RECENT DEVELOPMENTS IN LATTICE GAUGE THEORIES

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I. Introduction

Since 1980 most of the work on QCD has involved Monte Carlo simulations of lattice gauge theories. In this report I will review these works, emphasizing the most recent and/or the most accurate ones. The basic concepts of QCD, as well as a description of prior developments can be found (with all modesty) in my review [1]. This talk will be divided into three parts.

(i) Results from theories with no dynamical quarks. These include a determination of the string tension and the spectra of quarkless mesons (glueballs).

(ii) Inclusion of quarks in the quenched approximation. The topics discussed include chiral symmetry breaking, hadronic spectra and static properties of hadrons.

(iii) Caveats. Some of the studies on the validity of the quenched approximation and the problems brought on by the practical restrictions on lattice sizes are presented

Of course, time limiations prevent a complete, self contained discussion ; this report should be viewed as a synopsis of results and as a guide to the literature. This discussion will be restricted to color SU(3) and zero temperature.

Before proceeding we shall established some notation. The continuum QCD action is

$$S = \int d^4x \left\{ \frac{1}{2g^2} \text{Tr } F_{\mu\nu}F_{\mu\nu} + \sum_f \bar{q}^{(f)} (i\not{D} - m^{(f)}) q^{(f)} \right\} \qquad (I.1)$$

with

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + [A_\mu, A_\nu] \qquad (I.2)$$

$$A_\mu = A_\mu^\alpha \frac{\lambda^\alpha}{2} \quad (\lambda^\alpha \text{ are the SU(3) generators}).$$

$q^{(f)}$ is the quark field of flavour f and the covariant derivative D_μ is

$$D_\mu = \partial_\mu - iA_\mu \; .$$

The coupling constant g depends implicitely on a cut-off Λ (or renormalization point μ). This dependence is obtained from the renormalization group equation

$$- \Lambda \frac{\partial g}{\partial \Lambda} = \beta_o g^3 + \beta_1 g^5 + \ldots \qquad (I.3)$$

For SU(3)

$$\beta_o = \frac{11}{16\pi^2} \; ; \; \beta_1 = \frac{102}{(16\pi^2)^2} \qquad (I.4)$$

On the lattice, instead of the gauge potentials A_μ, we introduce link variables $U_{x,\varepsilon}$ defined on the link starting at point x and extending to $x+\varepsilon a$, with a the lattice parameter and ε a unit vector. For a pure gauge theory the lattice version of the action is

$$S = \beta \sum_P \mathrm{Tr} \prod_P U \quad ; \beta = 6/g^2 \tag{I.5}$$

The summation refers to all fundamental plaquettes of the lattice, and the product is over the U's defined on the four links bounding a plaquette.

Should this theory have a continuum limit, then renormalization group arguments indicate that for small g and a the following mass scale

$$\Lambda_o = \frac{1}{a} [\beta_o\, g^2(a)]^{-\beta_1/2\beta_o^2} \exp[-1/2\beta_o\, g^2(a)] \tag{I.6}$$

should be a independent. In fact, in the absence of other scales, such a quark masses, all dimensionful *physical* quantities should be expressible as pure numbers multiplied by appropriate powers of Λ_o. For example, we believe that the large distance quark-antiquark potential is linear

$$V(R) \sim K\, R. \tag{I.7}$$

The string tension K should, as we vary g, behave as

$$\sqrt{K} = A\, \Lambda_o \tag{I.8}$$

with A a number to be determined.

II. Theories without dynamical quarks

As QCD without matter fields is a non trivial interacting theory, we may study some of its consequences. These include a determination of the string tension K, (in fact, dynamical quarks would destroy such a linear term) and the spectrum of hadrons made exclusively out of gauge potential degrees of freedom ; these are commonly refered to as glueballs.

A. String tension

The string tension is most commonly determined by evaluating the expectation vlaue of

$$W[C] = \mathrm{Tr} \prod_{\ell \varepsilon C} U_\ell \tag{II.1}$$

where C is a large contour in the lattice. It A(C) is the minimal area of some surface bounded by C, we expect

$$W[C] \sim \exp[-KA(C)] \tag{II.2}$$

K is the same coefficient that appears in Eq.(I.7). The purpose of measuring K is threefold :

(i) We wish to check (I.6) and (I.8) thus determining whether the lattice theory approaches for small a what we believe the perturbative continuum theory to be.

(ii) If the above point is satisfied, we can use (I.6) and (I.8) together with the phenomenologically determined value of K

$$\sqrt{K} \sim 440 \text{ MeV} \tag{II.3}$$

to set the value of the lattice constant a for each value of the coupling constant g. In turn we will be able to use a to determine the values of other physical quantities, such as hadron masses.

(iii) The cut-off Λ_o can be related to a continuum cut-off which can in principle be determined from the phenomenology of high energy, high momentum transfer collisions.

