corresponding Fokker-Planck equation which reads

$$\frac{\partial}{\partial t} P(\{\theta_i\}, t) = \frac{kT}{\nu} \sum_i \left\{ \frac{1}{kT} \frac{\partial}{\partial \theta_i} \left[-\sum_j J_{ij} \operatorname{Sin}(\theta_i - \theta_j) P \right] + \frac{\partial^2 P}{\partial \theta^2} \right\} \tag{8}$$

It is easy to see that the stationary solution of this equation is the canonical distribution is

$$P_o = N \exp\left[- \frac{1}{kT} \sum_{ij} J_{ij} \operatorname{Cos}(\theta_i - \theta_j) \right] . \tag{9}$$

In order to understand the physics implied by (7), let us first consider the behaviour, when there are no interactions

$$\nu \dot{\theta}(t) = f(t) \tag{10}$$

$$\theta(t) - \theta(o) = \frac{1}{\nu} \int_o^t f(t') \, dt' \tag{11}$$

$$\langle (\theta(t)-\theta(o))^2 \rangle = \frac{2kT}{\nu} t \tag{12}$$

$$\langle \text{Cos}(\theta(t)-\theta(o)) \rangle = \text{Re}[\exp i(\theta(t)-\theta(o))] \tag{13}$$

To evaluate (13), we use cumulant expansion and note that (6) and (11) imply that $(\theta(o)-\theta(o))$ is a Gaussian variable, with zero average. Thus

$$\langle \text{Cos}(\theta(t)-\theta(o)) \rangle = \langle \vec{S}_i(t).\vec{S}_i(o) \rangle = \exp[-\frac{kT}{\nu} t] \tag{14}$$

Now let us see the role of interactions. The latter restrict the motion of each spin, so there must be some slowing down in general. The actual behaviour is indeed quite complex, but as a first level description EA proposed that the autocorrelation function $\langle\langle \vec{S}_i(t).\vec{S}_i(o) \rangle\rangle$ has the same form as (14) but with a renormalised coefficient of viscosity η in place of ν. Thus the entire complexity introduced by interactions is tackled by calculating selfconsistently a renormalised coefficient of viscosity, which now becomes a temperature dependent object.

The motion of a given spin θ_i depends upon its nearest neighbours θ_j's, whose motion in turn depend upon θ_i. This back reaction of the medium i.e. the other spins, is the principle factor which renormalises the self viscosity η. To isolate this, we write the equation of motion of one of the neighbours of θ_j in the following way

$$\nu\dot{\theta}_j + J_{ij} \text{ Sin}(\theta_j-\theta_i) = f_j - \sum_{k(\neq i)} J_{jk} \text{ Sin}(\theta_j-\theta_k) \tag{15}$$

Now the renormalised quantities are introduced through the definitions

$$\eta\dot{\theta}_j + J_{ij} \text{ Sin}(\theta_j-\theta_i) = F_j \tag{16}$$

In (16) the interactions on θ_j, apart from that due to θ_i have been lumped into the new coefficient of viscosity η, and a new fluctuating force F. In this way the major interdependence between θ_i and θ_j is identified and rest of the terms are regarded as part of the heat bath. Equation (16) is now integrated to give

$$\theta_j(t)-\theta_j(-\infty) = \frac{1}{\eta}\left[\int_{-\infty}^{t} F_j(\tau)d\tau - J_{ij}\int_{-\infty}^{t} \text{Sin}[\theta_j(\tau)-\theta_i(\tau)]\, d\tau\right]$$

$$\theta_j(t) = \tilde{\theta}_j(t) - \frac{J_{ij}}{\eta}\int_{-\infty}^{t} \text{Sin}(\theta_j(\tau) -\theta_i(\tau))d\tau \tag{17}$$

Note that $\tilde{\theta}_j(t)$ can be taken to be independent of θ_i. Putting this solution back into (16) gives

$$\nu\dot{\theta}_i + \sum_j J_{ij} \text{ Sin}[\theta_i-\tilde{\theta}_j + \frac{J_{ij}}{\eta}\int_{-\infty}^{t}\text{Sin}(\theta_j(\tau) -\theta_i(\tau))d\tau] = f_i \tag{18}$$

The second term in (17), being the contribution of a single spin, can be treated as small. Using this fact, (18) can be written as

$$\nu\dot{\theta}_i + \sum_j J_{ij}\text{Sin}(\theta_i-\tilde{\theta}_j) + \frac{1}{\eta}\sum_j J_{ij}^2\text{Cos}(\theta_i-\tilde{\theta}_j)\int_{-\infty}^{t}\text{Sin}(\theta_j(\tau) -\theta_j(\tau))d\tau = f_i \tag{19}$$