A recent Monte Carlo measurement of the string tension [2] is presented in Fig.1

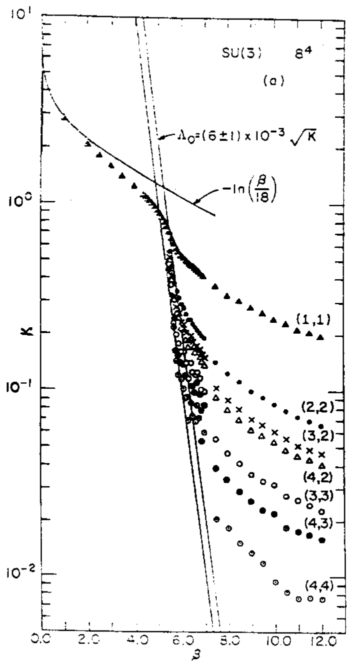

Fig.1 : String tension as a function of the coupling constant (From Ref. 2).

In this figure we are supposed to look at the envelope of curves for different size contours entering Eq.(II.1). To the extent this envelope is a straight line (for small g^2 or large β). With a slope determined by (I.6), to that extent is point (i) satisfied. The intercept of this line determines the constant A in Eq.(I.8). From Ref.2 we obtain A = 170 $\pm$ 30. A recent [3] evaluation of K on a large ($10^3\times20$) lattice, however restricted to one value of g^2 ($g^2=1$), gives a value of A about twice as large. Using (I.8) and the phenomenologically determined string tension we obtain :

$$\Lambda_o \sim \begin{array}{l} 2.6 \text{ MeV (Ref.2)} \\ 1.3 \text{ MeV (Ref.3)} \end{array} \qquad \text{(II.4)}$$

In order to compare these results to other ways of determining Λ we must relate this lattice cut-off Λ_o to a continuum one. This has been done [4] and we find $\Lambda_{MOM} = 83.5\ \Lambda_o$ yielding Λ_{MON} to lie between 100 MeV and 200 MeV in agreement with recent phenomenological determinations. The reason for the difference between the results of Ref.2 and Ref.3 may be due to the fact that the loops considered are not large enough. More comments will be made in the last section.

B. Gluonic Mesons

Hadronic masses may be obtained by evaluating the average of

$$\sum_{\vec{x}} \langle O_H(\vec{x},t)\ O_H(\vec{x},0)\rangle \sim e^{-m_H t} \qquad \text{(II.5)}$$

where O_H is an operator that creates the hadron of interest out of the vacuum. For example, for a gluonic meson with quantum numbers 0^{++} we could use $O_H \doteq \text{Tr } F^2$ while for 0^{-+}, $O_H = \text{TR } F\ \tilde{F}$ would do, etc. In Monte Carlo evaluations we use lattice analogous of these operators. The masses are given in terms of the inverse lattice cons tant and we must use (I.6) and (I.8) to obtain then in physical units. In Table 1 we present two sets of results for gluonic meson masses, based on $\Lambda_o = 2.4$ MeV (cf. Eq.(II.4))

TABLE 1

Glueball Masses (MeV)

	Ref.5	Ref.6
0^{++}	740 ± 40	730 ± 100
0^{-+}	1400 ± 200	
2^{++}	1600 ± 100	(1100 - 2200)
1^{-+}	1750 ± 200	

The wide variation for 2^{++} meson in the calculation of Ref.6 is a reflection of the fact that this mass depends on the value of the gauge coupling constant. *It has not yet reached the scaling limit.* It is interesting to note that the 0^{-+} and 2^{++} masses are in the region of recently discovered resonances [7]. Most interesting is the prediction for the 1^{-+} meson which is a state that cannot be made out of a quark and an antiquark.

III. Calculations with Dynamical Quarks

A quark field may be accomodated on the lattice by adding to the action the term

$$S_j = \sum_{\vec{r}} \bar{\psi}_{\vec{r}}\psi_{\vec{r}} - K \sum_{\hat{\varepsilon}} [\psi_{\vec{r}+\hat{\varepsilon}a} U_{\vec{r},\varepsilon} (r+\gamma_\mu) \psi_{\vec{r}} + h.c.] \qquad \text{(III.1)}$$

$$= \sum_{\vec{r},\vec{s}} \psi_{\vec{r}} Q_{rs} \psi_{\vec{s}}$$

The constant r plays no role in the continuum limit and is introduced, for technical reasons in order to eliminate a multiplication of fermionic degrees of freedom. The parameter K is related to the bare quark mass by

$$(r\,K)^{-1} = 8 + 2am_q^{(o)} \qquad \text{(III.2)}$$

It is refered to as the "hopping parameter".

In the case where r=1 the fermions are refered to as Wilson fermions ; where r=0 and the various spin components are placed on different lattice sites the fermions are refered to as Susskind fermions. In the latter case we still have some doubling of degrees of freedom but the chiral limit is easier to control.