The second term in the left of (19) is now brought together with f_i to define a new fluctuating force

$$F_i = f_i - \sum_j J_{ij} \sin(\theta_i - \tilde{\theta}_j) \tag{20}$$

and the third term is manipulated in the following manner

$$2 \cos(\theta_i - \tilde{\theta}_j)\sin(\theta_j(\tau) - \theta_i(\tau)) = \sin[\theta_i(t) - \theta_i(\tau) - \tilde{\theta}_j(t) + \theta_j(\tau)]$$

$$+\sin[\theta_i(t) + \theta_i(\tau) - \tilde{\theta}_j(t) - \theta_j(\tau)] \tag{21}$$

The second of these terms can be ignored as being small when averages with respect to J_{ij}'s are taken, and the first term can again be decomposed to write

$$\sin[\theta_i(t) - \theta_i(\tau) - \tilde{\theta}_j(t) + \theta_j(\tau)] = \sin(\theta_i(t) - \theta_i(\tau))\cos(\tilde{\theta}_j(t) - \theta_j(\tau))$$

$$+\cos(\theta_i(t) - \theta_i(\tau))\sin(\tilde{\theta}_j(t) - \theta_j(\tau)) \tag{22}$$

Now, following the mean field philosophy,we replace the j-dependent terms by the averaged quantities

$$\langle \cos(\tilde{\theta}_j(t) - \theta_j(\tau)) \rangle = q(t-\tau) \tag{23}$$

$$\langle \sin(\tilde{\theta}_j(t) - \theta_j(\tau)) \rangle \approx 0 \tag{24}$$

Substituting the result of these manipulations in (19), one obtains

$$\nu \dot{\theta}_i + \frac{1}{2\eta} \sum_j \bar{J}^2 \int_{-\infty}^{t} q(t-\tau) \sin(\theta_i(t) - \theta_i(\tau))\, d\tau = F_i(t) \tag{25}$$

The self-consistency is now imposed by requiring that the final equation (25) is linear in θ_i and $F_i(t)$ satisfies the fluctuation-dissipation relation

$$\langle F_i(t)\, F_i(t') \rangle = \eta kT\, \delta(t-t') \tag{26}$$

Thus

$$\nu \dot{\theta}_i + \frac{J_o^2}{2\eta} \int_{-\infty}^{t} q(t-\tau)\, (\theta_i(t) - \theta_i(\tau))\, d\tau = F_i(t) \tag{27}$$

where $J_o^2 = z\,\bar{J}^2$. Further, one can approximate

$$\theta_i(t) - \theta_i(\tau) = (t-\tau)\, \dot{\theta}_i + \ldots \tag{28}$$

and write for $q(t-\tau)$ + exp $[-\frac{kT}{\eta}(t-\tau)]$, to get

$$[\nu + \frac{J_o^2}{2\eta} \int_{-\infty}^{t} (t-\tau)\, e^{-(kT/\eta)(t-\tau)}\, d\tau]\dot{\theta}_i = F_i(t). \tag{29}$$

Use of (26) finally yields the following self-consistent equation for η

$$\eta = \nu + \frac{J_o^2}{2\eta} \int_0^\infty t' \, e^{-kT/\eta t'} \, dt' \qquad (30)$$

$$\eta = \nu \left[1 - \frac{1}{2} (J_o/kT)^2 \right]^{-1} \qquad (31)$$

The consistency of this approximation can be further checked by using (20) to evaluate (26). Equation (31) shows that at as $T \to T_g = \frac{J_o}{\sqrt{2}k}$, $\eta \to \infty$, which means that at this temperature the spin motion slows down to the extent of having a strong memory even for infinite time.

As mentioned in the beginning, EA picture outlines above has come under much scrutiny in recent work, but its idea remains essential for an understanding of spin-glass physics.

References

1. S.F. Edwards and P.W. Anderson, J. Phys. F6, 1927 (1976)
2. P.W. Anderson in "Ill-condensed Matter" p.162, Eds. R. Balian, R. Maynard and G. Toulouse (North-Holland Publishing Co., 1979)
3. A. Blandin, J.de.Physique 39, C6-1499 (1978)
4. A.P. Murani, J. Mag. Materials 5, 95 (1977)
5. D. Kumar in "Current Trends in Magnetism" p.388 Ed. N.S. Satyamurthy and L. Madhav Rao (Published by Indian Physics Association, 1981)
6. S.F. Edwards and P.W. Anderson, J. Phys. F5, 965 (1975)