As the quark fields are anticommuting variables we cannot perform a usual Monte Carlo calculation on them. As the fermion fields appear quadratically, a formal integration may be performed. The term that appears is a determinant of the operator Q. Most calculations performed up till now have set this determinant equal to one (quenched approximation). This corresponds to the neglect of closed fermion loops in a Feynman diagram expansion. Justification for this approach comes from the large N expansion of QCD where such loops do not contribute, and from the smallness of the ρ-ω mass difference, which is due to such loops. In the next section we shall discuss some further tests of this approximation.

A. Chiral Symmetry Breaking

We expect that QCD will spontaneously break chiral symmetry. This may be studied by examining the expectation value, $\langle\bar{\psi}\psi\rangle$ in the limit of vanishing quark mass. Figure 2 shows a recent evaluation (with Susskind fermions) of this quantity showing that for $\beta > 5.5$ we are in a scaling region

B. Hadron Spectrum

Analogous to the evaluation of the gluonic meson spectra we may evaluate the masses of hadrons containing quarks, both mesons and baryons. In addition to fixing the scale of the lattice spacing we have one more parameter to fix, namely K (cf. Eq.(III.1) and Eq.(II.2)) or equivalently the quark mass. Ideally the procedure one should follow (for Wilson fermions) is :

1. Fix β (presumably in the scaling region)

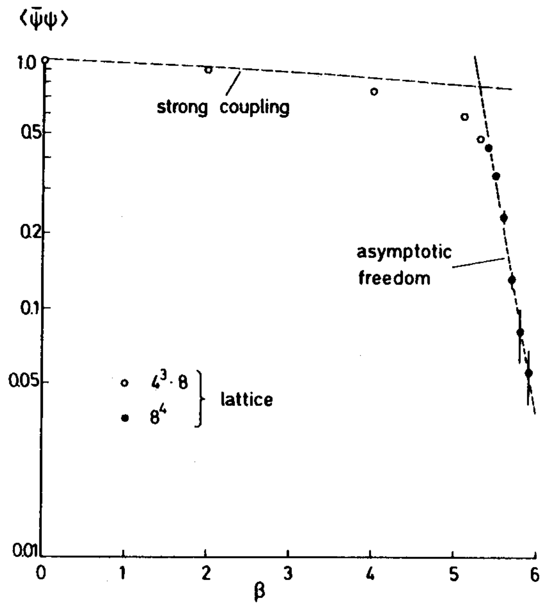

Fig.2 : Chiral symmetry parameter $<\bar{\psi}\psi>$.

2. Adjust K so that the pion is masless (chiral limit) and then tune it in order to obtain the correct pion mass
3. Measure other physical quantities
4. Repeat the above for a different value β in order to check scaling.

The second step is usually done by extrapolation from small values of K, while step four has not been performed extensively. Recent results are summarized in Table 2.

TABLE 2

Hadron Masses (MeV)

	Ref.9 (Susskind)	Ref.10 (Wilson)
ρ(770)	730 ± 90	820 ± 30
A_1(1270)	1190 ± 90	
N(940)	920 ± 100	1250 ± 150
Δ(1230)		1350 ± 200
f_π(130)	134	(100-200)
K(490)		500 ± 30
K^*(840)		930 + 40

C. Static Hadrons Properties

By placing a baryon in an external magnetic field one may measure its magnetic moment. A recent work gives [11]

$$g_p = 2.7 \pm 1.0 \qquad (2.79 \text{ exp})$$

$$g_n = -1.6 \pm 0.5 \qquad (-1.91 \text{ exp})$$

A quantity related to the decay rate of $\rho \to \pi\pi$ has also been determined [12]

$$g_{\rho\pi\pi} = 3.0 \pm 1.0 \qquad (6.1 \pm .1 \text{ exp})$$

IV. Caveats and Checks

The effects of two approximations must be checked. One concerns the finite size of the lattice and the other is the setting of the fermion determinant to one (quenched approximation). In addition to the effects of finite lattice size the scaling behavior of physical quantities, as measured on the lattice, should be verified by approaching the scaling limit as close as possible. Unfortunately these two effects work against each other. Going to smaller couplings, equivalently smaller lattice constants has the effect reducing the overall size of the lattice. Many calculations have been performed on lattices of the order of 10^4 at $\beta = 6/g^2 = 1$. At this coupling the lattice parameter is ~ 0.1 F. The hadrons contained therein are somewhat squeezed [13]. The effects of increasing the lattice to 16^4 and decreasing g^2 to 0.95 is discussed in Ref.14.

Improvements on the quenched approximation have been studied by two methods. The first consists of evaluating the fermion determinant by an expansion in the hopping parameter K (cf. Eq.III.1) [15]. Inclusion of fermion loops for one fermion flavor changes the physical value of the lattice spacing by 10%. In the second method, the dynamical fermions are included by considering an analogous dynamical boson problem (pseudofermion method) [16]. Fermion loops change the value of $\langle\bar{\psi}\psi\rangle$ by about 5% for each fermion species.

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